$$0=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{kz}{-a+k-z}-a-z \text{ for } a\in\mathbb{Z}\land z\in\mathbb{C}\land a\geq 0$$

$$1=-\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{2}$$

$$1=-\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-2k(-1+2k)}{1+4k}$$

$$1=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-2k(1+2k)}{3+4k}+3}$$

$$1=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-\frac{1+2k}{-1+2k}}{\frac{4k}{-1+2k}}+2}$$

$$1=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{k+z}{-1+k+z} \text{ for } z\in\mathbb{C}$$

$$1=\frac{a+z}{\mathrm{K}_{k=1}^{\infty}\frac{-a^2+(ak+z)^2}{a}+a} \text{ for } (a,z)\in\mathbb{C}^2\land -\frac{\pi}{2}<\arg(a)\leq\frac{\pi}{2}$$

$$2=-\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-k(1+k)^2(2+k)}{2\left(2+3k+k^2\right)}$$

$$2=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{6}{1}$$

$$\mathrm{Ai}(z)=\frac{1}{3^{2/3}\Gamma\left(\frac{2}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(-1+3k)}}{1+\frac{z^3}{3k(-1+3k)}}+1\right)}-\frac{z}{\sqrt[3]{3}\Gamma\left(\frac{1}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(1+3k)}}{1+\frac{z^3}{3k(1+3k)}}+1\right)} \text{ for } z\in\mathbb{C}$$

$$\mathrm{Ai}'(z)=\frac{z^2}{2\ 3^{2/3}\Gamma\left(\frac{2}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(2+3k)}}{1+\frac{z^3}{3k(2+3k)}}+1\right)}-\frac{1}{\sqrt[3]{3}\Gamma\left(\frac{1}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(-2+3k)}}{1+\frac{z^3}{3k(-2+3k)}}+1\right)} \text{ for } z\in\mathbb{C}$$

$$\mathrm{Bi}(z)=\frac{\sqrt[6]{3}z}{\Gamma\left(\frac{1}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(1+3k)}}{1+\frac{z^3}{3k(1+3k)}}+1\right)}+\frac{1}{\sqrt[6]{3}\Gamma\left(\frac{2}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(-1+3k)}}{1+\frac{z^3}{3k(-1+3k)}}+1\right)} \text{ for } z\in\mathbb{C}$$

$$\mathrm{Bi}'(z)=\frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(-2+3k)}}{1+\frac{z^3}{3k(-2+3k)}}+1\right)}+\frac{z^2}{2\sqrt[6]{3}\Gamma\left(\frac{2}{3}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^3}{3k(2+3k)}}{1+\frac{z^3}{3k(2+3k)}}+1\right)} \text{ for } z\in\mathbb{C}$$

$$\frac{(b+\beta)(d+e-\epsilon)}{\frac{2dU\left(\frac{d(d+2e)b^2+2d(d+2e)\beta b+d^2\beta^2+2de\beta^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{4d^2(b+\beta)^2},\frac{2b^2d^2+2\beta^2d^2+4b\beta d^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{2d^2(b+\beta)^2},\frac{b^2-\beta^2}{2d}\right)}{U\left(\frac{d(5d+2e)b^2+2d(5d+2e)\beta b+5d^2\beta^2+2de\beta^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{4d^2(b+\beta)^2},\frac{2b^2d^2+2\beta^2d^2+4b\beta d^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{2d^2(b+\beta)^2},\frac{b^2-\beta^2}{2d}\right)}-d-e+\epsilon}=\mathop{\mathrm{K}}_{k=1}^{\infty}$$

$$\frac{\beta(d+e-\epsilon)}{\frac{2dU\left(\frac{d^2\beta^2+2de\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2},\frac{2d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{2d^2\beta^2},-\frac{\beta^2}{2d}\right)}{U\left(\frac{5d^2\beta^2+2de\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2},\frac{2d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{2d^2\beta^2},-\frac{\beta^2}{2d}\right)}-d-e+\epsilon}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+dk+(-1)^k\epsilon}{(-1)^k\beta}\text{ for }(\beta,d,e,\epsilon)\in\mathbb{C}^4$$

$$\frac{(b+\beta)(d-\epsilon)U\left(\frac{5b^2d^2+5\beta^2d^2+10b\beta d^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{4d^2(b+\beta)^2},\frac{2b^2d^2+2\beta^2d^2+\cdots}{\cdots}\right.}{2dU\left(\frac{b^2d^2+\beta^2d^2+2b\beta d^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{4d^2(b+\beta)^2},\frac{2b^2d^2+2\beta^2d^2+4b\beta d^2+\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{2d^2(b+\beta)^2},\frac{b^2-\beta^2}{2d}\right)+(\epsilon-d)U\left(\frac{5b^2d^2+5\beta^2 d\cdots}{\cdots}\right.}$$

$$\frac{\beta(d-\epsilon)U\left(\frac{5d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2},\frac{2d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{2d^2\beta^2},-\frac{\beta^2}{2d}\right)}{2dU\left(\frac{d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2},\frac{2d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{2d^2\beta^2},-\frac{\beta^2}{2d}\right)+(\epsilon-d)U\left(\frac{5d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2},\frac{2d^2\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{2d^2\beta^2},-\cdots\right.}$$

$$\frac{(b+\beta)(e-\epsilon)}{b^2-\beta^2+\frac{2\epsilon^2-2e^2}{(b^2-\beta^2+2e)\left(\sqrt{\frac{b^4+\beta^4+b^2(4e-2\beta^2)-4\beta^2e+4\epsilon^2}{(b^2-\beta^2+2e)^2}}+1\right)}+e+\epsilon}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{b+(-1)^k\beta}\text{ for }(b,\beta,e,\epsilon)\in\mathbb{C}^4$$

$$\frac{\beta(e-\epsilon)}{-\beta^2+\frac{2(e^2-\epsilon^2)}{(\beta^2-2e)\left(\sqrt{\frac{\beta^4-4\beta^2e+4\epsilon^2}{(\beta^2-2e)^2}}+1\right)}+e+\epsilon}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{(-1)^k\beta}\text{ for }(\beta,e,\epsilon)\in\mathbb{C}^3$$

$$-\frac{\epsilon(b+\beta)^2}{(b+\beta)\left(b^2-\beta^2+\epsilon\right)+\frac{2\epsilon^2}{(b-\beta)\left(\sqrt{\frac{b^4+\beta^4-2\beta^2b^2+4\epsilon^2}{(b^2-\beta^2)^2}}+1\right)}}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1)^k\epsilon}{b+(-1)^k\beta}\text{ for }(b,\beta,\epsilon)\in\mathbb{C}^3$$

$$-\frac{\beta^2\epsilon}{-\frac{2\epsilon^2}{\beta\left(\sqrt{\frac{4\epsilon^2}{\beta^4}+1}+1\right)}-\beta^3+\beta\epsilon}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1)^k\epsilon}{(-1)^k\beta}\text{ for }(\beta,\epsilon)\in\mathbb{C}^2$$

$$\frac{b(d+e-\epsilon)}{\frac{2dU\left(\frac{d(d+2e)b^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{2b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{2b^2d^2},\frac{b^2}{2d}\right)}{U\left(\frac{d(5d+2e)b^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{2b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{2b^2d^2},\frac{b^2}{2d}\right)}-d-e+\epsilon}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+dk+(-1)^k\epsilon}{b}\text{ for }(b,d,e,\epsilon)\in\mathbb{C}^4$$

$$\frac{b(d-\epsilon)U\left(\frac{5b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{2b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{2b^2d^2},\frac{b^2}{2d}\right)}{2dU\left(\frac{b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{2b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{2b^2d^2},\frac{b^2}{2d}\right)+(\epsilon-d)U\left(\frac{5b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{2b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{2b^2d^2},\frac{b^2}{2d}\right)} =$$

$$\frac{b(e-\epsilon)}{b^2-\frac{2(e^2-\epsilon^2)}{(b^2+2e)\left(\sqrt{\frac{b^4+4b^2e+4\epsilon^2}{(b^2+2e)^2}}+1\right)}+e+\epsilon}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e+(-1)^k\epsilon}{b}\text{ for }(b,e,\epsilon)\in\mathbb{C}^3$$

$$-\frac{b^2\epsilon}{\frac{2\epsilon^2}{b\left(\sqrt{\frac{4\epsilon^2}{b^4}+1}+1\right)}+b^3+b\epsilon}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1)^k\epsilon}{b}\text{ for }(b,\epsilon)\in\mathbb{C}^2$$

$$-\frac{}{2b^3+4ab^2+2\beta b^2-2\beta^2b+6db+4eb+8a\beta b-2\beta^3+4a\beta^2+6d\beta+4e\beta-2(b+\beta)\left(b^2-\beta^2+2d+e+2\right.}$$

$$\frac{-b^3+\beta b^2+\beta^2b-db-2eb-\beta^3+d\beta+2e\beta+(b-\beta)(d+e-\epsilon)+\frac{(b-\beta)^2\left((d+2e)(b-\beta)\left(\sqrt{\frac{(ab+d-a\beta)^2}{a(b-\beta)(ab+2d-a\beta)}}-1\right.\right.}{}}{}$$

$$-\frac{}{\frac{\beta\left(a^2(-\beta)\left(d\left(3\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)+2e\left(\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)\right)-ad\left(d\left(6\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)+4e\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-\beta^2-2e\right)+\beta d^2\right)\,{}_2F_1\left(\frac{d^2\beta\ldots}{}\right.}{d(a\beta+d)\,{}_2F_1\left(\frac{5d^2\beta^2+2de\beta^2-\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2},\frac{5d^2\beta^2+2de\beta^2+\sqrt{d^2\beta^4(d-2\epsilon)^2}}{4d^2\beta^2};\frac{7d^2\ldots}{}\right.}}$$

$$-\beta\left(a^2\beta(d+2e)\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-1\right)+ad\left(-2d\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-4e\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}+\beta^2+d+2e\right)-\beta d^2\right)\,{}_2F_1\left(-\frac{d^2\beta^2-2de\beta^2+\sqrt{d^2\beta^4(d+2\epsilon)^2}}{4d^2\beta^2},\ldots\right.$$

$$\overline{d(d-a\beta)\,{}_2F_1\left(\frac{3d^2\beta^2+2de\beta^2-\sqrt{d^2\beta^4(d+2\epsilon)^2}}{4d^2\beta^2},\frac{3d^2\beta^2+2de\beta^2+\sqrt{d^2\beta^4(d+2\epsilon)^2}}{4d^2\beta^2};\frac{5d^2+\left(2e+\beta\left(a\left(\sqrt{\ldots}\right.\right.\right.}{\ldots}\right.}$$

$$b(d\ldots$$

$$b\left(a^2b\left(2e\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+d\left(3\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)\right)+ad\left(4e\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}+6d\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}+b^2-d-2e\right)+bd^2\right)\,{}_2F_1\left(\frac{b^2d(d+2e)-\sqrt{b^4d^2\ldots}}{4b^2d^2}\right.$$

$$\overline{d(ab+d)\,{}_2F_1\left(\frac{b^2d(5d+2e)-\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{d(5d+2e)b^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2};\frac{d\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}\,b^2\ldots\right.}{\ldots}\right.}$$

$$-\frac{b\left(a^2b(d+2e)\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+ad\left((d+2e)\left(2\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+b^2\right)+bd^2\right)\,{}_2F_1\left(\frac{\sqrt{b^4d^2(d+2\epsilon)^2}-b^2d(d-2e)}{4b^2d^2},-\frac{d(d-2e)b^2+\sqrt{b^4d^2\ldots}}{4b^2d^2}\right.}{d(ab+d)\,{}_2F_1\left(\frac{b^2d(3d+2e)-\sqrt{b^4d^2(d+2\epsilon)^2}}{4b^2d^2},\frac{d(3d+2e)b^2+\sqrt{b^4d^2(d+2\epsilon)^2}}{4b^2d^2};\frac{d\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}\,b^2+5d+2e\right)-ab\left(d\left(\sqrt{\ldots}\right.\right.}{4d(ab+\ldots}\right.}$$

$$-\frac{\ldots}{b\ldots}$$

$$-\frac{\ldots}{\ldots}$$

$$-\frac{2(b+\beta)^2\left(a^2(b+\beta)\left(3\sqrt{\frac{(a(b+\beta)+d)^2}{a(b+\beta)(a(b+\beta)+2d)}}-1\right)+a\left(6d\sqrt{\frac{(a(b+\beta)+d)^2}{a(b+\beta)(a(b+\beta)+2d)}}+b^2-\beta^2-d\right)+d(b-\beta)\right)\,{}_2F_1\left(\frac{b^2d^2+\beta^2d^2+2b\beta d^2-\sqrt{d^2(\ldots}}{4d^2(b+\beta)^2}\right.}{(a(b+\beta)+d)\,{}_2F_1\left(\frac{5b^2d^2+5\beta^2d^2+10b\beta d^2-\sqrt{d^2(b+\beta)^4(d-2\epsilon)^2}}{4d^2(b+\beta)^2},\frac{5b^2d^2+5\beta^2d^2+10b\beta d^2+\sqrt{d^2(b+\ldots}}{4d^2(b+\beta)^2}\right.}$$

$$\frac{(b-\beta)^2\left(a^2(b-\beta)\left(\sqrt{\frac{(ab-a\beta+d)^2}{a(b-\beta)(ab-a\beta+2d)}}-1\right)+a\left(d\left(2\sqrt{\frac{(ab-a\beta+d)^2}{a(b-\beta)(ab-a\beta+2d)}}-1\right)+b^2-\beta^2\right)+d(b+\beta)\right)\,{}_2F_1\left(\frac{-b^2d^2-\beta^2d^2+2b\beta d^2+\sqrt{d^2(b-\ldots}}{4d^2(b-\beta)^2}\right.}{(a(b-\beta)+d)\,{}_2F_1\left(-\frac{-3b^2d^2-3\beta^2d^2+6b\beta d^2+\sqrt{d^2(b-\beta)^4(d+2\epsilon)^2}}{4d^2(b-\beta)^2},\frac{3b^2d^2+3\beta^2d^2-6b\beta d^2+\sqrt{d^2(b-\beta)\ldots}}{4d^2(b-\beta)^2}\right.}$$

$$\frac{\beta(d-e)}{-\dfrac{\beta\left(a^2(-\beta)\left(3\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)+a\left(-6d\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}+\beta^2+d\right)+\beta d\right)\,{}_2F_1\left(\frac{d^2\beta^2-\sqrt{d^2(d-2e)^2\beta^4}}{4d^2\beta^2},\frac{d^2\beta^2+\sqrt{d^2(d-2e)^2\beta^4}}{4d^2\beta^2};\frac{3d-\beta\left(\sqrt{\frac{(d+a}{a\beta(2d}}\right.}{}\right.}{(a\beta+d)\,{}_2F_1\left(-\frac{\sqrt{d^2(d-2e)^2\beta^4}-5d^2\beta^2}{4d^2\beta^2},\frac{5d^2\beta^2+\sqrt{d^2(d-2e)^2\beta^4}}{4d^2\beta^2};\frac{7d-\beta\left(\sqrt{\frac{(d+a\beta)^2}{a\beta(2d+a\beta)}}\beta+a\left(\sqrt{\frac{(d+a\beta)^2}{a\beta(2d+a\beta)}}-7\right)\right)}{4(d+a\beta)};\right.}}$$

$$-\frac{\beta\left(a^2\beta\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-1\right)+a\left(-2d\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}+\beta^2+d\right)-\beta d\right)\,{}_2F_1\left(\frac{\sqrt{d^2(d+2e)^2\beta^4}-d^2\beta^2}{4d^2\beta^2},-\frac{d^2\beta^2+\sqrt{d^2(d+2e)^2\beta^4}}{4d^2\beta^2};\frac{d-\beta\left(-\sqrt{\frac{(d-a\beta)}{a\beta(a\beta-2}}\right.}{4}\right.}{(d-a\beta)\,{}_2F_1\left(-\frac{\sqrt{d^2(d+2e)^2\beta^4}-3d^2\beta^2}{4d^2\beta^2},\frac{3d^2\beta^2+\sqrt{d^2(d+2e)^2\beta^4}}{4d^2\beta^2};\frac{5d+\beta\left(a\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-5\right)-\beta\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}\right)}{4(d-a\beta)};\frac{1}{2}\left(\right.\right.}$$

$$\beta$$

$$\frac{b(d-\epsilon)}{\dfrac{b\left(a^2b\left(3\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+a\left(d\left(6\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+b^2\right)+bd\right)\,{}_2F_1\left(\frac{b^2d^2-\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2},\frac{b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2};\frac{\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2-a\left(\sqrt{\frac{(a}{ab(}}\right.}{4(ab+d)}\right.}{(ab+d)\,{}_2F_1\left(-\frac{\sqrt{b^4d^2(d-2\epsilon)^2}-5b^2d^2}{4b^2d^2},\frac{5b^2d^2+\sqrt{b^4d^2(d-2\epsilon)^2}}{4b^2d^2};\frac{\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2-a\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-7\right)b+7d}{4(ab+d)};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{(}{ab}}\right.}}$$

$$-\frac{b\left(a^2b\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+a\left(d\left(2\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+b^2\right)+bd\right)\,{}_2F_1\left(\frac{\sqrt{b^4d^2(d+2\epsilon)^2}-b^2d^2}{4b^2d^2},-\frac{b^2d^2+\sqrt{b^4d^2(d+2\epsilon)^2}}{4b^2d^2};\frac{\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2+ab}{4ab}\right.}{(ab+d)\,{}_2F_1\left(-\frac{\sqrt{b^4d^2(d+2\epsilon)^2}-3b^2d^2}{4b^2d^2},\frac{3b^2d^2+\sqrt{b^4d^2(d+2\epsilon)^2}}{4b^2d^2};\frac{\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2-a\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-5\right)b+5d}{4(ab+d)};\frac{1}{2}-\frac{1}{2}\sqrt{\right.}}$$

$$b$$

$$\frac{(b+\beta)^2(e-\epsilon)}{\dfrac{\sqrt{-(b+\beta)^2(e^2-\epsilon^2)}\,I_{\frac{b^2-\beta^2+2e-2a(b+\beta)}{4a(b+\beta)}}\left(\frac{\sqrt{-(b+\beta)^2(e^2-\epsilon^2)}}{2a(b+\beta)^2}\right)}{I_{\frac{b^2-\beta^2+2e+2a(b+\beta)}{4a(b+\beta)}}\left(\frac{\sqrt{-(b+\beta)^2(e^2-\epsilon^2)}}{2a(b+\beta)^2}\right)}+(-b-\beta)(e-\epsilon)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{b+ak+(-1)^k(-ak+\beta)}\ \text{for}\ (\ldots$$

$$\frac{\dfrac{\sqrt{-(b-\beta)^2(e^2-\epsilon^2)}\,I_{-\frac{-b^2+4ab+\beta^2-2e-4a\beta}{4ab-4a\beta}}\left(\frac{\sqrt{-(b-\beta)^2(e^2-\epsilon^2)}}{2a(b-\beta)^2}\right)}{I_{\frac{b^2-\beta^2+2e}{4ab-4a\beta}}\left(\frac{\sqrt{-(b-\beta)^2(e^2-\epsilon^2)}}{2a(b-\beta)^2}\right)}+(\beta-b)\left(b^2-\beta^2+e+\epsilon\right)}{(b-\beta)^2}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{b+ak+(-1)^k(ak\ldots}$$

$$\frac{\beta^2(e-\epsilon)}{\frac{\sqrt{-\beta^2(e^2-\epsilon^2)}\,I_{\frac{2e-\beta(2a+\beta)}{4a\beta}}\left(\frac{\sqrt{-\beta^2(e^2-\epsilon^2)}}{2a\beta^2}\right)}{I_{\frac{-\beta^2+2a\beta+2e}{4a\beta}}\left(\frac{\sqrt{-\beta^2(e^2-\epsilon^2)}}{2a\beta^2}\right)}+\beta(\epsilon-e)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{ak+(-1)^k(-ak+\beta)}\ \text{ for }(a,\beta,e,\epsilon)\in\mathbb{C}^4$$

$$\frac{\frac{\sqrt{-\beta^2(e^2-\epsilon^2)}\,I_{\frac{\beta^2-2e}{4a\beta}-1}\left(\frac{\sqrt{-\beta^2(e^2-\epsilon^2)}}{2a\beta^2}\right)}{I_{\frac{\beta^2-2e}{4a\beta}}\left(\frac{\sqrt{-\beta^2(e^2-\epsilon^2)}}{2a\beta^2}\right)}+\beta\left(-\beta^2+e+\epsilon\right)}{\beta^2}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{ak+(-1)^k(ak+\beta)}\ \text{ for }(a,\beta,e,\epsilon)\in\mathbb{C}^4$$

$$\frac{b^2(e-\epsilon)}{\frac{\sqrt{-b^2(e^2-\epsilon^2)}\,I_{\frac{b^2-2ab+2e}{4ab}}\left(\frac{\sqrt{-b^2(e^2-\epsilon^2)}}{2ab^2}\right)}{I_{\frac{b^2+2ab+2e}{4ab}}\left(\frac{\sqrt{-b^2(e^2-\epsilon^2)}}{2ab^2}\right)}+b(\epsilon-e)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{b-\left(-1+(-1)^k\right)ak}\ \text{ for }(a,b,e,\epsilon)\in\mathbb{C}^4$$

$$\frac{\frac{\sqrt{-b^2(e^2-\epsilon^2)}\,I_{\frac{b^2+2e}{4ab}-1}\left(\frac{\sqrt{-b^2(e^2-\epsilon^2)}}{2ab^2}\right)}{I_{\frac{b^2+2e}{4ab}}\left(\frac{\sqrt{-b^2(e^2-\epsilon^2)}}{2ab^2}\right)}-b\left(b^2+e+\epsilon\right)}{b^2}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+(-1)^k\epsilon}{b+\left(1+(-1)^k\right)ak}\ \text{ for }(a,b,e,\epsilon)\in\mathbb{C}^4$$

$$-\frac{\epsilon(b+\beta)^2 I_{\frac{2a+b-\beta}{4a}}\left(\frac{\epsilon^2}{2a\sqrt{(b+\beta)^2\epsilon^2}}\right)}{\epsilon(b+\beta)I_{\frac{2a+b-\beta}{4a}}\left(\frac{\epsilon^2}{2a\sqrt{(b+\beta)^2\epsilon^2}}\right)+\sqrt{\epsilon^2(b+\beta)^2}\,I_{-\frac{2a-b+\beta}{4a}}\left(\frac{\epsilon^2}{2a\sqrt{(b+\beta)^2\epsilon^2}}\right)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1)^k\epsilon}{b+ak+(-1)^k(-ak+\beta)}\ \text{ f}$$

$$\frac{\frac{\sqrt{\epsilon^2(b-\beta)^2}\,I_{\frac{-4a+b+\beta}{4a}}\left(\frac{\epsilon^2}{2a\sqrt{(b-\beta)^2\epsilon^2}}\right)}{I_{\frac{b+\beta}{4a}}\left(\frac{\epsilon^2}{2a\sqrt{(b-\beta)^2\epsilon^2}}\right)}+(\beta-b)\left(b^2-\beta^2+\epsilon\right)}{(b-\beta)^2}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1)^k\epsilon}{b+ak+(-1)^k(ak+\beta)}\ \text{ for }(a,b,\beta,\epsilon)\in\mathbb{C}^4$$

$$-\frac{\beta^2\epsilon I_{\frac{1}{2}-\frac{\beta}{4a}}\left(\frac{\sqrt{\beta^2\epsilon^2}}{2a\beta^2}\right)}{\sqrt{\beta^2\epsilon^2}\,I_{-\frac{2a+\beta}{4a}}\left(\frac{\sqrt{\beta^2\epsilon^2}}{2a\beta^2}\right)+\beta\epsilon I_{\frac{1}{2}-\frac{\beta}{4a}}\left(\frac{\sqrt{\beta^2\epsilon^2}}{2a\beta^2}\right)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1)^k\epsilon}{ak+(-1)^k(-ak+\beta)}\ \text{ for }(a,\beta,\epsilon)\in\mathbb{C}^3$$

$$\frac{\frac{\sqrt{\beta^2\epsilon^2} I_{\frac{\beta}{4a}-1}\left(\frac{\sqrt{\beta^2\epsilon^2}}{2a\beta^2}\right)}{I_{\frac{\beta}{4a}}\left(\frac{\sqrt{\beta^2\epsilon^2}}{2a\beta^2}\right)} - \beta^3 + \beta\epsilon}{\beta^2} = \operatorname*{K}_{k=1}^{\infty} \frac{(-1)^k \epsilon}{ak + (-1)^k (ak+\beta)} \text{ for } (a,\beta,\epsilon) \in \mathbb{C}^3$$

$$-\frac{b^2 \epsilon I_{\frac{2a+b}{4a}}\left(\frac{\sqrt{b^2\epsilon^2}}{2ab^2}\right)}{\sqrt{b^2\epsilon^2} I_{\frac{b-2a}{4a}}\left(\frac{\sqrt{b^2\epsilon^2}}{2ab^2}\right) + b\epsilon I_{\frac{2a+b}{4a}}\left(\frac{\sqrt{b^2\epsilon^2}}{2ab^2}\right)} = \operatorname*{K}_{k=1}^{\infty} \frac{(-1)^k \epsilon}{b - \left(-1 + (-1)^k\right) ak} \text{ for } (a,b,\epsilon) \in \mathbb{C}^3$$

$$\frac{\frac{\sqrt{b^2\epsilon^2} I_{\frac{b}{4a}-1}\left(\frac{\sqrt{b^2\epsilon^2}}{2ab^2}\right)}{I_{\frac{b}{4a}}\left(\frac{\sqrt{b^2\epsilon^2}}{2ab^2}\right)} - b\left(b^2+\epsilon\right)}{b^2} = \operatorname*{K}_{k=1}^{\infty} \frac{(-1)^k \epsilon}{b + \left(1 + (-1)^k\right) ak} \text{ for } (a,b,\epsilon) \in \mathbb{C}^3$$

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z\sqrt{1-z^2}}{\operatorname*{K}_{k=1}^{\infty} \frac{-2z^2 \left\lfloor \frac{1+k}{2} \right\rfloor \left(-1 + 2\left\lfloor \frac{1+k}{2} \right\rfloor\right)}{1+2k} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \le -1 \lor 1 \le z < \infty))$$

$$\cos^{-1}(z) = \frac{\sqrt{1-z^2}}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{k^2\left(-1+\frac{1}{z^2}\right)}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 1 \le z < \infty) \land \Re(z) > 0$$

$$\cos^{-1}(z) = \frac{z\sqrt{1-z^2}}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\left((-1)^k - k\right)k\left(1-z^2\right)}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 1 \le z < \infty) \land \Re(z) > 0 \land \left|\arg\left(\frac{1}{z^2}\right)\right| < \pi$$

$$\cos^{-1}(z) = \frac{\sqrt{1-z^2}}{\operatorname*{K}_{k=1}^{\infty} \frac{k^2(1-z^2)}{(1+2k)z} + z} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 1 \le z < \infty) \land \Re(z) > 0 \land \left|\arg\left(1-z^2\right)\right| < \pi$$

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{\sqrt{1-z^2}\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k^2 z^2}{1-z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \le -1 \lor 1 \le z < \infty))$$

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z\sqrt{1-z^2}}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k\left(-(-1)^k + k\right)z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \le -1 \lor 1 \le z < \infty))$$

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2 z^2}{(1+2k)(1-z^2)^{\frac{1}{2}(1+(-1)^k)}} - z^2 + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\cos^{-1}(z) = \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k(-(-1)^k + k)(1-z^2)}{1+2k} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2 z^2}{2k(1+2k)}}{1+\frac{(1-2k)^2 z^2}{2k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty)) \land |z| < 1$$

$$\cos^{-1}(1-z) = \frac{\sqrt{2}\sqrt{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2 z}{4k(1+2k)}}{1+\frac{(1-2k)^2 z}{4k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < 1-z \leq -1 \lor 1 \leq z < \infty)) \land |z| < 1$$

$$\cos^{-1}(z-1) = \pi - \frac{\sqrt{2}\sqrt{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2 z}{4k(1+2k)}}{1+\frac{(1-2k)^2 z}{4k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z-1 \leq -1 \lor 1 \leq z < \infty)) \land |z| < 1$$

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z\left(\log\left(-4z^2\right) - \frac{1}{2z^2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k(1+2k)}{2(1+k)^2 z^2}}{1+\frac{k(1+2k)}{2(1+k)^2 z^2}} + 1\right)}\right)}{2\sqrt{-z^2}} \text{ for } z \in \mathbb{C} \land \neg\left(z \in \mathbb{R} \land \left(-\infty < \frac{1}{z} \leq -1 \lor 1 \leq\right.\right.$$

$$\cos^{-1}(z)^2 = \frac{\pi^2}{4} - \frac{\pi z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{2z\Gamma\left(\frac{1+k}{2}\right)^2}{(1+k)\Gamma\left(\frac{k}{2}\right)^2}}{1-\frac{2z\Gamma\left(\frac{1+k}{2}\right)^2}{(1+k)\Gamma\left(\frac{k}{2}\right)^2}} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty)) \land |z| < 1$$

$$\cosh^{-1}(z) = \frac{\sqrt{z-1}\left(\frac{\pi}{2} - \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-2z^2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k} + 1}\right)}{\sqrt{1-z}} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z <$$

$$\cosh^{-1}(z) = \frac{z\sqrt{z^2-1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k(-(-1)^k + k)(-1+z^2)}{3+4(-1+k)(1+k)}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 1 \leq z < \infty) \land \Re(z) > 0$$

$$\cosh^{-1}(z) = \frac{\sqrt{z^2-1}}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k^2(-1+z^2)}{(-1+4k^2)z^2}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge 1 \leq z < \infty) \wedge \Re(z) > 0 \wedge \left|\arg\left(\frac{1}{z^2}\right)\right| < \pi$$

$$\cosh^{-1}(z) = \frac{\sqrt{z-1}\sqrt{z+1}}{\operatorname*{K}_{k=1}^{\infty} \frac{k^2(1-z^2)}{(1+2k)z} + z} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge 1 \leq z < \infty) \wedge \Re(z) > 0 \wedge \left|\arg\left(1-z^2\right)\right| < \pi$$

$$\cosh^{-1}(z) = \frac{\sqrt{\frac{z-1}{z+1}}z}{(z-1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k^2z^2}{1-z^2}}{1+2k} + 1\right)} + \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty))$$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} - \frac{\sqrt{z-1}z\sqrt{z+1}}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k\left(-(-1)^k+k\right)z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty))$$

$$\cosh^{-1}(z) = \frac{\pi\sqrt{z-1}}{2\sqrt{1-z}} - \frac{\sqrt{z-1}z\sqrt{z+1}}{\operatorname*{K}_{k=1}^{\infty} \frac{k^2z^2}{(1+2k)(1-z^2)^{\frac{1}{2}\left(1+(-1)^k\right)}} - z^2 + 1} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z <$$

$$\cosh^{-1}(z) = \frac{\sqrt{z-1}\left(\frac{\pi}{2} - \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2z^2}{2k(1+2k)}}{1+\frac{(1-2k)^2z^2}{2k(1+2k)}} + 1}\right)}{\sqrt{1-z}} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty)) \wedge |z| <$$

$$\cosh^{-1}(1-z) = \frac{\sqrt{2}\sqrt{-z}}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2z}{4k(1+2k)}}{1+\frac{(1-2k)^2z}{4k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < 1-z \leq -1 \vee 1 \leq z < \infty)) \wedge |z| < 1$$

$$\cosh^{-1}(z-1) = i(2\theta(\Im(z))-1)\left(\pi - \frac{\sqrt{2}\sqrt{z}}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2z}{4k(1+2k)}}{1+\frac{(1-2k)^2z}{4k(1+2k)}} + 1}\right) \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z-1 \leq -1 \vee 1 \leq z$$

$$\cosh^{-1}(z) = \log(2z) - \frac{1}{4z^2\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k(1+2k)}{2(1+k)^2z^2}}{1+\frac{k(1+2k)}{2(1+k)^2z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg\left(z \in \mathbb{R} \wedge \left(-\infty < \frac{1}{z} \leq -1 \vee 1 \leq z < \infty\right)\right.$$

$$\cosh^{-1}(z)^2 = \frac{\pi z}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{2z\Gamma\left(\frac{1+k}{2}\right)^2}{(1+k)\Gamma\left(\frac{k}{2}\right)^2}}{1-\frac{2z\Gamma\left(\frac{1+k}{2}\right)^2}{(1+k)\Gamma\left(\frac{k}{2}\right)^2}} + 1} - \frac{\pi^2}{4} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty)) \wedge |z| < 1$$

$$\cot^{-1}(z) = \frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k^2}{z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\cot^{-1}(z) = \frac{1}{z} - \frac{1}{z^3\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(1-(-1)^k+k)^2}{z^2}}{3+2k} + 3\right)} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\cot^{-1}(z) = \frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k^2}{(-1+4k^2)z^2}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\cot^{-1}(z) = \frac{z}{(z^2+1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k(-(-1)^k+k)}{(-1+4k^2)(1+z^2)}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\cot^{-1}(z) = \frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(-1+2k)^2}{z^2}}{1+2k-\frac{-1+2k}{z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\cot^{-1}(z) = \frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{2\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{z^2}}{(1+2k)\left(1+\frac{1+(-1)^k}{2z^2}\right)} + \frac{1}{z^2} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\cot^{-1}(z) = \frac{1}{2z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(-1+2k)^2}{4z^2}}{\frac{1}{2}(1+2k)-\frac{-1+2k}{2z^2}} + \frac{1}{2}\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\cot^{-1}(z) = \frac{1}{2z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\left(\frac{1}{2}-k\right)k(1+z^2)}{z^4}}{\frac{1}{2}+k+\frac{1+4k}{2z^2}} + \frac{z^2+1}{2z^2}\right)} \text{ for } z \in \mathbb{C} \wedge |\Im(z)| > \sqrt{2}$$

$$\cot^{-1}(z) = \frac{1}{2}\pi\sqrt{\frac{1}{z^2}}z - \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{(-1+2k)z^2}{1+2k}}{1+\frac{z^2-2kz^2}{1+2k}} + 1} \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$-i\coth^{-1}(1-iz) = \frac{1}{2}i\left(\frac{iz}{2\left(1+\mathrm{K}_{k=1}^{\infty} \frac{-\frac{ikz}{2(1+k)}}{1+\frac{ikz}{2(1+k)}}\right)} + \log(-iz) - \log(2)\right) \text{ for } z \in \mathbb{C}\wedge|z| < 1$$

$$i\coth^{-1}(1+iz) = \frac{1}{2}i\left(\frac{iz}{2\left(1+\mathrm{K}_{k=1}^{\infty} \frac{\frac{ikz}{2(1+k)}}{1-\frac{ikz}{2(1+k)}}\right)} - \log(iz) + \log(2)\right) \text{ for } z \in \mathbb{C}\wedge|z| < 1$$

$$\cot^{-1}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{-1+2k}{(1+2k)z^2}}{1-\frac{-1+2k}{(1+2k)z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\coth^{-1}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k^2}{z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq z \leq 1)$$

$$\coth^{-1}(z) = \frac{1}{z^3\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{\left(1-(-1)^k+k\right)^2}{z^2}}{3+2k} + 3\right)} + \frac{1}{z} \text{ for } z \in \mathbb{C}\wedge\neg(z \in \mathbb{R}\wedge-1 \leq z \leq 1)$$

$$\coth^{-1}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k^2}{(-1+4k^2)z^2}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq z \leq 1)$$

$$\coth^{-1}(z) = \frac{z}{(z^2-1)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{k\left(-(-1)^k+k\right)}{(-1+4k^2)(-1+z^2)}}{1} + 1\right)} \text{ for } z \in \mathbb{C}\wedge\neg(z \in \mathbb{R}\wedge-1 \leq z \leq 1)$$

$$\coth^{-1}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+2k)^2}{z^2}}{1+2k+\frac{-1+2k}{z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\coth^{-1}(z) = \cfrac{1}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{\frac{2\left\lfloor \frac{1+k}{2} \right\rfloor\left(-1+2\left\lfloor \frac{1+k}{2} \right\rfloor\right)}{z^2}}{(1+2k)\left(1-\frac{1+(-1)^k}{2z^2}\right)} - \frac{1}{z^2} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq z \leq 1)$$

$$\coth^{-1}(z) = \cfrac{1}{2z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{-\frac{(-1+2k)^2}{4z^2}}{\frac{1}{2}(1+2k)+\frac{-1+2k}{2z^2}} + \frac{1}{2}\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\coth^{-1}(z) = \cfrac{1}{2z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{\frac{\left(-\frac{1}{2}+k\right)k(-1+z^2)}{z^4}}{\frac{1}{2}+k-\frac{1+4k}{2z^2}} + \frac{z^2-1}{2z^2}\right)} \text{ for } z \in \mathbb{C} \wedge |\Re(z)| > \sqrt{2}$$

$$\coth^{-1}(z) = \cfrac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{-\frac{(-1+2k)z^2}{1+2k}}{1+\frac{(-1+2k)z^2}{1+2k}} + 1} - \frac{1}{2}\pi\sqrt{-\frac{1}{z^2}}z \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\coth^{-1}(z+1) = \frac{1}{2}\left(\cfrac{z}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{\frac{kz}{2(1+k)}}{1-\frac{kz}{2(1+k)}} + 1\right)} - \log(z) + \log(2)\right) \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$-\coth^{-1}(1-z) = \frac{1}{2}\left(\cfrac{z}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{-\frac{kz}{2(1+k)}}{1+\frac{kz}{2(1+k)}} + 1\right)} + \log(-z) - \log(2)\right) \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\coth^{-1}(z) = \cfrac{1}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{-\frac{-1+2k}{(1+2k)z^2}}{1+\frac{-1+2k}{(1+2k)z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\csc^{-1}(z) = \cfrac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{-\frac{2\left\lfloor \frac{1+k}{2} \right\rfloor\left(-1+2\left\lfloor \frac{1+k}{2} \right\rfloor\right)}{z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq z \leq 1)$$

$$\csc^{-1}(z) = \cfrac{1}{\sqrt{1-\frac{1}{z^2}}\,z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \cfrac{\frac{k^2}{-1+z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge -1 \leq z \leq 1)$$

$$\csc^{-1}(z)=\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k\left(-(-1)^k+k\right)}{\left(-1+4k^2\right)z^2}}{1}+1\right)}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land-1\leq z\leq 1)$$

$$\csc^{-1}(z)=\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{k^2}{z^2}}{(1+2k)\left(1-\frac{1}{z^2}\right)^{\frac{1}{2}\left(1+(-1)^k\right)}}-\frac{1}{z^2}+1\right)}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land-1\leq z\leq 1)$$

$$\csc^{-1}(z)=\frac{\pi\sqrt{z^2}}{2z}-\frac{\sqrt{1-\frac{1}{z^2}}z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2\left(-1+z^2\right)}{1+2k}+1}\text{ for }z\in\mathbb{C}\land\Re(z)\neq 0$$

$$\csc^{-1}(z)=\frac{\pi}{2}-\frac{\sqrt{1-\frac{1}{z^2}}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2\left(1-\frac{1}{z^2}\right)}{\frac{1+2k}{z}}+\frac{1}{z}}\text{ for }z\in\mathbb{C}\land\Re(z)>0$$

$$\csc^{-1}(z)=\frac{1}{2}\pi\sqrt{\frac{1}{z^2}}z-\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left((-1)^k-k\right)k\left(1-\frac{1}{z^2}\right)}{-1+4k^2}}{1}+1\right)}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land-1\leq z\leq 1)$$

$$\csc^{-1}(z)=\frac{1}{2}\sqrt{-\frac{1}{z^2}}z\left(\frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k(1+2k)z^2}{2(1+k)^2}}{1+\frac{k(1+2k)z^2}{2(1+k)^2}}+1\right)}-\log\left(-\frac{4}{z^2}\right)\right)\text{ for }z\in\mathbb{C}\land|z|<1$$

$$\csc^{-1}(z+1)=\frac{\pi}{2}-\frac{\sqrt{2}\sqrt{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}+1}\text{ for }z\in\mathbb{C}\land|z|<1$$

$$-\csc^{-1}(1-z)=\frac{\sqrt{2}\sqrt{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}+1}-\frac{\pi}{2}\text{ for }z\in\mathbb{C}\land|z|<1$$

$$\csc^{-1}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2}{2k(1+2k)z^2}}{1+\frac{(1-2k)^2}{2k(1+2k)z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\csc^{-1}(z)^2 = \frac{z^2}{2\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k^2(1+2k)z^2}{2(1+k)^3}}{1+\frac{k^2(1+2k)z^2}{2(1+k)^3}} + 1\right)} + \frac{z^2 \log\left(-\frac{1}{z^2}\right)}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2}{2(1+k)^2}}{1+\frac{k(1+2k)z^2}{2(1+k)^2}} + 1\right)} - \frac{z^2\left(\gamma + \psi^{(0)}\left(-\frac{1}{2}\right)\right)}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right)-\psi}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right)-\psi}}{1+\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right)-\psi}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right)-\psi}}\right.}$$

$$\operatorname{csch}^{-1}(z) = \frac{\sqrt{\frac{1}{z^2}+1}}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\operatorname{csch}^{-1}(z) = \frac{1}{\sqrt{\frac{1}{z^2}+1}\,z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k^2}{1+z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\operatorname{csch}^{-1}(z) = \frac{1}{\sqrt{\frac{1}{z^2}+1}\,z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k^2}{1+z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\operatorname{csch}^{-1}(z) = \frac{\sqrt{\frac{1}{z^2}+1}}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k^2}{z^2}}{(1+2k)\left(1+\frac{1}{z^2}\right)^{\frac{1}{2}\left(1+(-1)^k\right)}} + \frac{1}{z^2} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\operatorname{csch}^{-1}(z) = \frac{\sqrt{\frac{1}{z^2}+1}\,z}{\mathrm{K}_{k=1}^{\infty} \frac{-k^2(1+z^2)}{1+2k} + 1} + \frac{\pi\sqrt{-z^2}}{2z} \text{ for } z \in \mathbb{C} \wedge \Im(z) \neq 0$$

$$\operatorname{csch}^{-1}(z) = -\frac{1}{2}\pi\sqrt{-\frac{1}{z^2}}\,z - \frac{i\sqrt{\frac{1}{z^2}+1}}{\mathrm{K}_{k=1}^{\infty} \frac{k^2\left(1+\frac{1}{z^2}\right)}{-\frac{i(1+2k)}{z}} - \frac{i}{z}} \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\operatorname{csch}^{-1}(z) = -\frac{\sqrt{\frac{1}{z^2}+1}}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{((-1)^k-k)k\left(1+\frac{1}{z^2}\right)}{-1+4k^2}}{1} + 1\right)} - \frac{1}{2}\pi\sqrt{-\frac{1}{z^2}}\,z \text{ for } z \in \mathbb{C} \wedge \neg(iz \in \mathbb{R} \wedge -1 \leq iz \leq 1)$$

$$\operatorname{csch}^{-1}(z)=\frac{1}{2}\sqrt{\frac{1}{z^2}}z\left(\frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{k(1+2k)z^2}{2(1+k)^2}}{1-\frac{k(1+2k)z^2}{2(1+k)^2}}+1\right)}+\log\left(\frac{4}{z^2}\right)\right)\text{ for }z\in\mathbb{C}\land|z|<1$$

$$-i\csc^{-1}(1-iz)=\frac{i\sqrt{2}\sqrt{-iz}}{1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{i(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{i(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}}-\frac{i\pi}{2}\text{ for }z\in\mathbb{C}\land|z|<1$$

$$i\csc^{-1}(1+iz)=\frac{i\pi}{2}-\frac{i\sqrt{2}\sqrt{iz}}{1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{i(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{i(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}}\text{ for }z\in\mathbb{C}\land|z|<1$$

$$\operatorname{csch}^{-1}(z)=\frac{1}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1-2k)^2}{2k(1+2k)z^2}}{1-\frac{(1-2k)^2}{2k(1+2k)z^2}}+1\right)}\text{ for }z\in\mathbb{C}\land|z|>1$$

$$\operatorname{csch}^{-1}(z)^2=\frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{k^2(1+2k)z^2}{2(1+k)^3}}{1-\frac{k^2(1+2k)z^2}{2(1+k)^3}}+1\right)}+\frac{z^2\log\left(\frac{1}{z^2}\right)}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{k(1+2k)z^2}{2(1+k)^2}}{1-\frac{k(1+2k)z^2}{2(1+k)^2}}+1\right)}-\frac{z^2\left(\gamma+\psi^{(0)}\left(-\frac{1}{2}\right)\right.}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right)-\right.}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right)-\psi\right.}}{1-\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right)\right.}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right)-\right.}}\right.}$$

$$\sec^{-1}(z)=\frac{\pi}{2}-\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{z^2}}{1+2k}+1\right)}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land-1\leq z\leq 1)$$

$$\sec^{-1}(z)=\frac{\sqrt{1-\frac{1}{z^2}}z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2(-1+z^2)}{1+2k}+1}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land 0<z<1)\land\Re(z)>0$$

$$\sec^{-1}(z)=\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left((-1)^k-k\right)k\left(1-\frac{1}{z^2}\right)}{-1+4k^2}}{1}+1\right)}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land 0<z<1)\land\Re(z)>0\land\left|\arg\left(z^2\right)\right|<\pi$$

$$\sec^{-1}(z)=\frac{\sqrt{1-\frac{1}{z^2}}}{\mathrm{K}_{k=1}^{\infty}\frac{k^2\left(1-\frac{1}{z^2}\right)}{\frac{1+2k}{z}}+\frac{1}{z}}\text{ for } z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land 0<z<1)\land\Re(z)>0\land\left|\arg\left(1-\frac{1}{z^2}\right)\right|<\pi$$

$$\sec^{-1}(z)=\frac{\pi}{2}-\frac{1}{\sqrt{1-\frac{1}{z^2}}\,z\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{k^2}{-1+z^2}}{1+2k}+1\right)}\text{ for } z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land -1\leq z\leq 1)$$

$$\sec^{-1}(z)=\frac{\pi}{2}-\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{k\left(-(-1)^k+k\right)}{\left(-1+4k^2\right)z^2}}{1}+1\right)}\text{ for } z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land -1\leq z\leq 1)$$

$$\sec^{-1}(z)=\frac{\pi}{2}-\frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{k^2}{z^2}}{(1+2k)\left(1-\frac{1}{z^2}\right)^{\frac{1}{2}\left(1+(-1)^k\right)}}-\frac{1}{z^2}+1\right)}\text{ for } z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land -1\leq z\leq 1)$$

$$\sec^{-1}(z)=\frac{1}{2}\sqrt{-\frac{1}{z^2}}\,z\left(\log\left(-\frac{4}{z^2}\right)-\frac{z^2}{2\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{k(1+2k)z^2}{2(1+k)^2}}{1+\frac{k(1+2k)z^2}{2(1+k)^2}}+1\right)}\right)+\frac{\pi}{2}\text{ for } z\in\mathbb{C}\land|z|<1$$

$$\sec^{-1}(z+1)=\frac{\sqrt{2}\sqrt{z}}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}+1}\text{ for } z\in\mathbb{C}\land|z|<1$$

$$\sec^{-1}(z-1)=\pi-\frac{\sqrt{2}\sqrt{-z}}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}+1}\text{ for } z\in\mathbb{C}\land|z|<1$$

$$\sec^{-1}(z)=\frac{\pi}{2}-\frac{1}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2}{2k(1+2k)z^2}}{1+\frac{(1-2k)^2}{2k(1+2k)z^2}}+1\right)}\text{ for } z\in\mathbb{C}\land|z|>1$$

$$\sec^{-1}(z)^2 = \frac{z^2}{2\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k^2(1+2k)z^2}{2(1+k)^3}}{1+\frac{k^2(1+2k)z^2}{2(1+k)^3}} + 1\right)} + \frac{z^2\left(\log\left(-\frac{1}{z^2}\right) - \pi\sqrt{-\frac{1}{z^2}}z\right)}{4\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2}{2(1+k)^2}}{1+\frac{k(1+2k)z^2}{2(1+k)^2}} + 1\right)} - \frac{z^2\left(\gamma + \psi^{(0)}\left(-\frac{1}{2}\right)\right) \cdots}{4\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right)-\psi \cdots\right)}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right)-\psi \cdots\right)}}{1+\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right)- \cdots\right)}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right)-\psi \cdots\right)}} \cdots\right.}$$

$$\operatorname{sech}^{-1}(z) = \frac{\sqrt{\frac{1}{z}-1}\left(\frac{\pi}{2} - \frac{\sqrt{1-\frac{1}{z^2}}}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{z^2}}{1+2k} + 1\right)}\right)}{\sqrt{1-\frac{1}{z}}} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land -1 \leq z \leq 1)$$

$$\operatorname{sech}^{-1}(z) = \frac{\sqrt{\frac{1}{z^2}-1}}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k\left(-(-1)^k+k\right)\left(-1+\frac{1}{z^2}\right)}{3+4(-1+k)(1+k)}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 1 \leq z < \infty) \land \Re(z) > 0$$

$$\operatorname{sech}^{-1}(z) = \frac{\sqrt{\frac{1}{z^2}-1}\,z}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k^2\left(1-z^2\right)}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 0 < z < 1) \land \Re(z) > 0 \land \left|\arg\left(z^2\right)\right| < \pi$$

$$\operatorname{sech}^{-1}(z) = \frac{\sqrt{\frac{1}{z}-1}\sqrt{\frac{1}{z}+1}}{\operatorname*{K}_{k=1}^{\infty} \frac{k^2\left(1-\frac{1}{z^2}\right)}{\frac{1+2k}{z}} + \frac{1}{z}} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land 0 < z < 1) \land \Re(z) > 0 \land \left|\arg\left(1-\frac{1}{z^2}\right)\right| < \pi$$

$$\operatorname{sech}^{-1}(z) = \frac{\pi\sqrt{\frac{1}{z}-1}}{2\sqrt{1-\frac{1}{z}}} - \frac{\sqrt{\frac{1-z}{z+1}}}{(z-1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k^2}{-1+z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land -1 \leq z \leq 1)$$

$$\operatorname{sech}^{-1}(z) = \frac{\pi\sqrt{\frac{1}{z}-1}}{2\sqrt{1-\frac{1}{z}}} - \frac{\sqrt{\frac{1}{z}-1}\sqrt{\frac{1}{z}+1}}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k\left(-(-1)^k+k\right)}{\left(-1+4k^2\right)z^2}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land -1 \leq z \leq 1)$$

$$\operatorname{sech}^{-1}(z) = \frac{\pi\sqrt{\frac{1}{z}-1}}{2\sqrt{1-\frac{1}{z}}} - \frac{\sqrt{\frac{1}{z}-1}\sqrt{\frac{1}{z}+1}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k^2}{z^2}}{(1+2k)\left(1-\frac{1}{z^2}\right)^{\frac{1}{2}\left(1+(-1)^k\right)}} - \frac{1}{z^2}+1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land -1 \le z \le 1)$$

$$\operatorname{sech}^{-1}(z) = \log\left(\frac{2}{z}\right) - \frac{z^2}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2}{2(1+k)^2}}{1+\frac{k(1+2k)z^2}{2(1+k)^2}}+1\right)} \text{ for } z \in \mathbb{C} \land |z| < 1$$

$$\operatorname{sech}^{-1}(z+1) = \frac{\sqrt{2}\sqrt{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{(-1+2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}+1} \text{ for } z \in \mathbb{C} \land |z| < 1$$

$$\operatorname{sech}^{-1}(z-1) = i\left(2\theta\left(\Im\left(\frac{1}{z-1}\right)\right)-1\right)\left(\pi - \frac{\sqrt{2}\sqrt{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}{1-\frac{(1-2k)z\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}+k;\frac{3}{2};-1\right)}{2k\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+k;\frac{3}{2};-1\right)}}+1}\right) \text{ for } z \in \mathbb{C} \land |z| < 1$$

$$\operatorname{sech}^{-1}(z) = i\left(2\theta\left(\Im\left(\frac{1}{z}\right)\right)-1\right)\left(\frac{\pi}{2} - \frac{1}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2}{2k(1+2k)z^2}}{1+\frac{(1-2k)^2}{2k(1+2k)z^2}}+1\right)}\right) \text{ for } z \in \mathbb{C} \land |z| > 1$$

$$\operatorname{sech}^{-1}(z)^2 = -\frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k^2(1+2k)z^2}{2(1+k)^3}}{1+\frac{k^2(1+2k)z^2}{2(1+k)^3}}+1\right)} - \frac{z^2\log\left(\frac{1}{z}\right)}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2}{2(1+k)^2}}{1+\frac{k(1+2k)z^2}{2(1+k)^2}}+1\right)} + \frac{z^2\left(\gamma+\psi^{(0)}\left(-\frac{1}{2}\right.\right.}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right.\right.}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right.\right.}}{1+\frac{k(1+2k)z^2\left(\psi^{(0)}\left(-\frac{1}{2}-k\right.\right.}{2(1+k)^2\left(\psi^{(0)}\left(\frac{1}{2}-k\right.\right.}}\right.}$$

$$\sin^{-1}(z) = \frac{z}{\sqrt{1-z^2}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k^2z^2}{1-z^2}}{1+2k}+1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \le -1 \lor 1 \le z < \infty))$$

$$\sin^{-1}(z) = \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k\left(-(-1)^k+k\right)z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty))$$

$$\sin^{-1}(z) = \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-2z^2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k} + 1} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty))$$

$$\sin^{-1}(z) = \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2z^2}{(1+2k)(1-z^2)^{\frac{1}{2}\left(1+(-1)^k\right)}} - z^2 + 1} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge (-\infty < z \leq -1 \vee 1 \leq z < \infty))$$

$$\sin^{-1}(z) = \frac{1}{2}\pi\sqrt{\frac{1}{z^2}}z - \frac{\sqrt{1-z^2}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2\left(-1+\frac{1}{z^2}\right)}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \Re(z) \neq 0$$

$$\sin^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2(1-z^2)}{(1+2k)z} + z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\sin^{-1}(z) = \frac{\pi\sqrt{z^2}}{2z} - \frac{z\sqrt{1-z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left((-1)^k-k\right)k\left(1-z^2\right)}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \wedge \Re(z) \neq 0$$

$$\sin^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2z^2}{2k(1+2k)}}{1+\frac{(1-2k)^2z^2}{2k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \wedge \Re(z) \neq 0 \wedge |z| < 1$$

$$\sin^{-1}(1-z) = \frac{\pi}{2} - \frac{\sqrt{2}\sqrt{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2z}{4k(1+2k)}}{1+\frac{(1-2k)^2z}{4k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \wedge \Re(z) \neq 0 \wedge |z| < 1$$

$$-\sin^{-1}(1-z) = \frac{\sqrt{2}\sqrt{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(1-2k)^2z}{4k(1+2k)}}{1+\frac{(1-2k)^2z}{4k(1+2k)}} + 1} - \frac{\pi}{2} \text{ for } z \in \mathbb{C} \wedge \Re(z) \neq 0 \wedge |z| < 1$$

$$\sin^{-1}(z) = \frac{z\left(\log\left(-4z^2\right) - \frac{1}{2z^2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k(1+2k)}{2(1+k)^2z^2}}{1+\frac{k(1+2k)}{2(1+k)^2z^2}} + 1\right)}\right)}{2\sqrt{-z^2}} \text{ for } z \in \mathbb{C} \wedge \Re(z) \neq 0 \wedge |z| > 1$$

$$\sin^{-1}(z)^2 = \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{2k^2 z^2}{(1+k)(1+2k)}}{1+\frac{2k^2 z^2}{(1+k)(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \land \Re(z) \neq 0 \land |z| < 1$$

$$\sinh^{-1}(z) = \frac{z}{\sqrt{z^2+1}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k^2 z^2}{1+z^2}}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(iz \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\sinh^{-1}(z) = \frac{z\sqrt{z^2+1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k\left(-(-1)^k+k\right)z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(iz \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\sinh^{-1}(z) = \frac{z\sqrt{z^2+1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2z^2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k} + 1} \text{ for } z \in \mathbb{C} \land \neg(iz \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\sinh^{-1}(z) = \frac{z\sqrt{z^2+1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2 z^2}{(1+2k)(1+z^2)^{\frac{1}{2}(1+(-1)^k)}} + z^2 + 1} \text{ for } z \in \mathbb{C} \land \neg(iz \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\sinh^{-1}(z) = \frac{\sqrt{z^2+1}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2\left(1+\frac{1}{z^2}\right)}{1+2k} + 1\right)} + \frac{1}{2}\pi\sqrt{-\frac{1}{z^2}}z \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0$$

$$\sinh^{-1}(z) = \frac{i\pi}{2} - \frac{i\sqrt{z^2+1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2(1+z^2)}{-i(1+2k)z} - iz} \text{ for } z \in \mathbb{C} \land \Im(z) > 0$$

$$\sinh^{-1}(z) = -\frac{\sqrt{z^2+1}z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left((-1)^k-k\right)k\left(1+z^2\right)}{-1+4k^2}}{1} + 1} - \frac{\pi\sqrt{-z^2}}{2z} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0$$

$$\sinh^{-1}(z) = \frac{z\sqrt{z^2+1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1-(-1)^k)(1+k)z^2+\frac{1}{2}(1+(-1)^k)k(1+z^2)}{1} + 1} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0$$

$$\sinh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(1-2k)^2 z^2}{2k(1+2k)}}{1-\frac{(1-2k)^2 z^2}{2k(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0 \land |z| < 1$$

$$i \sin^{-1}(1-iz) = \frac{i\pi}{2} - \frac{i\sqrt{2}\sqrt{iz}}{1 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{i(1-2k)^2 z}{4k(1+2k)}}{1+\frac{i(1-2k)^2 z}{4k(1+2k)}}} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0 \land |z| < 1$$

$$-i \sin^{-1}(1+iz) = \frac{i\sqrt{2}\sqrt{-iz}}{1 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{i(1-2k)^2 z}{4k(1+2k)}}{1-\frac{i(1-2k)^2 z}{4k(1+2k)}}} - \frac{i\pi}{2} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0 \land |z| < 1$$

$$\sinh^{-1}(z) = \frac{z\left(\frac{1}{2z^2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k(1+2k)}{2(1+k)^2 z^2}}{1-\frac{k(1+2k)}{2(1+k)^2 z^2}}+1\right)} + \log\left(4z^2\right)\right)}{2\sqrt{z^2}} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0 \land |z| > 1$$

$$\sinh^{-1}(z)^2 = \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{2k^2 z^2}{(1+k)(1+2k)}}{1-\frac{2k^2 z^2}{(1+k)(1+2k)}} + 1} \text{ for } z \in \mathbb{C} \land \Im(z) \neq 0 \land |z| < 1$$

$$\tan^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2 z^2}{1+2k} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\tan^{-1}(z) = z - \frac{z^3}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(1-(-1)^k+k)^2 z^2}{3+2k} + 3} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\tan^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k^2 z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\tan^{-1}(z) = \frac{z}{(z^2+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k\left(-(-1)^k+k\right) z^2}{\left(-1+4k^2\right)\left(1+z^2\right)}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\tan^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2 z^2}{1+2k-(-1+2k) z^2} + 1} \text{ for } z \in \mathbb{C} \land |z| < 1$$

$$\tan^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2z^2\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{(1+2k)\left(1+\frac{1}{2}(1+(-1)^k) z^2\right)} + z^2 + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < iz \leq -1 \lor 1 \leq iz < \infty))$$

$$\tan^{-1}(z) = \frac{z}{2\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{4}(-1+2k)^2 z^2}{\frac{1}{2}(1+2k)-\frac{1}{2}(-1+2k)z^2} + \frac{1}{2}\right)} \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\tan^{-1}(z) = \frac{z}{2\left(\operatorname*{K}_{k=1}^{\infty} \frac{\left(\frac{1}{2}-k\right)kz^2(1+z^2)}{\frac{1}{2}+k+\frac{1}{2}(1+4k)z^2} + \frac{1}{2}\left(z^2+1\right)\right)} \text{ for } z \in \mathbb{C} \wedge |\Im(z)| < \frac{1}{\sqrt{2}}$$

$$\tan^{-1}(z) = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1-(-1)^k)kz^2+\frac{1}{2}(1+(-1)^k)k(1+z^2)}{1} + 1} \text{ for } z \in \mathbb{C} \wedge |\Im(z)| < \frac{1}{\sqrt{2}}$$

$$\tan^{-1}(z) = \frac{z}{\operatorname*{K}_{k=0}^{\infty} \frac{\left(\frac{1}{2}+k\right)^2 z^2}{2+2k+\sqrt{1+z^2}} + \frac{1}{2}\left(\sqrt{z^2+1}+1\right)} \text{ for } z \in \mathbb{C} \wedge |\Im(z)| < \frac{1}{\sqrt{2}}$$

$$\tan^{-1}\left(\frac{x}{y}\right) = \frac{x}{\operatorname*{K}_{k=1}^{\infty} \frac{k^2 x^2}{(1+2k)y} + y} \text{ for } (x,y) \in \mathbb{C}^2 \wedge \left|\Im\left(\frac{x}{y}\right)\right| < \frac{1}{\sqrt{2}}$$

$$\tan^{-1}\left(\frac{x}{y}\right) = \frac{xy}{\operatorname*{K}_{k=1}^{\infty} \frac{(-1+2k)^2 x^2 y^2}{-(-1+2k)x^2+(1+2k)y^2} + y^2} \text{ for } (x,y) \in \mathbb{C}^2 \wedge \left|\Im\left(\frac{x}{y}\right)\right| < \frac{1}{\sqrt{2}}$$

$$\tan^{-1}(z) = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(-1+2k)z^2}{1+2k}}{1-\frac{(-1+2k)z^2}{1+2k}} + 1} \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$i\tanh^{-1}(1-iz) = \frac{1}{2}i\left(-\frac{iz}{2\left(1+\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{ikz}{2(1+k)}}{1+\frac{ikz}{2(1+k)}}\right)} - \log(iz) + \log(2)\right) \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$-i\tanh^{-1}(1+iz) = \frac{1}{2}i\left(-\frac{iz}{2\left(1+\operatorname*{K}_{k=1}^{\infty} \frac{\frac{ikz}{2(1+k)}}{1-\frac{ikz}{2(1+k)}}\right)} + \log(-iz) - \log(2)\right) \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\tan^{-1}(z) = \frac{\pi z}{2\sqrt{z^2}} - \frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{-1+2k}{(1+2k)z^2}}{1-\frac{-1+2k}{(1+2k)z^2}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$\tanh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2 z^2}{1+2k} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\tanh^{-1}(z) = \frac{z^3}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-(1-(-1)^k+k)^2 z^2}{3+2k} + 3} + z \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\tanh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k^2 z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\tanh^{-1}(z) = \frac{z}{(1-z^2)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k\left(-(-1)^k+k\right) z^2}{\left(-1+4k^2\right)\left(1-z^2\right)}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\tanh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-(-1+2k)^2 z^2}{1+2k+(-1+2k)z^2} + 1} \text{ for } z \in \mathbb{C} \land |z| < 1$$

$$\tanh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2z^2 \left\lfloor \frac{1+k}{2} \right\rfloor \left(-1+2\left\lfloor \frac{1+k}{2} \right\rfloor\right)}{(1+2k)\left(1-\frac{1}{2}(1+(-1)^k) z^2\right)} - z^2 + 1} \text{ for } z \in \mathbb{C} \land \neg(z \in \mathbb{R} \land (-\infty < z \leq -1 \lor 1 \leq z < \infty))$$

$$\tanh^{-1}(z) = \frac{z}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{4}(-1+2k)^2 z^2}{\frac{1}{2}(1+2k)+\frac{1}{2}(-1+2k)z^2} + \frac{1}{2}\right)} \text{ for } z \in \mathbb{C} \land |z| < 1$$

$$\tanh^{-1}(z) = \frac{z}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}+k\right) k z^2 (1-z^2)}{\frac{1}{2}+k-\frac{1}{2}(1+4k)z^2} + \frac{1}{2}\left(1-z^2\right)\right)} \text{ for } z \in \mathbb{C} \land |\Re(z)| < \frac{1}{\sqrt{2}}$$

$$\tanh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{2}(1-(-1)^k) k z^2 + \frac{1}{2}(1+(-1)^k) k (1-z^2)}{1} + 1} \text{ for } z \in \mathbb{C} \land |\Re(z)| < \frac{1}{\sqrt{2}}$$

$$\tanh^{-1}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=0}^{\infty} \frac{-\left(\frac{1}{2}+k\right)^2 z^2}{2+2k+\sqrt{1-z^2}} + \frac{1}{2}\left(\sqrt{1-z^2}+1\right)} \text{ for } z \in \mathbb{C} \land |\Re(z)| < \frac{1}{\sqrt{2}}$$

$$\tanh^{-1}\left(\frac{x}{y}\right) = \frac{x}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2 x^2}{(1+2k)y} + y} \text{ for } (x,y) \in \mathbb{C}^2 \land \left|\Re\left(\frac{x}{y}\right)\right| < \frac{1}{\sqrt{2}}$$

$$\tanh^{-1}\left(\frac{x}{y}\right) = \frac{xy}{\mathrm{K}_{k=1}^{\infty} \frac{-(-1+2k)^2x^2y^2}{(-1+2k)x^2+(1+2k)y^2} + y^2} \text{ for } (x,y) \in \mathbb{C}^2 \wedge \left|\Re\left(\frac{x}{y}\right)\right| < \frac{1}{\sqrt{2}}$$

$$\tanh^{-1}(z) = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+2k)z^2}{1+2k}}{1+\frac{(-1+2k)z^2}{1+2k}} + 1} \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\tanh^{-1}(z+1) = \frac{1}{2}\left(\frac{z}{2\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{kz}{2(1+k)}}{1-\frac{kz}{2(1+k)}} + 1\right)} - \log(-z) + \log(2)\right) \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$-\tanh^{-1}(1-z) = \frac{1}{2}\left(\frac{z}{2\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{kz}{2(1+k)}}{1+\frac{kz}{2(1+k)}} + 1\right)} + \log(z) - \log(2)\right) \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\tanh^{-1}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{-1+2k}{(1+2k)z^2}}{1+\frac{-1+2k}{(1+2k)z^2}} + 1\right)} + \frac{\pi z}{2\sqrt{-z^2}} \text{ for } z \in \mathbb{C} \wedge |z| > 1$$

$$I_\nu(z) = \frac{2^{-\nu} z^\nu}{\Gamma(\nu+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z^2}{4k(k+\nu)}}{1+\frac{z^2}{4k(k+\nu)}} + 1\right)} \text{ for } (\nu,z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$I_{-m}(z) = \frac{2^{-m} z^m}{m!\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z^2}{4k(k+m)}}{1+\frac{z^2}{4k(k+m)}} + 1\right)} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m > 0$$

$$\frac{I_1(2)}{I_0(2)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{k}$$

$$\frac{I_\nu(z)}{I_{\nu-1}(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{z^2}{2(k+\nu)} + 2\nu} \text{ for } (\nu,z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{z}{(2\nu+2)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4(k+\nu)(1+k+\nu)}}{1} + 1\right)} \text{ for } (\nu,z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{z(-1-2k-2\nu)}{2+k+2z+2\nu} + 2\nu + z + 2} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = i \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-1}{-\frac{2i(k+\nu)}{z}} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{I_\nu\left(2\sqrt{z}\right)}{I_{\nu+1}\left(2\sqrt{z}\right)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{z}{1+k+\nu} + \nu + 1}{\sqrt{z}} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{\frac{\partial I_\nu(z)}{\partial z}}{I_\nu(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{z(-1-2k-2\nu)}{2+k+2z+2\nu} + 2\nu + z + 2} + \frac{\nu}{z} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{\frac{\partial I_\nu(z)}{\partial z}}{I_\nu(z)} = \frac{z}{(2\nu+2)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4(k+\nu)(1+k+\nu)}}{1} + 1\right)} + \frac{\nu}{z} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$J_\nu(z) = \frac{2^{-\nu} z^\nu}{\Gamma(\nu+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+\nu)}}{1-\frac{z^2}{4k(k+\nu)}} + 1\right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$J_{-m}(z) = \frac{2^{-m}(-z)^m}{m!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+m)}}{1-\frac{z^2}{4k(k+m)}} + 1\right)} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m > 0$$

$$\frac{J_\nu(z)}{J_{\nu-1}(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{-z^2}{2(k+\nu)} + 2\nu} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{z}{(2\nu+2)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z^2}{4(k+\nu)(1+k+\nu)}}{1} + 1\right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{iz(1+2k+2\nu)}{2+k-2iz+2\nu} + 2\nu - iz + 2} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = -\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-1}{\frac{2(k+\nu)}{z}} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(\nu \in \mathbb{Z} \wedge \nu \leq 0)$$

$$\frac{J_\nu\left(2i\sqrt{z}\right)}{J_{\nu-1}\left(2i\sqrt{z}\right)}=\frac{i\sqrt{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{z}{k+\nu}+\nu}\text{ for }(\nu,z)\in\mathbb{C}^2\land\neg(\nu\in\mathbb{Z}\land\nu\leq 0)$$

$$\frac{\frac{\partial J_\nu(z)}{\partial z}}{J_\nu(z)}=\frac{\nu}{z}-\frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{iz(1+2k+2\nu)}{2+k-2iz+2\nu}+2\nu-iz+2}\text{ for }(\nu,z)\in\mathbb{C}^2\land\neg(\nu\in\mathbb{Z}\land\nu\leq 0)$$

$$\frac{\frac{\partial J_\nu(z)}{\partial z}}{J_\nu(z)}=\frac{\nu}{z}-\frac{z}{(2\nu+2)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2}{4(k+\nu)(1+k+\nu)}}{1}+1\right)}\text{ for }(\nu,z)\in\mathbb{C}^2\land\neg(\nu\in\mathbb{Z}\land\nu\leq 0)$$

$$K_\nu(z)=\frac{1}{2}\pi\csc(\pi\nu)\left(\frac{2^\nu z^{-\nu}}{\Gamma(1-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2}{4k(k-\nu)}}{1+\frac{z^2}{4k(k-\nu)}}+1\right)}-\frac{2^{-\nu}z^\nu}{\Gamma(\nu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2}{4k(k+\nu)}}{1+\frac{z^2}{4k(k+\nu)}}+1\right)}\right)\text{ for }(\nu,z)\in$$

$$K_0(z)=-\frac{\log\left(\frac{z}{2}\right)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2}{4k^2}}{1+\frac{z^2}{4k^2}}+1}-\frac{\gamma}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2\psi^{(0)}(1+k)}{4k^2\psi^{(0)}(k)}}{1+\frac{z^2\psi^{(0)}(1+k)}{4k^2\psi^{(0)}(k)}}+1}\text{ for }z\in\mathbb{C}$$

$$K_m(z)=\frac{2^{m-1}(m-1)!z^{-m}}{\mathop{\mathrm{K}}_{k=1}^{-1+m}\frac{-\frac{z^2}{4k(k-m)}}{1+\frac{z^2}{4k(k-m)}}+1}+\frac{(-1)^{m-1}2^{-m}z^m\log\left(\frac{z}{2}\right)}{m!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2}{4k(k+m)}}{1+\frac{z^2}{4k(k+m)}}+1\right)}+\frac{(-1)^m2^{-m-1}z^m(\psi^{(0)}(m+1)-\gamma)}{m!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2(\psi^{(0)}(1+k)+\psi^{(0)}(1+k+m))}{4k(k+m)(\psi^{(0)}(k)+\psi^{(0)}(k+m))}}{1+\frac{z^2(\psi^{(0)}(1+k)+\psi^{(0)}(1+k+m))}{4k(k+m)(\psi^{(0)}(k)+\psi^{(0)}(k+m))}}+1\right.}$$

$$\frac{K_{\nu+1}(z)}{K_\nu(z)}=\frac{1}{1-\frac{2\nu+1}{2z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{4}\left(1+(-1)^k\right)(-1+k-2\nu)+\frac{1}{2}\left(1-(-1)^k\right)\left(1+\frac{k}{2}+\nu\right)}{\frac{2z}{1}}+1\right)}}\text{ for }(\nu,z)\in\mathbb{C}^2$$

$$\frac{K_{\nu+1}(z)}{K_\nu(z)}=\frac{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{4}(-1+2k)^2+\nu^2}{2(k+z)}}{z}+\frac{2\nu+1}{2z}+1\text{ for }(\nu,z)\in\mathbb{C}^2$$

$$\frac{\frac{\partial K_\nu(z)}{\partial z}}{K_\nu(z)}=\frac{\nu}{z}-\frac{1}{1-\frac{2\nu+1}{2z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{4}\left(1+(-1)^k\right)(-1+k-2\nu)+\frac{1}{2}\left(1-(-1)^k\right)\left(1+\frac{k}{2}+\nu\right)}{\frac{2z}{1}}+1\right)}}\text{ for }(\nu,z)\in\mathbb{C}^2$$

$$\frac{\frac{\partial K_\nu(z)}{\partial z}}{K_\nu(z)} = -\frac{\mathrm{K}_{k=1}^{\infty} \frac{-\frac{1}{4}(-1+2k)^2+\nu^2}{2(k+z)}}{z} - \frac{1}{2z} - 1 \text{ for } (\nu, z) \in \mathbb{C}^2$$

$$Y_\nu(z) = \csc(\pi\nu) \left(\frac{2^{-\nu}\cos(\pi\nu) z^\nu}{\Gamma(\nu+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+\nu)}}{1-\frac{z^2}{4k(k+\nu)}} + 1\right)} - \frac{2^\nu z^{-\nu}}{\Gamma(1-\nu)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k-\nu)}}{1-\frac{z^2}{4k(k-\nu)}} + 1\right)} \right) \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge$$

$$Y_0(z) = \frac{2\log\left(\frac{z}{2}\right)}{\pi\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k^2}}{1-\frac{z^2}{4k^2}} + 1\right)} + \frac{2\gamma}{\pi\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2\psi^{(0)}(1+k)}{4k^2\psi^{(0)}(k)}}{1-\frac{z^2\psi^{(0)}(1+k)}{4k^2\psi^{(0)}(k)}} + 1\right)} \text{ for } z \in \mathbb{C}$$

$$Y_m(z) = -\frac{2^m(m-1)!z^{-m}}{\pi\left(\mathrm{K}_{k=1}^{-1+m} \frac{-\frac{z^2(-1+k)!(-1-k+m)!}{4k!(-k+m)!}}{1+\frac{z^2(-1+k)!(-1-k+m)!}{4k!(-k+m)!}} + 1\right)} + \frac{2^{1-m}z^m\log\left(\frac{z}{2}\right)}{\pi m!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+m)}}{1-\frac{z^2}{4k(k+m)}} + 1\right)} - \frac{2^{-m}z^m(\psi^{(0)}(}{\pi m!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2(\psi^{(0)}(1+}{4k(k+m)(\psi^{(}}}{1-\frac{z^2(\psi^{(0)}(1}{4k(k+m)(}}\right.}$$

$$B_z(a,b) = \frac{z^a(1-z)^b}{a\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)\right)\left(a+b+\frac{1}{2}(-1+k)\right)z}{2(-1+a+k)(a+k)} + \frac{\left(1+(-1)^k\right)(2b-k)kz}{8(-1+a+k)(a+k)}}{1} + 1\right)} \text{ for } (z,a,b) \in \mathbb{C}^3 \wedge |\arg(1-z$$

$$B_z(a,b) = \frac{z^a(1-z)^b}{\mathrm{K}_{k=1}^{\infty} \frac{(b-k)kz}{a+k-(-1+a+b-k)z} + z(-a-b+1) + a} \text{ for } (z,a,b) \in \mathbb{C}^3 \wedge |\arg(1-z)| < \pi \wedge \Re(z) < 1$$

$$B_z(a,b) = \frac{z^a(1-z)^b}{\mathrm{K}_{k=1}^{\infty} \frac{k(-1+a+b+k)(1-z)z}{a+k-(a+b+2k)z} - z(a+b) + a} \text{ for } (z,a,b) \in \mathbb{C}^3 \wedge |\arg(1-z)| < \pi$$

$$B_z(a,b) = \frac{z^a(1-z)^b}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{(b-k)k(-1+a+k)(-1+a+b+k)z^2}{(-1+a+2k)^2}}{a+2k+\left(\frac{(b-k)k}{-1+a+2k} - \frac{(a+k)(a+b+k)}{1+a+2k}\right)z} - \frac{az(a+b)}{a+1} + a} \text{ for } (z,a,b) \in \mathbb{C}^3 \wedge |\arg(1-z)| < \pi$$

$$B_z(a,b) = \frac{z^a}{a\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{(b-k)(-1+a+k)z}{k(a+k)}}{1-\frac{(b-k)(-1+a+k)z}{k(a+k)}} + 1\right)} \text{ for } (z,a,b) \in \mathbb{C}^3 \wedge |z| < 1$$

$$B_{1-z}(a,b)=B(a,b)-\frac{(1-z)^a z^b}{b\left(\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{(-1+a+b+k)z}{b+k}}{1+\frac{(-1+a+b+k)z}{b+k}}+1\right)}\text{ for }(z,a,b)\in\mathbb{C}^3\land\neg(b\in\mathbb{Z}\land-b\geq 0)\land|z|<1$$

$$B_{1-z}(a,0)=\frac{(1-z)^a(-\psi^{(0)}(a)-\gamma)}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{(-1+a+k)z(-\psi^{(0)}(1+k)+\psi^{(0)}(a+k))}{k(\psi^{(0)}(k)-\psi^{(0)}(-1+a+k))}}{1-\frac{(-1+a+k)z(-\psi^{(0)}(1+k)+\psi^{(0)}(a+k))}{k(\psi^{(0)}(k)-\psi^{(0)}(-1+a+k))}}+1}-\log(z)\text{ for }(z,a)\in\mathbb{C}^2\land|z|<1$$

$$B_{1-z}(a,-m)=\frac{(1-z)^a(-\psi^{(0)}(a)-\gamma)(1-a)_m}{m!\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{(-1+a+k)z(-\psi^{(0)}(1+k)+\psi^{(0)}(a+k))}{k(\psi^{(0)}(k)-\psi^{(0)}(-1+a+k))}}{1-\frac{(-1+a+k)z(-\psi^{(0)}(1+k)+\psi^{(0)}(a+k))}{k(\psi^{(0)}(k)-\psi^{(0)}(-1+a+k))}}+1\right)}+\frac{(-1)^{m-1}\Gamma(a)\log(z)}{m!\Gamma(a-m)}+\frac{(1-z)^{a}\cdots}{m\left(\operatorname*{K}_{k=1}^{-1+m}\frac{-\cdots}{1+\cdots}\right.}$$

$$B_z(a,b)=\frac{z^a(-z)^{b-1}}{(a+b-1)\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{(b-k)(a+b-k)}{(-1+a+b-k)kz}}{1-\frac{(b-k)(a+b-k)}{(-1+a+b-k)kz}}+1\right)}+\frac{z^a(-z)^{-a}\Gamma(a)\Gamma(-a-b+1)}{\Gamma(1-b)}\text{ for }(z,a,b)\in\mathbb{C}^3\land\neg(a+b\cdots$$

$$B_z(a,1-a)=(-z)^{-a}z^a(-\psi^{(0)}(a)+\log(-z)-\gamma)-\frac{a(-z)^{-a}z^{a-1}}{\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{k(a+k)}{(1+k)^2z}}{1+\frac{k(a+k)}{(1+k)^2z}}+1}\text{ for }(z,a)\in\mathbb{C}^2\land|z|>1$$

$$B_z(a,-a+m+1)=\frac{(-z)^{-a}z^{a-1}(-a)_{m+1}}{(m+1)!\left(\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{k(a+k)}{(1+k)(1+k+m)z}}{1+\frac{k(a+k)}{(1+k)(1+k+m)z}}+1\right)}+\frac{(-z)^{-a}z^a(1-a)_m(-\psi^{(0)}(a)+\psi^{(0)}(m+1)+\cdots}{m!}$$

$$I_z(a,b)=\frac{z^a(1-z)^b}{aB(a,b)\left(\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{(1-(-1)^k)\left(a+\frac{1}{2}(-1+k)\right)\left(a+b+\frac{1}{2}(-1+k)\right)z}{2(-1+a+k)(a+k)}+\frac{(1+(-1)^k)(2b-k)kz}{8(-1+a+k)(a+k)}}{1}+1\right)}\text{ for }(z,a,b)\in\mathbb{C}^3\land|\arg\cdots$$

$$I_z(a,b)=\frac{z^a(1-z)^b}{B(a,b)\left(\operatorname*{K}_{k=1}^{\infty}\frac{(b-k)kz}{a+k-(-1+a+b-k)z}+z(-a-b+1)+a\right)}\text{ for }(z,a,b)\in\mathbb{C}^3\land|\arg(1-z)|<\pi$$

$$I_z(a,b)=\frac{z^a(1-z)^b}{B(a,b)\left(\operatorname*{K}_{k=1}^{\infty}\frac{k(-1+a+b+k)(1-z)z}{a+k-(a+b+2k)z}-z(a+b)+a\right)}\text{ for }(z,a,b)\in\mathbb{C}^3\land|\arg(1-z)|<\pi$$

$$I_z(a,b)=\frac{z^a(1-z)^b}{B(a,b)\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(b-k)k(-1+a+k)(-1+a+b+k)z^2}{(-1+a+2k)^2}}{a+2k+\left(\frac{(b-k)k}{-1+a+2k}-\frac{(a+k)(a+b+k)}{1+a+2k}\right)z}-\frac{az(a+b)}{a+1}+a\right)} \text{ for } (z,a,b)\in\mathbb{C}^3\wedge|\arg(1-z)|<\pi$$

$$I_z(a,b)=\frac{z^a}{aB(a,b)\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(b-k)(-1+a+k)z}{k(a+k)}}{1-\frac{(b-k)(-1+a+k)z}{k(a+k)}}+1\right)} \text{ for } (z,a,b)\in\mathbb{C}^3\wedge|z|<1$$

$$I_{1-z}(a,b)=1-\frac{(1-z)^a z^b}{bB(a,b)\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(-1+a+b+k)z}{b+k}}{1+\frac{(-1+a+b+k)z}{b+k}}+1\right)} \text{ for } (z,a,b)\in\mathbb{C}^3\wedge\neg(b\in\mathbb{Z}\wedge -b\geq 0)\wedge|z|<1$$

$$I_z(a,b)=\frac{z^a(-z)^{b-1}}{(a+b-1)B(a,b)\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(b-k)(a+b-k)}{(-1+a+b-k)kz}}{1-\frac{(b-k)(a+b-k)}{(-1+a+b-k)kz}}+1\right)}+z^a(-z)^{-a}\sin(\pi b)\csc(\pi(a+b)) \text{ for } (z,a,b)\in\mathbb{C}^3$$

$$I_z(a,1-a)=\frac{(-z)^{-a}z^a\sin(\pi a)(-\psi^{(0)}(a)+\log(-z)-\gamma)}{\pi}-\frac{a(-z)^{-a}z^{a-1}\sin(\pi a)}{\pi\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k(a+k)}{(1+k)^2z}}{1+\frac{k(a+k)}{(1+k)^2z}}+1\right)} \text{ for } (z,a)\in\mathbb{C}^2\wedge|z|>$$

$$I_z(a,-a+m+1)=-\frac{a(-z)^{-a}z^{a-1}\sin(\pi a)}{\pi(m+1)\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k(a+k)}{(1+k)(1+k+m)z}}{1+\frac{k(a+k)}{(1+k)(1+k+m)z}}+1\right)}+\frac{z^a(-z)^{m-a}}{mB(a,-a+m+1)\left(\mathop{\Huge\mathrm{K}}_{k=1}^{-1+m}\frac{\frac{(-1+a+k-m)(1}{k(k-m)}}{1-\frac{(-1+a+k-m)}{k(k-m}}\right.}$$

$$\frac{I_z(a+1,b)}{I_z(a,b)}=\frac{z(a+b)}{\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{2}(1+(-1)^k)\left(a+\frac{k}{2}\right)\left(a+b+\frac{k}{2}\right)z+\frac{1}{4}(1-(-1)^k)(-1-k)\left(-b+\frac{1+k}{2}\right)z}{1+a+k}+a+1} \text{ for } (z,a,b)\in\mathbb{C}^3\wedge|\arg$$

$$\frac{I_z(a+1,b)}{I_z(a,b)}=\frac{z(a+b)}{\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{(-a-k)(a+b+k)z}{1+a+k+(a+b+k)z}+z(a+b)+a+1} \text{ for } (z,a,b)\in\mathbb{C}^3\wedge|z|<1$$

$$C=1-\frac{1}{2\left(\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{4\left\lfloor\frac{1+k}{2}\right\rfloor^2}{2+(-1)^k}+3\right)}$$

$$C=\frac{1}{2\mathop{\Huge\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{16}\left((-1+(-1)^k)^2(1+k)^2+2(1+(-1)^k)k(2+k)\right)}{\frac{1}{2}}+1}+\frac{1}{2}$$

$$C = \frac{13}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{16(1-2k)^4 k^4 (29-48k+20k^2)(13+32k+20k^2)}{7+16k-156k^2-384k^3+2064k^4+5632k^5+3520k^6} + 7\right)}$$

$$C = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(1-2k)^2}{(1+2k)^2}}{\frac{8k}{(1+2k)^2}} + 1}$$

$$T_\nu(z) = \cos\left(\frac{\pi\nu}{2}\right) \left(\frac{\nu z \sin\left(\frac{\pi\nu}{2}\right)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{4^{-2+k} z \nu^2 \Gamma\left(-\frac{1}{2}+\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}+\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(-\frac{1}{2}+\frac{k}{2}+\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}+\frac{k}{2}+\frac{\nu}{2}\right)\sin^2(\pi\nu)}{\pi^2\Gamma(k)\Gamma(2+k)}}{-\frac{(-1)^k 2^{-2+k} \nu \Gamma\left(\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{k}{2}+\frac{\nu}{2}\right)\sin(\pi\nu)}{\pi\Gamma(1+k)} + \frac{(-1)^k 2^{-1+k} z \nu \Gamma\left(\frac{1}{2}+\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}+\frac{k}{2}+\frac{\nu}{2}\right)\sin(\pi\nu)}{\pi\Gamma(2+k)}} + \cos\left(\frac{\pi\nu}{2}\right.} \right.$$

$$T_\nu(1-2z) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{2z(-1+k-\nu)(-1+k+\nu)}{k(-1+2k)}}{1+\frac{2z(-1+k-\nu)(-1+k+\nu)}{k(-1+2k)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| < 1$$

$$T_\nu(2z-1) = \frac{2\nu\sqrt{z}\sin(\pi\nu)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(-1-4(-1+k)k+4\nu^2\right)}{2k(1+2k)}}{1-\frac{z\left(-1-4(-1+k)k+4\nu^2\right)}{2k(1+2k)}} + 1} + \frac{\cos(\pi\nu)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{2z(-1+k-\nu)(-1+k+\nu)}{k(-1+2k)}}{1+\frac{2z(-1+k-\nu)(-1+k+\nu)}{k(-1+2k)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| < 1$$

$$T_\nu(z) = \frac{2^{-\nu-1} z^{-\nu}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-2+2k+\nu)(-1+2k+\nu)}{4kz^2(k+\nu)}}{1+\frac{(-2+2k+\nu)(-1+2k+\nu)}{4kz^2(k+\nu)}} + 1} + \frac{2^{\nu-1} z^{\nu}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(k-\nu)}}{1+\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(k-\nu)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| > 1$$

$$T_\nu(z) = \frac{2^{\nu-1} z^{\nu}}{\mathop{\mathrm{K}}_{k=1}^{\left\lfloor \frac{\nu}{2} \right\rfloor} \frac{-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(k-\nu)}}{1+\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(k-\nu)}} + 1} \text{ for } \nu \in \mathbb{Z} \wedge \nu > 0$$

$$U_\nu(z) = \cos\left(\frac{\pi\nu}{2}\right) \left(\frac{(\nu+1) z \sin\left(\frac{\pi\nu}{2}\right)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{4^{-1+k} z \Gamma\left(-\frac{1}{2}+\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}+\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}+\frac{k}{2}+\frac{\nu}{2}\right)\Gamma\left(\frac{3}{2}+\frac{k}{2}+\frac{\nu}{2}\right)\sin^2(\pi\nu)}{\pi^2\Gamma(k)\Gamma(2+k)}}{-\frac{(-1)^k 2^{-1+k} \Gamma\left(\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(1+\frac{k}{2}+\frac{\nu}{2}\right)\sin(\pi\nu)}{\pi\Gamma(1+k)} + \frac{(-2)^k z \Gamma\left(\frac{1}{2}+\frac{k}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{3}{2}+\frac{k}{2}+\frac{\nu}{2}\right)\sin(\pi\nu)}{\pi\Gamma(2+k)}} + \cos\left(\frac{\pi\nu}{2}\right)} + \right.$$

$$U_\nu(1-2z) = \frac{\nu+1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{2z(-1+k-\nu)(1+k+\nu)}{k(1+2k)}}{1+\frac{2z(-1+k-\nu)(1+k+\nu)}{k(1+2k)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| < 1$$

$$U_\nu(2z-1) = \frac{(\nu+1)\cos(\pi\nu)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{2z(-1+k-\nu)(1+k+\nu)}{k(1+2k)}}{1+\frac{2z(-1+k-\nu)(1+k+\nu)}{k(1+2k)}} + 1} - \frac{\sin(\pi\nu)}{2\sqrt{z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z(3-4(-1+k)k+4\nu(2+\nu))}{2k(-1+2k)}}{1-\frac{z(3-4(-1+k)k+4\nu(2+\nu))}{2k(-1+2k)}} + 1\right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| <$$

$$U_\nu(z) = \frac{2^\nu z^\nu}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(-1+k-\nu)}}{1+\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(-1+k-\nu)}} + 1} - \frac{2^{-\nu-2} z^{-\nu-2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(2k+\nu)(1+2k+\nu)}{4kz^2(1+k+\nu)}}{1+\frac{(2k+\nu)(1+2k+\nu)}{4kz^2(1+k+\nu)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| > 1$$

$$U_\nu(z) = \frac{2^\nu z^\nu}{\mathop{\mathrm{K}}_{k=1}^{\lfloor \frac{\nu}{2} \rfloor} \frac{-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(-1+k-\nu)}}{1+\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(-1+k-\nu)}} + 1} \text{ for } \nu \in \mathbb{Z} \wedge \nu > 0$$

$$\frac{e(b+\beta)}{-\frac{2e^2}{(b^2-\beta^2+2e)\left(\sqrt{\frac{(b^2-\beta^2)(b^2-\beta^2+4e)}{(b^2-\beta^2+2e)^2}}+1\right)} + b^2 - \beta^2 + e} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e}{b+(-1)^k\beta} \text{ for } (b, \beta, e) \in \mathbb{C}^3$$

$$\frac{\beta^2 e}{-\beta^3 + \frac{2\beta e^2}{(\beta^2-2e)\left(\sqrt{\frac{\beta^2(\beta^2-4e)}{(\beta^2-2e)^2}}+1\right)} + \beta e} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e}{(-1)^k\beta} \text{ for } (\beta, e) \in \mathbb{C}^2$$

$$\sqrt{\frac{e\pi}{2}}\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{1} + 1}$$

$$\log(2) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{1} + 1}$$

$$\log(2) = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\lfloor \frac{1+k}{2} \rfloor}{\frac{1}{2}(3+(-1)^k)+(1+(-1)^k)k} + 2}$$

$$\log(2) = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2}{3(1+2k)} + 3}$$

$$\sqrt[3]{2} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^k+3\lfloor \frac{1+k}{2} \rfloor}{\frac{1}{2}(5+(-1)^k+3(1+(-1)^k)k)} + 3} + 1$$

$$\sqrt[3]{2} = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1-9k^2}{9(1+2k)} + 8} + 1$$

$$\psi^{(2)}(2) = -\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^6}{(1+2k)(5+k+k^2)} + 5}$$

$$\sqrt{2} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{2} + 1$$

$$\sqrt{5} = 2 \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{1} + 1$$

$$\tan(1) = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{\frac{1}{2}\left(1 + (-1)^k\right) + \frac{1}{2}\left(1 - (-1)^k\right) k} + 1$$

$$\tanh(1) = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{-1 + 2k}$$

$$\frac{\pi^2}{6} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{8}(1-(-1)^k+4k+2k^2)}{1} + 1} + 1$$

$$\frac{\pi^2}{6} = \frac{2}{\mathrm{K}_{k=1}^{\infty} \frac{k^4}{1+2k} + 1}$$

$$\zeta(3) = \frac{6}{\mathrm{K}_{k=1}^{\infty} \frac{-k^6}{5+27k+51k^2+34k^3} + 5}$$

$$\zeta(3) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{-k^6}{(1+2k)(5+k+k^2)} + 5} + 1$$

$$\zeta(3) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor^3}{\frac{1}{2}(1-(-1)^k)+2(1+(-1)^k)(1+k)} + 4} + 1$$

$$\frac{2e}{b\left(\sqrt{\frac{4e}{b^2} + 1} + 1\right)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e}{b} \text{ for } (b, e) \in \mathbb{C}^2$$

$$\frac{\sqrt{e} I_{\frac{b}{a}-1}\left(\frac{2\sqrt{e}}{a}\right)}{I_{\frac{b}{a}}\left(\frac{2\sqrt{e}}{a}\right)} - b = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e}{b + ak} \text{ for } (a, b, e) \in \mathbb{C}^3$$

$$\frac{\sqrt{e} I_1\left(\frac{2\sqrt{e}}{a}\right)}{I_0\left(\frac{2\sqrt{e}}{a}\right)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e}{ak} \text{ for } (a, e) \in \mathbb{C}^2$$

$$\frac{\frac{\sqrt{-e^2(b-\beta)^2}\, I_{-\frac{-b^2+4ab+\beta^2-2e-4a\beta}{4ab-4a\beta}}\left(-\frac{e^2}{2a\sqrt{-e^2(b-\beta)^2}}\right)}{I_{\frac{b^2-\beta^2+2e}{4ab-4a\beta}}\left(-\frac{e^2}{2a\sqrt{-e^2(b-\beta)^2}}\right)} + (\beta - b)\left(b^2 - \beta^2 + e\right)}{(b-\beta)^2} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e}{b + ak + (-1)^k(ak + \beta)} \text{ for } (\text{}$$

$$-\frac{e(b+\beta)^2 I_{\frac{b^2-\beta^2+2e+2a(b+\beta)}{4a(b+\beta)}}\left(-\frac{e^2}{2a\sqrt{-e^2(b+\beta)^2}}\right)}{e(b+\beta) I_{\frac{b^2-\beta^2+2e+2a(b+\beta)}{4a(b+\beta)}}\left(-\frac{e^2}{2a\sqrt{-e^2(b+\beta)^2}}\right)-\sqrt{-e^2(b+\beta)^2} I_{\frac{b^2-\beta^2+2e-2a(b+\beta)}{4a(b+\beta)}}\left(-\frac{e^2}{2a\sqrt{-e^2(b+\beta)^2}}\right)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{}{b}$$

$$\frac{\beta^2 e}{\frac{\sqrt{\beta^2(-e^2)} I_{\frac{2e-\beta(2a+\beta)}{4a\beta}}\left(\frac{\sqrt{-e^2\beta^2}}{2a\beta^2}\right)}{I_{\frac{-\beta^2+2a\beta+2e}{4a\beta}}\left(\frac{\sqrt{-e^2\beta^2}}{2a\beta^2}\right)}-\beta e} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{ak+(-1)^k(-ak+\beta)} \text{ for } (a,\beta,e)\in\mathbb{C}^3$$

$$\frac{\frac{\sqrt{\beta^2(-e^2)} I_{\frac{\beta^2-2e}{4a\beta}-1}\left(\frac{\sqrt{-e^2\beta^2}}{2a\beta^2}\right)}{I_{\frac{\beta^2-2e}{4a\beta}}\left(\frac{\sqrt{-e^2\beta^2}}{2a\beta^2}\right)}-\beta^3+\beta e}{\beta^2} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{ak+(-1)^k(ak+\beta)} \text{ for } (a,\beta,e)\in\mathbb{C}^3$$

$$\frac{b^2 e}{\frac{\sqrt{-b^2e^2} I_{\frac{b^2-2ab+2e}{4ab}}\left(\frac{\sqrt{-b^2e^2}}{2ab^2}\right)}{I_{\frac{b^2+2ab+2e}{4ab}}\left(\frac{\sqrt{-b^2e^2}}{2ab^2}\right)}-be} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{b+ak-(-1)^k ak} \text{ for } (a,b,e)\in\mathbb{C}^3$$

$$\frac{\frac{\sqrt{-b^2e^2} I_{\frac{b^2+2e}{4ab}-1}\left(\frac{\sqrt{-b^2e^2}}{2ab^2}\right)}{I_{\frac{b^2+2e}{4ab}}\left(\frac{\sqrt{-b^2e^2}}{2ab^2}\right)}-b\left(b^2+e\right)}{b^2} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{b+ak+(-1)^k ak} \text{ for } (a,b,e)\in\mathbb{C}^3$$

$$\frac{\left(a+\frac{2e}{\sqrt{b^2+4e}+b}+b\right)\text{QHypergeometricPFQ}\left(\{\},\left\{-\frac{a\left(\sqrt{b^2+4e}+b\right)}{b\sqrt{b^2+4e}+b^2+2e}\right\},q,\frac{2eq}{b\sqrt{b^2+4e}+b^2+2e}\right)}{\text{QHypergeometricPFQ}\left(\{\},\left\{-\frac{aq\left(\sqrt{b^2+4e}+b\right)}{b\sqrt{b^2+4e}+b^2+2e}\right\},q,\frac{2eq}{b\sqrt{b^2+4e}+b^2+2e}\right)}-a-b = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{b+aq}$$

$$\frac{e\text{QHypergeometricPFQ}\left(\{\},\{0\},\frac{1}{q^2},\frac{e}{a^2q^5}\right)}{aq\text{QHypergeometricPFQ}\left(\{\},\{0\},\frac{1}{q^2},\frac{e}{a^2q^3}\right)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{aq^k} \text{ for } (a,e,q)\in\mathbb{C}^3 \wedge 0<|q|<1$$

$$\frac{a\left(q^4;q^{10}\right)_\infty\left(q^6;q^{10}\right)_\infty}{\sqrt{q}\left(q^2;q^{10}\right)_\infty\left(q^8;q^{10}\right)_\infty}-\frac{a}{\sqrt{q}}-a = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{-\frac{a^2}{\sqrt{q}}}{a+\frac{a}{\sqrt{q}}+aq^k} \text{ for } (a,q)\in\mathbb{C}^2 \wedge 0<|q|<1$$

$$\cos(z) = 1 - \frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{2(1+k)(1+2k)}}{1-\frac{z^2}{2(1+k)(1+2k)}} + 1\right)} \text{ for } z \in \mathbb{C}$$

$$\cos(z) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{2k(-1+2k)}}{1-\frac{z^2}{2k(-1+2k)}} + 1} \text{ for } z \in \mathbb{C}$$

$$\cos\left(\frac{\pi z}{2}\right) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{2}(1+(-1)^k)\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-1-z+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)+\frac{1}{2}(1-(-1)^k)\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-1+z+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{\frac{1}{2}(1+(-1)^k)k+z} + 1} + 1 \text{ for } z$$

$$\cos\left(\frac{\pi z}{2}\right) = 1 - \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-(-1+2k)^2((-1+2k)^2-z^2)}{2+8k^2-z^2} + 1} \text{ for } z \in \mathbb{C}$$

$$\cos\left(\frac{\pi z}{2}\right) = 1 - \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2(1-2k)k(4k^2-z^2)}{4k^2+(1+2k)(2+2k)-z^2} + 2} \text{ for } z \in \mathbb{C}$$

$$\cos^m(z) = 1 - 2^{-m} z^2 \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} \frac{(-2i+m)^2\binom{m}{i}}{1 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2 \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-2i+m)^{2+2k}\binom{m}{i}}{2(1+k)(1+2k)\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-2i+m)^{2k}\binom{m}{i}}}{1-\frac{z^2 \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-2i+m)^{2+2k}\binom{m}{i}}{2(1+k)(1+2k)\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-2i+m)^{2k}\binom{m}{i}}}} \text{ for } m \in \mathbb{Z} \land z \in \mathbb{C} \land m > 0$$

$$\cosh(z) = \frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z^2}{2(1+k)(1+2k)}}{1+\frac{z^2}{2(1+k)(1+2k)}} + 1\right)} + 1 \text{ for } z \in \mathbb{C}$$

$$\cosh(z) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z^2}{2k(-1+2k)}}{1+\frac{z^2}{2k(-1+2k)}} + 1} \text{ for } z \in \mathbb{C}$$

$$\cosh\left(\frac{\pi z}{2}\right) = 1 + \frac{iz}{1 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{2}(1+(-1)^k)\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-1-iz+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)+\frac{1}{2}(1-(-1)^k)\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-1+iz+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{\frac{1}{2}(1+(-1)^k)k+iz}} \text{ fo}$$

$$\cosh\left(\frac{\pi z}{2}\right) = \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-(-1+2k)^2((-1+2k)^2+z^2)}{2+8k^2+z^2} + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$\cosh\left(\frac{\pi z}{2}\right) = \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2(1-2k)k(4k^2+z^2)}{4k^2+(1+2k)(2+2k)+z^2} + 2} + 1 \text{ for } z \in \mathbb{C}$$

$$\cosh^m(z) = 2^{-m} z^2 \left(\sum_{i=0}^{\left\lfloor \frac{1}{2}(-1+m) \right\rfloor} \frac{(-2i+m)^2 \binom{m}{i}}{1 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z^2 \sum_{i=0}^{\left\lfloor \frac{1}{2}(-1+m) \right\rfloor} (-2i+m)^{2+2k} \binom{m}{i}}{2(1+k)(1+2k) \sum_{i=0}^{\left\lfloor \frac{1}{2}(-1+m) \right\rfloor} (-2i+m)^{2k} \binom{m}{i}}}{1 + \frac{z^2 \sum_{i=0}^{\left\lfloor \frac{1}{2}(-1+m) \right\rfloor} (-2i+m)^{2+2k} \binom{m}{i}}{2(1+k)(1+2k) \sum_{i=0}^{\left\lfloor \frac{1}{2}(-1+m) \right\rfloor} (-2i+m)^{2k} \binom{m}{i}}}} \right) + 1 \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m >$$

$$\mathrm{Chi}(z) = \frac{z^2}{4 \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{kz^2}{2(1+k)^2(1+2k)}}{1 + \frac{kz^2}{2(1+k)^2(1+2k)}} + 1 \right)} + \log(z) + \gamma \text{ for } z \in \mathbb{C} \wedge |\arg(z)| < \pi$$

$$\mathrm{Ci}(z) = -\frac{z^2}{4 \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{kz^2}{2(1+k)^2(1+2k)}}{1 - \frac{kz^2}{2(1+k)^2(1+2k)}} + 1 \right)} + \log(z) + \gamma \text{ for } z \in \mathbb{C} \wedge |\arg(z)| < \pi$$

$$\cot(z) = \frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-z^2}{1+2k}}{z} + \frac{1}{z} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\cot(z) = \frac{1}{z} - \frac{4z}{\pi^2 \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2 \left(k^2 - \frac{4z^2}{\pi^2} \right)}{1+2k} + 1 \right)} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\cot(z) = \frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z^2}{-1+4k^2}}{1} + 1}{z} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\cot(z) = \frac{\pi \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2 - \frac{16z^2}{\pi^2}}{2} + 1 \right)}{4z} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\cot(z) = \frac{\frac{\pi}{z} - \frac{4z}{\pi}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2 - \frac{16z^2}{\pi^2}}{6} + 3} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\cot(z) = \frac{1}{z} - \frac{4z}{\pi^2 \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2 \left(k^2 - \frac{4z^2}{\pi^2} \right)}{1+2k} + 1 \right)} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\cot(z)=\frac{z}{2\,\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^2}{4}}{-\frac{3}{2}-k}-3}+\frac{1}{z}\text{ for }\frac{z}{\pi}\notin\mathbb{Z}$$

$$\cot(z)=\frac{1}{z}-\frac{z}{3\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{2z^2B_{2(1+k)}}{(1+k)(1+2k)B_{2k}}}{1-\frac{2z^2B_{2(1+k)}}{(1+k)(1+2k)B_{2k}}}+1\right)}\text{ for }\frac{z}{\pi}\notin\mathbb{Z}$$

$$\cot(z)=\frac{1}{z}-\frac{z}{3\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^2\zeta(2(1+k))}{\pi^2\zeta(2k)}}{1+\frac{z^2\zeta(2(1+k))}{\pi^2\zeta(2k)}}+1\right)}\text{ for }z\in\mathbb{C}\wedge\frac{z}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{\mathrm{K}_{k=1}^{\infty}\frac{z^2}{1+2k}}{z}+\frac{1}{z}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{4z}{\pi^2\left(\mathrm{K}_{k=1}^{\infty}\frac{k^2\left(k^2+\frac{4z^2}{\pi^2}\right)}{1+2k}+1\right)}+\frac{1}{z}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{-1+4k^2}}{1}+1}{z}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{\pi\left(\mathrm{K}_{k=1}^{\infty}\frac{(-1+2k)^2+\frac{16z^2}{\pi^2}}{2}+1\right)}{4z}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{\frac{4z}{\pi}+\frac{\pi}{z}}{\mathrm{K}_{k=1}^{\infty}\frac{(-1+2k)^2+\frac{16z^2}{\pi^2}}{6}+3}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{4z}{\pi^2\left(\mathrm{K}_{k=1}^{\infty}\frac{k^4+\frac{4k^2z^2}{\pi^2}}{1+2k}+1\right)}+\frac{1}{z}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{1}{z}-\frac{z}{2\,\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{4}}{-\frac{3}{2}-k}-3}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth(z)=\frac{z}{3\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{2z^2B_{2(1+k)}}{(1+k)(1+2k)B_{2k}}}{1+\frac{2z^2B_{2(1+k)}}{(1+k)(1+2k)B_{2k}}}+1\right)}+\frac{1}{z}\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth\left(\frac{1}{z}\right)=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{1}{(1+2k)z}+z\text{ for }\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\coth\left(\frac{\pi z}{2}\right) = \frac{2\left(\frac{z^2}{\mathrm{K}_{k=1}^{\infty} \frac{k^2(k^2+z^2)}{1+2k} + 1} + 1\right)}{\pi z} \text{ for } \frac{\pi z}{2} \notin \mathbb{Z}$$

$$\coth(z) = \frac{z}{3\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2\zeta(2(1+k))}{\pi^2\zeta(2k)}}{1-\frac{z^2\zeta(2(1+k))}{\pi^2\zeta(2k)}} + 1\right)} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\csc(z) = \frac{z}{6\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2(1+k)(3+2k)}}{1-\frac{z^2}{2(1+k)(3+2k)}} - \frac{z^2}{6} + 1\right)} + \frac{1}{z} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\csc(z) = \frac{1}{\pi\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1-(-1)^k+k}{2+k} + \frac{\left(-1+3(-1)^k+2(-1)^k k\right)z}{(1+k)(2+k)\pi}}{\frac{1+(-1)^k}{2+k} + \frac{\left(1-3(-1)^k-2(-1)^k k\right)z}{(1+k)(2+k)\pi}} - \frac{z}{\pi} + 1\right)} + \frac{1}{z} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\csc(z) = \frac{1}{\pi\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{8}(1+2k+2k^2-(-1)^k(1+2k)) + \frac{\left(-1+(-1)^k+2(-1)^k k\right)z}{4\pi}}{\frac{1}{2}\left(1+(-1)^k - \frac{2(-1)^k z}{\pi}\right)} - \frac{z}{\pi} + 1\right)} + \frac{1}{z} \text{ for } \frac{z}{\pi} \notin \mathbb{Z}$$

$$\csc(z) = \frac{z}{6\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{\left(-1+2^{1+2k}\right)z^2\zeta(2(1+k))}{2\left(-2+4^k\right)\pi^2\zeta(2k)}}{\frac{1}{4}\left(4+\frac{\left(-2+4^{1+k}\right)z^2\zeta(2(1+k))}{\left(-2+4^k\right)\pi^2\zeta(2k)}\right)} + 1\right)} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \frac{z}{\pi} \notin \mathbb{Z}$$

$$\operatorname{csch}(z) = \frac{1}{z} - \frac{z}{6\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z^2}{2(1+k)(3+2k)}}{1+\frac{z^2}{2(1+k)(3+2k)}} + \frac{z^2}{6} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\operatorname{csch}(z) = \frac{1}{z} + \frac{i}{\pi\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1-(-1)^k+k}{2+k} + \frac{i\left(-1+3(-1)^k+2(-1)^k k\right)z}{(1+k)(2+k)\pi}}{\frac{1+(-1)^k}{2+k} + \frac{i\left(1-3(-1)^k-2(-1)^k k\right)z}{(1+k)(2+k)\pi}} - \frac{iz}{\pi} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\operatorname{csch}(z) = \frac{1}{z} + \frac{i}{\pi\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{8}(1+2k+2k^2-(-1)^k(1+2k)) + \frac{i\left(-1+(-1)^k+2(-1)^k k\right)z}{4\pi}}{\frac{1}{2}\left(1+(-1)^k - \frac{2i(-1)^k z}{\pi}\right)} - \frac{iz}{\pi} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\operatorname{csch}(z) = \frac{e^z \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left(-1-(-1)^k+2(-1)^k(1+k)\right)z}{2k(1+k)}}{1} + 1 \right)}{z} \text{ for } z \in \mathbb{C} \land \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\operatorname{csch}(z) = \frac{1}{z} - \frac{z}{6 \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left(-1+2^{1+2k}\right)z^2\zeta(2(1+k))}{2\left(-2+4^k\right)\pi^2\zeta(2k)}}{1-\frac{\left(-2+4^{1+k}\right)z^2\zeta(2(1+k))}{4\left(-2+4^k\right)\pi^2\zeta(2k)}} + 1 \right)} \text{ for } z \in \mathbb{C} \land \frac{iz}{\pi} \notin \mathbb{Z}$$

$$F(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{2(-1)^k k z^2}{-1+4k^2}}{1} + 1} \text{ for } z \in \mathbb{C}$$

$$F(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{4kz^2}{-1+4k^2}}{1+\frac{2z^2}{1+2k}} + 2z^2 + 1} \text{ for } z \in \mathbb{C}$$

$$F(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{2z^2}{1+2k}}{1-\frac{2z^2}{1+2k}} + 1} \text{ for } z \in \mathbb{C}$$

$$e = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{\frac{1}{250}(23+36k)(k \bmod 5) + \frac{3}{125}(3+16k)((1+k) \bmod 5) + \frac{1}{125}(49+108k)((2+k) \bmod 5) - \frac{6}{125}(11 +}$$

$$e = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{\begin{cases} \frac{2(1+k)}{3} & (k \bmod 3) = 2 \\ 1 & \text{(otherwise)} \end{cases}} + 2$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^{1+k}}{1+(-1)^{1+k}-\frac{1}{2}(-1+(-1)^{1+k})(1+k)}} + 1$$

$$e = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^{-1+k}}{1+(-1)^k+\frac{1}{2}\left(1-(-1)^k\right)k} + 1$$

$$e = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{2(1+2k)} + 1} + 1$$

$$e = \frac{1}{1 - \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{2+4k} + 3}}$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1-(-1)^k)+\frac{1}{4}(1+(-1)^k)(2+k)}{1} + 1} + 2$$

$$e = \frac{2}{\frac{1}{6\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4(1+2k)(3+2k)}}{1}+1\right)} + 1} + 1$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{-1+(-1)^k(1+2k)}{4k(1+k)}}{1} + 1} + 1$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{k}} + 1$$

$$e = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1+k}{1+k} + 2$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{1+k} + 1} + 2$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k}{2+k} + 1} + 1$$

$$e = \frac{1}{1 - \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k}{2+k}+2}}$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-1-k}{3+k} + 2} + 2$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}}{1+2k} + \frac{1}{2}} + 1$$

$$e = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{-1+4k^2}}{2} + 1} + 1$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{k}}{1+\frac{1}{k}} + 1}$$

$$e = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{1+k}}{1+\frac{1}{1+k}} + 1} + 1$$

$$\frac{1}{e-2} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{1+k} + 1$$

$$\frac{e}{e-2} = 2 \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{1+k} + 3$$

$$\frac{1}{e-1} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{k}{k}$$

$$1 - \frac{1}{e} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{k}{k} + 1}$$

$$\frac{e}{e-1} = 2 - \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{-1-k}{3+k} + 3}$$

$$\frac{e}{e-1} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)+\frac{1}{4}(1-(-1)^k)(1+k)}{1} + 1} + 1$$

$$\frac{1+e}{e-1} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1}{2(1+2k)} + 2$$

$$e^2 = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1}{\frac{1}{5}\left(\left\{ \begin{array}{cc} 30+12k & (k \bmod 5) = 0 \\ 7+3k & (k \bmod 5) = 1 \\ 3+3k & (k \bmod 5) = 4 \\ 5 & (\text{otherwise}) \end{array} \right. \right)} + 7$$

$$e^2 = \frac{2}{\mathrm{K}_{k=1}^{\infty} \frac{1}{5+2k} + 5} + 7$$

$$\frac{e^2-1}{1+e^2} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{1}{1+2k} + 1}$$

$$\frac{1+e^2}{e^2-1} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1}{1+2k} + 1$$

$$\sqrt{e} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{1}{1+\left(\left\{ \begin{array}{cc} \frac{4k}{3} & (k \bmod 3) = 0 \\ 0 & (\text{otherwise}) \end{array} \right. \right)} + 1} + 1$$

$$\frac{1}{\sqrt{e}-1} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1}{\frac{1}{27}((9-8k)(k \bmod 3) + (9+4k)((1+k) \bmod 3) + (9+16k)((2+k) \bmod 3))} + 1$$

$$\sqrt[3]{e} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{1}{\frac{1}{9}(2(1+k)(k \bmod 3) + (-1+8k)((1+k) \bmod 3) + (5-4k)((2+k) \bmod 3))} + 1$$

$$E(z) = \frac{\pi}{2\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{(-3-4(-2+k)k)z}{4k^2}}{1+\frac{(3+4(-2+k)k)z}{4k^2}} + 1 \right)} \quad \text{for } z \in \mathbb{C} \wedge |z| < 1$$

$$E(1-z) = -\frac{z\log(z)}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{\left(-1+4k^2\right)z}{4k(1+k)}}{1+\frac{\left(-1+4k^2\right)z}{4k(1+k)}}+1\right)} - \frac{z\left(1+2\gamma+2\psi^{(0)}\left(\frac{1}{2}\right)\right)}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1-2k)^2z\left(-1-2(1+k)(1+2k)\psi^{(0)}\left(\frac{1}{2}+k\right)+2(1+k)(1+2k)\psi^{(0)}(1+k)\right)}{4(1+k)^2\left(1+2k(-1+2k)\left(\psi^{(0)}\left(-\frac{1}{2}+k\right)-\psi^{(0)}(k)\right)\right)}}{1-\frac{(1-2k)^2z\left(-1-2(1+k)(1+2k)\psi^{(0)}\left(\frac{1}{2}+k\right)+2(1+k)(1+2k)\psi^{(0)}(1+k)\right)}{4(1+k)^2\left(1+2k(-1+2k)\left(\psi^{(0)}\left(-\frac{1}{2}+k\right)-\psi^{(0)}(k)\right)\right)}} \cdots$$

$$E(z) = \frac{\log(-z)}{4\sqrt{-z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2}{4k(1+k)z}}{1+\frac{(1-2k)^2}{4k(1+k)z}}+1\right)} + \frac{1+4\log(2)}{4\sqrt{-z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2\left(-1+2(1+k)\psi^{(0)}\left(\frac{1}{2}+k\right)-2(1+k)\psi^{(0)}(1+k)\right)}{4(1+k)^2z\left(-1+2k\psi^{(0)}\left(-\frac{1}{2}+k\right)-2k\psi^{(0)}(k)\right)}}{1+\frac{(1-2k)^2\left(-1+2(1+k)\psi^{(0)}\left(\frac{1}{2}+k\right)-2(1+k)\psi^{(0)}(1+k)\right)}{4(1+k)^2z\left(-1+2k\psi^{(0)}\left(-\frac{1}{2}+k\right)-2k\psi^{(0)}(k)\right)}}+1\right)} + \cdots$$

$$K(z) = \frac{\pi}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2z}{4k^2}}{1+\frac{(1-2k)^2z}{4k^2}}+1\right)} \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$K(1-z) = \frac{2\log(2)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1-2k)^2z\left(-\psi^{(0)}\left(\frac{1}{2}+k\right)+\psi^{(0)}(1+k)\right)}{4k^2\left(\psi^{(0)}\left(-\frac{1}{2}+k\right)-\psi^{(0)}(k)\right)}}{1-\frac{(1-2k)^2z\left(-\psi^{(0)}\left(\frac{1}{2}+k\right)+\psi^{(0)}(1+k)\right)}{4k^2\left(\psi^{(0)}\left(-\frac{1}{2}+k\right)-\psi^{(0)}(k)\right)}}+1} - \frac{\log(z)}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2z}{4k^2}}{1+\frac{(1-2k)^2z}{4k^2}}+1\right)} \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$K(z) = \frac{\log(-z)}{2\sqrt{-z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2}{4k^2z}}{1+\frac{(1-2k)^2}{4k^2z}}+1\right)} + \frac{2\log(2)}{\sqrt{-z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)^2\left(\psi^{(0)}\left(\frac{1}{2}+k\right)-\psi^{(0)}(1+k)\right)}{4k^2z\left(\psi^{(0)}\left(-\frac{1}{2}+k\right)-\psi^{(0)}(k)\right)}}{1+\frac{(1-2k)^2\left(\psi^{(0)}\left(\frac{1}{2}+k\right)-\psi^{(0)}(1+k)\right)}{4k^2z\left(\psi^{(0)}\left(-\frac{1}{2}+k\right)-\psi^{(0)}(k)\right)}}+1\right)} \text{ for } z\in\mathbb{C}\wedge|z|>$$

$$\vartheta_2\left(0,\sqrt{z}\right) = \frac{2\sqrt[8]{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{2}(1-(-1)^k)z^k-\frac{1}{2}(1+(-1)^k)\left(-z^{k/2}+z^k\right)}{1}+1} \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$\vartheta_2\left(0,\sqrt{z}\right) = 2\sqrt[8]{z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{2}\left(1+(-1)^k\right)z+\frac{1}{2}\left(1-(-1)^k\right)z^{\frac{1+k}{2}}}{\frac{1}{2}\left(1-(-1)^k\right)(1-z)+\frac{1}{2}\left(1+(-1)^k\right)\left(1+z^{k/2}\right)}+1\right) \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$\vartheta_2\left(0,\sqrt{z}\right) = \frac{2\sqrt[8]{z}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{2}(1-(-1)^k)z-\frac{1}{2}(1+(-1)^k)z^{\frac{2+k}{2}}}{\frac{1}{2}(1-(-1)^k)(1+z)+\frac{1}{2}(1+(-1)^k)\left(1+z^{k/2}\right)}+1} \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$\vartheta_2\left(0,\sqrt{z}\right) = \frac{2\sqrt[8]{z}(z+1)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-z^{2+k}}{1+z^k}+1} \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$\operatorname{erf}(z) = 1 - \frac{e^{-z^2}}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k}{2}}{z} + z\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = 1 - \frac{2e^{-z^2}}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{2k}{2z} + 2z\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = 1 - \frac{\sqrt{\frac{2}{\pi}}e^{-z^2}}{\operatorname*{K}_{k=1}^{\infty} \frac{k}{\sqrt{2}z} + \sqrt{2}z} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = 1 - \frac{2e^{-z^2}}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{k}{\frac{1}{2}(3+(-1)^k)z} + 2z\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = 1 - \frac{e^{-z^2}z}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k}{2}}{z^{1+(-1)^k}} + z^2\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = 1 - \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{k}{\frac{1}{2}(1-(-1)^k)+(1+(-1)^k)z^2} + 2z^2\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{2(-1)^k k z^2}{-1+4k^2}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{2(-1)^k k z^2}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{4kz^2}{1+2k-2z^2} - 2z^2 + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{4kz^2}{-1+4k^2}}{1-\frac{2z^2}{1+2k}} - 2z^2 + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = 1 - \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-2k(-1+2k)}{1+4k+2z^2} + 2z^2 + 1\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = \frac{e^{-z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{kz^2}{\frac{1}{2}+k-z^2} - z^2 + \frac{1}{2}\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erf}(z) = \frac{2z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{(-1+2k)z^2}{k(1+2k)}}{1-\frac{(-1+2k)z^2}{k(1+2k)}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{e^{-z^2}}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{k}{2}}{z} + z\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{2e^{-z^2}}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{2k}{2z} + 2z\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{\sqrt{\frac{2}{\pi}}e^{-z^2}}{\mathrm{K}_{k=1}^{\infty}\frac{k}{\sqrt{2}z} + \sqrt{2}z} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{2e^{-z^2}}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{k}{\frac{1}{2}(3+(-1)^k)z} + 2z\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{e^{-z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{k}{2}}{z^{1+(-1)^k}} + z^2\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{k}{\frac{1}{2}(1-(-1)^k)+(1+(-1)^k)z^2} + 2z^2\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = 1 - \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{2(-1)^k kz^2}{-1+4k^2}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = 1 - \frac{2e^{-z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{2(-1)^k kz^2}{1+2k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = 1 - \frac{2e^{-z^2} z}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{4kz^2}{1+2k-2z^2} - 2z^2 + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = 1 - \frac{2e^{-z^2} z}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{4kz^2}{-1+4k^2}}{1-\frac{2z^2}{1+2k}} - 2z^2 + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = \frac{2e^{-z^2} z}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-2k(-1+2k)}{1+4k+2z^2} + 2z^2 + 1\right)} - \frac{z}{\sqrt{z^2}} + 1 \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = 1 - \frac{e^{-z^2} z}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{kz^2}{\frac{1}{2}+k-z^2} - z^2 + \frac{1}{2}\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfc}(z) = 1 - \frac{2z}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(-1+2k)z^2}{k(1+2k)}}{1-\frac{(-1+2k)z^2}{k(1+2k)}} + 1\right)} \text{ for } z \in \mathbb{C} \land -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\operatorname{erfi}(z) = \frac{ie^{z^2}}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k}{2}}{iz} + iz\right)} + i \text{ for } z \in \mathbb{C} \land 0 < \arg(z) \leq \pi$$

$$\operatorname{erfi}(z) = \frac{2ie^{z^2}}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2k}{2iz} + 2iz\right)} - i \text{ for } z \in \mathbb{C} \land -\pi < \arg(z) \leq 0$$

$$\operatorname{erfi}(z) = \frac{i\sqrt{\frac{2}{\pi}}e^{z^2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{i\sqrt{2}z} + i\sqrt{2}z} + i \text{ for } z \in \mathbb{C} \land 0 < \arg(z) \leq \pi$$

$$\operatorname{erfi}(z) = \frac{2ie^{z^2}}{\sqrt{\pi}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k}{\frac{1}{2}i(3+(-1)^k)z} + 2iz\right)} + i \text{ for } z \in \mathbb{C} \land 0 < \arg(z) \leq \pi$$

$$\operatorname{erfi}(z) = i - \frac{e^{z^2} z}{\sqrt{\pi}\left(-z^2 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{k}{2}}{(iz)^{1+(-1)^k}}\right)} \text{ for } z \in \mathbb{C} \land 0 < \arg(z) \leq \pi$$

$$\operatorname{erfi}(z) = -\frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{k}{\frac{1}{2}(1-(-1)^k)+(-1-(-1)^k)z^2}-2z^2\right)}+i \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = \frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{2(-1)^k kz^2}{-1+4k^2}}{1}+1\right)} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = \frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{2(-1)^{-1+k}kz^2}{1+2k}+1\right)} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = \frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-4kz^2}{1+2k+2z^2}+2z^2+1\right)} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = -\frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-2k(-1+2k)}{1+4k-2z^2}-2z^2+1\right)}-\frac{\sqrt{-z}}{\sqrt{z}} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = -\frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-2k(-1+2k)}{1+4k-2z^2}-2z^2+1\right)}+i \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = \frac{2e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{4kz^2}{-1+4k^2}}{1+\frac{2z^2}{1+2k}}+2z^2+1\right)} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = \frac{e^{z^2}z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-kz^2}{\frac{1}{2}+k+z^2}+z^2+\frac{1}{2}\right)} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\operatorname{erfi}(z) = \frac{2z}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{(-1+2k)z^2}{k(1+2k)}}{1+\frac{(-1+2k)z^2}{k(1+2k)}}+1\right)} \text{ for } z\in\mathbb{C}\wedge 0<\arg(z)\leq\pi$$

$$\gamma = \frac{\pi^2}{12\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{(1+k)\zeta(2+k)}{(2+k)\zeta(1+k)}}{1-\frac{(1+k)\zeta(2+k)}{(2+k)\zeta(1+k)}}+1\right)}$$

$$\gamma = \frac{1}{2\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(1+k)\log(2+k)}{(2+k)\log(1+k)}}{1-\frac{(1+k)\log(2+k)}{(2+k)\log(1+k)}} + 1\right)} + \frac{\log(2)}{2}$$

$$\gamma = \log(2) - \frac{\zeta(3)}{12\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(1+2k)\zeta(3+2k)}{4(3+2k)\zeta(1+2k)}}{1+\frac{(1+2k)\zeta(3+2k)}{4(3+2k)\zeta(1+2k)}} + 1\right)}$$

$$e^z = \frac{1}{\operatorname*{K}_{k=1}^{\infty} \frac{(-1)^k z}{1+(-1)^k+\frac{1}{2}(1-(-1)^k)k} + 1} \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{1}{1 - \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{(-1)^{-1+k} z^{\left\lfloor \frac{1+k}{2} \right\rfloor}}{1+k} + 1}} \text{ for } z \in \mathbb{C}$$

$$e^z = \operatorname*{K}_{k=1}^{\infty} \frac{(-1)^{-1+k} z}{1 + (-1)^k + \frac{1}{2}\left(1 - (-1)^k\right) k} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{2z}{\operatorname*{K}_{k=1}^{\infty} \frac{z^2}{2(1+2k)} - z + 2} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{2z}{\frac{z^2}{6\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4(1+2k)(3+2k)}}{1} + 1\right)} - z + 2} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\left(-1+(-1)^k(1+2k)\right)z}{4k(1+k)}}{1} + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{kz}{1+k-z} - z + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{1}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z}{k}}{1+\frac{z}{k}} + 1} \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{1}{1 - \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{-kz}{1+k+z} + z + 1}} \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{-kz}{1+k+z} + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z}{1+k}}{1+\frac{z}{1+k}} + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$e^z = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4}}{1+2k} - \frac{z}{2} + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$e^{\sqrt{z}} = \frac{2\sqrt{z}}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z}{-1+4k^2}}{2} - \sqrt{z} + 2} + 1 \text{ for } z \in \mathbb{C}$$

$$e^{\frac{2z}{y}} = \frac{2z}{\operatorname*{K}_{k=1}^{\infty} \frac{z^2}{(1+2k)y} + y - z} + 1 \text{ for } (y,z) \in \mathbb{C}^2$$

$$e^{\frac{1}{z}} = \operatorname*{K}_{k=1}^{\infty} \frac{1}{\frac{1}{9}\left(1 + z\left(-1 + 2\left\lfloor\frac{2+k}{3}\right\rfloor\right)\right)(k \bmod 3) + \frac{1}{9}\left(-1 + 4\left(-1 + z\left(-1 + 2\left\lfloor\frac{2+k}{3}\right\rfloor\right)\right)\right)((1+k) \bmod 3) + \frac{1}{9}(5}$$

$$e^{\frac{1}{m}} = \frac{m}{\frac{1}{\operatorname*{K}_{k=1}^{\infty} \frac{1}{\frac{1}{27}((17+8k-12m)(k \bmod 3)+(5-4k+6m)((1+k) \bmod 3)+2(-8+k+12m)((2+k) \bmod 3))} + 2m} + m - 1} \text{ for } m \in \mathbb{Z} \land m > 0$$

$$e^{\frac{1}{m}} = \operatorname*{K}_{k=1}^{\infty} \frac{1}{\frac{1}{27}(3 + (1+2k)m)(k \bmod 3) + \frac{1}{27}(-15 + 4(1+2k)m)((1+k) \bmod 3) - \frac{1}{27}(-21 + 2(1+2k)m)((}$$

$$e^{\frac{1}{m}} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{1}{\frac{1}{27}((-1+8k+6m)(k \bmod 3)+(-13-4k+24m)((1+k) \bmod 3)+2(10+k-6m)((2+k) \bmod 3))}}{m} + \frac{m+1}{m} \text{ for } m \in \mathbb{Z}/$$

$$e^{2/m} = \operatorname*{K}_{k=1}^{\infty} \frac{}{\frac{1}{250}(5 + 9(1+2k)m)(k \bmod 5) + \frac{1}{125}(-35 + 2(11+12k)m)((1+k) \bmod 5) + \frac{1}{125}(15 + (17 + 54}$$

$$e^{\frac{2p}{m}} = 1 - \frac{2p}{-\operatorname*{K}_{k=1}^{\infty} \frac{p^2}{m+2km} - m + p} \text{ for } (m,p) \in \mathbb{Z}^2 \land m > 1 \land p > 0$$

$$e^{2\alpha \tan^{-1}\left(\frac{1}{z}\right)} = \frac{2\alpha}{\operatorname*{K}_{k=1}^{\infty} \frac{k^2+\alpha^2}{(1+2k)z} - \alpha + z} + 1 \text{ for } (\alpha, z) \in \mathbb{C}^2$$

$$\frac{e^z - 1}{e^z + 1} = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{z^2}{2(1+2k)} + 2} \text{ for } z \in \mathbb{C}$$

$$\frac{e^z - 1}{e^z + 1} = \operatorname*{K}_{k=1}^{\infty} \frac{1}{\frac{-2+4k}{z}} \text{ for } z \in \mathbb{C}$$

$$\frac{e^z - e^{-z}}{e^{-z} + e^z} = \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{z^2}{1+2k} + 1} \text{ for } z \in \mathbb{C}$$

$$\frac{e^{\frac{2p}{m}} - 1}{e^{\frac{2p}{m}} + 1} = \frac{p}{\operatorname*{K}_{k=1}^{\infty} \frac{p^2}{(1+2k)m} + m} \text{ for } (m,p) \in \mathbb{Z}^2 \land m > 1 \land p > 0$$

$$E_\nu(z) = \Gamma(1-\nu) z^{\nu-1} - \frac{e^{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^k z \left((1-\nu)^{\frac{1}{2}\left(1-(-1)^k\right)} + \left\lfloor \frac{1}{2}(-1+k) \right\rfloor\right)}{1+k-\nu} - \nu + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = \frac{e^{-z}}{z \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(\frac{1}{2}(-1+k)+\nu\right)}{\frac{z}{1}} + 1 \right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = e^{-z} \left(\frac{z^{r-1}}{(1-\nu)_r \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(\frac{1}{2}(-1+k)-r+\nu\right)}{\frac{z}{1}} + 1 \right)} - \sum_{k=0}^{-1+r} \frac{z^k}{(1-\nu)_{1+k}} \right) \text{ for } r \in \mathbb{Z} \wedge (\nu$$

$$E_\nu(z) = e^{-z} \left(\frac{(-1)^r z^{-r-1} (\nu)_r}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(\frac{1}{2}(-1+k)+r+\nu\right)}{\frac{z}{1}} + 1} + \frac{\sum_{k=0}^{-1+r} (-1)^k z^{-k} (\nu)_k}{z} \right) \text{ for } r \in \mathbb{Z} \wedge (\nu, z) \in$$

$$E_\nu(z) = \Gamma(1-\nu) z^{\nu-1} - \frac{e^{-z}}{(1-\nu) \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z \left(\frac{\left(1+(-1)^k\right)k}{4(k-\nu)(1+k-\nu)} - \frac{\left(1-(-1)^k\right)\left(\frac{1+k}{2}-\nu\right)}{2(k-\nu)(1+k-\nu)} \right)}{1} + 1 \right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z$$

$$E_\nu(z) = \Gamma(1-\nu) z^{\nu-1} - e^{-z} \left(\frac{z^r}{(1-\nu)_{r+1} \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z \left(\frac{\left(1+(-1)^k\right)k}{4(k+r-\nu)(1+k+r-\nu)} - \frac{\left(1-(-1)^k\right)\left(\frac{3+k}{2}-\nu\right)}{2(k+r-\nu)(1+k+r-\nu)} \right)}{1} + 1 \right)} + \sum_{k=0}^{-1+r} \text{[illegible]} \right.$$

$$E_\nu(z) = \Gamma(1-\nu) z^{\nu-1} - e^{-z} \left(\sum_{k=1}^{r} (-z)^{-k} (\nu)_{-1+k} - \frac{(-1)^r z^{-r} (\nu)_{r-1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z \left(\frac{\left(1+(-1)^k\right)k}{4(k-r-\nu)(1+k-r-\nu)} - \frac{\left(1-(-1)^k\right)\left(\frac{1+k}{2}-r-\nu\right)}{2(k-r-\nu)(1+k-r-\nu)} \right)}{1} + 1} \right) \text{ fo}$$

$$E_\nu(z) = \frac{e^{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}(1+(-1)^k)k+\frac{1}{2}(1-(-1)^k)\left(\frac{1}{2}(-1+k)+\nu\right)}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)z} + z} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = \frac{e^{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k(-1+k+\nu)}{2k+z+\nu} + \nu + z} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = \frac{e^{-z}\left(1 - \frac{\nu}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k(k+\nu)}{1+2k+z+\nu} + \nu + z + 1}\right)}{z} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = \Gamma(1-\nu)z^{\nu-1} - \frac{e^{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z(-k+\nu)}{1+k+z-\nu} - \nu + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = \Gamma(1-\nu)z^{\nu-1} - \frac{e^{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{kz}{1+k-z-\nu} - \nu - z + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_\nu(z) = \Gamma(1-\nu)z^{\nu-1} - e^{-z}\left(\frac{z^r}{(1-\nu)_r\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{kz}{1+k+r-z-\nu} - \nu + r - z + 1\right)} + \sum_{k=0}^{-1+r} \frac{z^k}{(1-\nu)_{1+k}}\right) \text{ for } r \in \mathbb{Z}\wedge$$

$$E_\nu(z) = \Gamma(1-\nu)z^{\nu-1} - e^{-z}\left(\frac{(-1)^r z^{-r}(\nu)_r}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{kz}{1+k-r-z-\nu} - \nu - r - z + 1} + \sum_{k=1}^{r}(-z)^{-k}(\nu)_{-1+k}\right) \text{ for } r \in \mathbb{Z} \wedge (\nu, z) \in \mathbb{C}$$

$$E_{-z}(z) = \frac{e^{-z}\left(\frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(-k+z)}{1+2k} + 1} + 1\right)}{z} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_{-z}(z) = \frac{e^{-z}\left(\frac{z-1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(-1-k+z)}{2+2k} + 2} + 2\right)}{z} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_{-z}(z) = z^{-z-1}\Gamma(z+1) - \frac{2e^{-z}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(2+k)z}{2+k} + 2} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$E_{\nu}(z)=\Gamma(1-\nu)z^{\nu-1}-\frac{1}{(1-\nu)\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z(k-\nu)}{k(1+k-\nu)}}{1+\frac{z(-k+\nu)}{k(1+k-\nu)}}+1\right)}\text{ for }(\nu,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{Z}\wedge z<0)$$

$$E_m(z)=-\frac{(-z)^m}{m!\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{kz}{(1+k)(k+m)}}{1-\frac{kz}{(1+k)(k+m)}}+1\right)}+\frac{(-z)^{m-1}(\psi^{(0)}(m)-\log(z))}{(m-1)!}-\frac{1}{(1-m)\left(\mathrm{K}_{k=1}^{-2+m}\frac{\frac{(k-m)z}{k(1+k-m)}}{1-\frac{(k-m)z}{k(1+k-m)}}+\right.}$$

$$\mathrm{Ei}(z)=\frac{e^z}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{-\left\lfloor\frac{1+k}{2}\right\rfloor}{z}}{1}+1\right)}+\frac{1}{2}\left(-\log\left(\frac{1}{z}\right)-2\log(-z)+\log(z)\right)\text{ for }z\in\mathbb{C}\wedge|\arg(z)|<\pi$$

$$\mathrm{Ei}(z)=2\mathrm{Shi}(z)-\frac{e^{-z}}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{\left\lfloor\frac{1+k}{2}\right\rfloor}{z}}{1}+1\right)}\text{ for }z\in\mathbb{C}\wedge|\arg(z)|<\pi$$

$$\mathrm{Ei}(z)=e^z\left(\frac{r!z^{-r-1}}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{-\frac{1}{4}\left(1+(-1)^k\right)k-\frac{1}{2}\left(1-(-1)^k\right)\left(\frac{1+k}{2}+r\right)}{z}}{1}+1}+\sum_{k=1}^{r}z^{-k}(-1+k)!\right)+i\pi\mathrm{sgn}(\Im(z))\text{ for }r\in\mathbb{Z}\wedge z\in\mathbb{C}$$

$$\mathrm{Ei}(z)=-\frac{e^z}{\mathrm{K}_{k=1}^{\infty}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor}{(-z)^{\frac{1}{2}\left(1+(-1)^k\right)}}-z}+i\pi\mathrm{sgn}(\Im(z))\text{ for }z\in\mathbb{C}\wedge|\arg(-z)|<\pi$$

$$\mathrm{Ei}(z)=-\frac{e^z}{\mathrm{K}_{k=1}^{\infty}\frac{-k^2}{1+2k-z}-z+1}+i\pi\mathrm{sgn}(\Im(z))\text{ for }z\in\mathbb{C}\wedge|\arg(-z)|<\pi$$

$$\mathrm{Ei}(z)=\frac{e^z\left(1-\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-k(1+k)}{2+2k-z}-z+2}\right)}{z}+i\pi\mathrm{sgn}(\Im(z))\text{ for }z\in\mathbb{C}\wedge|\arg(-z)|<\pi$$

$$\mathrm{Ei}(z)=-e^z\left(\frac{r!z^{-r}}{\mathrm{K}_{k=1}^{\infty}\frac{-k(k+r)}{1+2k+r-z}+r-z+1}-\sum_{k=0}^{-1+r}z^{-1-k}k!\right)+i\pi\mathrm{sgn}(\Im(z))\text{ for }r\in\mathbb{Z}\wedge z\in\mathbb{C}\wedge\neg(z\in\mathbb{R}\wedge z>$$

$$\operatorname{Ei}(z)=\frac{z}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{kz}{(1+k)^2}}{1+\frac{kz}{(1+k)^2}}+1}+\frac{1}{2}\left(\log(z)-\log\left(\frac{1}{z}\right)\right)+\gamma \text{ for } z\in\mathbb{C}\wedge|\arg(z)|<\pi$$

$$\frac{((2z)!!)^2}{((2z-1)!!)^2}=\pi z\left(\frac{2}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1+4k^2}{8z}+8z-1}+1\right) \text{ for } z\in\mathbb{Z}\wedge z>0$$

$$\frac{((2z-1)!!)^2}{((2z)!!)^2}=\frac{(2z-1)\left(\frac{2}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1+4k^2}{4(-1+2z)}+8z-5}+1\right)}{2\pi z^2} \text{ for } z\in\mathbb{Z}\wedge z>0$$

$$\frac{((2z)!)^2}{(z!)^4}=\frac{4^{2z+1}}{\pi\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1+2k)^2}{2(1+4z)}+4z+1\right)} \text{ for } z\in\mathbb{C}\wedge|\arg(z)|<\pi$$

$$F_\nu=\frac{2\nu\operatorname{csch}^{-1}(2)}{\sqrt{5}\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{\nu\left(-\left(-i\pi-\operatorname{csch}^{-1}(2)\right)^{1+k}-\left(i\pi-\operatorname{csch}^{-1}(2)\right)^{1+k}+2\operatorname{csch}^{-1}(2)^{1+k}\right)}{(1+k)\left(\left(-i\pi-\operatorname{csch}^{-1}(2)\right)^k+\left(i\pi-\operatorname{csch}^{-1}(2)\right)^k-2\operatorname{csch}^{-1}(2)^k\right)}}{1-\frac{\nu\left(-\left(-i\pi-\operatorname{csch}^{-1}(2)\right)^{1+k}-\left(i\pi-\operatorname{csch}^{-1}(2)\right)^{1+k}+2\operatorname{csch}^{-1}(2)^{1+k}\right)}{(1+k)\left(\left(-i\pi-\operatorname{csch}^{-1}(2)\right)^k+\left(i\pi-\operatorname{csch}^{-1}(2)\right)^k-2\operatorname{csch}^{-1}(2)^k\right)}}\right)} \text{ for } \nu\in\mathbb{C}$$

$$F_\nu(z)=\frac{2\nu\log\left(\frac{1}{2}\left(\sqrt{z^2+4}+z\right)\right)}{\sqrt{z^2+4}\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{\nu k!\left(1+\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}\right)\right)\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\cdots\right.}{(1+k)!\left(1-\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^k+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^k\right)\right)}}{1+\frac{\nu k!\left(1+\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}\right)\right)\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\cdots\right.}{(1+k)!\left(1-\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^k+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^k\right)\right)}}\right.}\cdots$$

$$F_v(z)=\frac{\sin^2\left(\frac{\pi\text{CalculateData`Private`nu}}{2}\right)}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{z\Gamma\left(\frac{1}{2}(1+k-\text{CalculateData`Private`nu})\right)\Gamma\left(\frac{1}{2}(1+k+\text{CalculateData`Private`nu})\right)\tan\left(\frac{1}{2}(k-\text{CalculateData`Private`nu})\pi\right)}{k\Gamma\left(\frac{k-\text{CalculateData`Private`nu}}{2}\right)\Gamma\left(\frac{k+\text{CalculateData`Private`nu}}{2}\right)}}{1+\frac{z\Gamma\left(\frac{1}{2}(1+k-\text{CalculateData`Private`nu})\right)\Gamma\left(\frac{1}{2}(1+k+\text{CalculateData`Private`nu})\right)\tan\left(\frac{1}{2}(k-\text{CalculateData`Private`nu})\pi\right)}{k\Gamma\left(\frac{k-\text{CalculateData`Private`nu}}{2}\right)\Gamma\left(\frac{k+\text{CalculateData`Private`nu}}{2}\right)}}+1} \text{ for}\cdots$$

$$C(z)=\frac{z}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{(3-4k)\pi^2z^4}{8k(-1+2k)(1+4k)}}{1+\frac{(3-4k)\pi^2z^4}{8k(-1+2k)(1+4k)}}+1} \text{ for } z\in\mathbb{C}$$

$$C(z) + iS(z) = \frac{e^{\frac{1}{2} i\pi z^2} z}{1 + \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{i\left(1-(-1)^k\right)k\pi}{2\left(-1+4k^2\right)} - \frac{i\left(1+(-1)^k\right)k\pi}{2\left(-1+4k^2\right)}\right) z^2}{1}} \text{ for } z \in \mathbb{C}$$

$$C(z) + iS(z) = \frac{e^{\frac{1}{2} i\pi z^2} z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{2ik\pi z^2}{1-4k^2}}{1+\frac{i\pi z^2}{1+2k}} + i\pi z^2 + 1} \text{ for } z \in \mathbb{C}$$

$$S(z) = \frac{\pi z^3}{6\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(-1+4k)\pi^2 z^4}{8k(1+2k)(3+4k)}}{1-\frac{(-1+4k)\pi^2 z^4}{8k(1+2k)(3+4k)}} + 1\right)} \text{ for } z \in \mathbb{C}$$

$$\Gamma(a,z) = \Gamma(a) - \frac{e^{-z} z^a}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^k z \left(a^{\frac{1}{2}\left(1-(-1)^k\right)} + \left\lfloor \frac{1}{2}(-1+k) \right\rfloor\right)}{a+k} + a} \text{ for } (a,z) \in \mathbb{C}^2 \land \neg(z \in \mathbb{R} \land z < 0)$$

$$\Gamma(a,z) = \frac{e^{-z} z^{a-1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}\right)}{z}}{1} + 1} \text{ for } (a,z) \in \mathbb{C}^2 \land \neg(z \in \mathbb{R} \land z < 0)$$

$$\Gamma(a,z) = e^{-z} z^a \left(\frac{z^{r-1}}{(a)_r \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}-r\right)}{z}}{1} + 1\right)} - \sum_{k=0}^{-1+r} \frac{z^k}{(a)_{1+k}} \right) \text{ for } r \in \mathbb{Z} \land (a,z) \in$$

$$\Gamma(a,z) = e^{-z} z^a \left(\frac{(-1)^r z^{-r-1} (1-a)_r}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}+r\right)}{z}}{1} + 1} - \sum_{k=1}^{r} (-z)^{-k} (1-a)_{-1+k} \right) \text{ for } r \in \mathbb{Z} \land (a,z)$$

$$\Gamma(a,z) = \Gamma(a) - \frac{e^{-z} z^a}{a\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)\right)}{2(-1+a+k)(a+k)} + \frac{\left(1+(-1)^k\right)k}{4(-1+a+k)(a+k)}\right) z}{1} + 1\right)} \text{ for } (a,z) \in \mathbb{C}^2 \land \neg(z \in \mathbb{R} \land z < 0)$$

$$\Gamma(a,z) = \Gamma(a) - e^{-z} z^a \left(\frac{z^r}{(a)_{r+1} \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1+(-1)^k\right)k}{4(-1+a+k+r)(a+k+r)} - \frac{\left(1-(-1)^k\right)\left(a+\frac{1+k}{2}\right)}{2(-1+a+k+r)(a+k+r)}\right) z}{1} + 1\right)} + \sum_{k=0}^{-1+r} \frac{z^k}{(a)_{1+k}} \right) \text{ fo}$$

$$\Gamma(a,z)=\Gamma(a)-e^{-z}z^{a}\left(\sum_{k=1}^{r}(-z)^{-k}(1-a)_{-1+k}-\frac{(-1)^{r}z^{-r}(1-a)_{r-1}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(\frac{\left(1+(-1)^{k}\right)k}{4(-1+a+k-r)(a+k-r)}-\frac{\left(1-(-1)^{k}\right)\left(a+\frac{1}{2}(-1+k)-r\right)}{2(-1+a+k-r)(a+k-r)}\right)z}{1}+1}\right)$$

$$\Gamma(a,z)=\frac{e^{-z}z^{a}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{2^{\frac{1}{2}\left(-1-(-1)^{k}\right)}k^{\frac{1}{2}\left(1+(-1)^{k}\right)}\left(-a+\frac{1+k}{2}\right)^{\frac{1}{2}\left(1-(-1)^{k}\right)}}{z^{\frac{1}{2}\left(1+(-1)^{k}\right)}}+z}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,z)=\frac{e^{-z}z^{a}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k(-a+k)}{1-a+2k+z}-a+z+1}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,z)=e^{-z}z^{a-1}\left(\frac{a-1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k(1-a+k)}{2-a+2k+z}-a+z+2}+1\right)\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,z)=\Gamma(a)-\frac{e^{-z}z^{a}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-z}+a-z}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,z)=\Gamma(a)-\frac{e^{-z}z^{a}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(1-a-k)z}{a+k+z}+a}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,z)=\Gamma(a)-e^{-z}z^{a}\left(\frac{z^{r}}{(a)_{r}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k+r-z}+a+r-z\right)}+\sum_{k=0}^{-1+r}\frac{z^{k}}{(a)_{1+k}}\right)\text{ for }r\in\mathbb{Z}\land(a,z)\in\mathbb{C}^{2}\land\neg(z\in$$

$$\Gamma(a,z)=\Gamma(a)-e^{-z}z^{a}\left(\frac{(-1)^{r}z^{-r}(1-a)_{r}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-r-z}+a-r-z}+\sum_{k=1}^{r}(-z)^{-k}(1-a)_{-1+k}\right)\text{ for }r\in\mathbb{Z}\land(a,z)\in\mathbb{C}^{2}\land\neg($$

$$\Gamma(0,z)=\frac{e^{-z}}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left\lfloor\frac{1+k}{2}\right\rfloor}{z}}{1}+1\right)}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)$$

$$e^{z}\Gamma(0,z)=\frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}\left(1-(-1)^{k}+z+(-1)^{k}z\right)}+z}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)$$

$$e^{z}\Gamma(0,z)=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-k^{2}}{1+2k+z}+z+1}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(0,z)=e^{-z}\left(\frac{(-1)^{r}r!z^{-r}}{\mathrm{K}_{k=1}^{\infty}\frac{-k(k+r)}{1+2k+r+z}+r+z+1}+\sum_{k=0}^{-1+r}(-1)^{k}z^{-1-k}k!\right)\text{ for }r\in\mathbb{Z}\land z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)\land r$$

$$\Gamma(z+1,z)=e^{-z}z^{z}\left(\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{k(-k+z)}{1+2k}+1}+1\right)\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(z+1,z)=e^{-z}z^{z}\left(\frac{z-1}{\mathrm{K}_{k=1}^{\infty}\frac{k(-1-k+z)}{2+2k}+2}+2\right)\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(z+1,z)=\Gamma(z+1)-\frac{2e^{-z}z^{z+1}}{\mathrm{K}_{k=1}^{\infty}\frac{(2+k)z}{2+k}+2}\text{ for }z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,z)=\Gamma(a)-\frac{z^{a}}{a\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{(-1+a+k)z}{k(a+k)}}{1-\frac{(-1+a+k)z}{k(a+k)}}+1\right)}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(-m,z)=\frac{(-1)^{m}z}{(m+1)!\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{kz}{(1+k)(1+k+m)}}{1-\frac{kz}{(1+k)(1+k+m)}}+1\right)}+\frac{(-1)^{m}(\psi^{(0)}(m+1)-\log(z))}{m!}+\frac{z^{-m}}{m\left(\mathrm{K}_{k=1}^{-1+m}\frac{\frac{(-1+k-m)}{k(k-m)}}{1-\frac{(-1+k-m}{k(k-m}}\right.}$$

$$\frac{1}{\Gamma(a)-\Gamma(a,z)}=e^{z}z^{-a}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-z}+a-z\right)\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,0,z)=\frac{e^{-z}z^{a}}{\mathrm{K}_{k=1}^{\infty}\frac{(-1)^{k}z\left(a^{\frac{1}{2}(1-(-1)^{k})}+\left\lfloor\frac{1}{2}(-1+k)\right\rfloor\right)}{a+k}+a}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,0,z)=\Gamma(a)-\frac{e^{-z}z^{a-1}}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{\frac{1}{4}(1+(-1)^{k})k+\frac{1}{2}(1-(-1)^{k})\left(-a+\frac{1+k}{2}\right)}{z}}{1}+1}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$\Gamma(a,0,z) = \Gamma(a) - e^{-z} z^a \left(\frac{z^{r-1}}{(a)_r \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}-r\right)}{\frac{z}{1}} + 1 \right)} - \sum_{k=0}^{-1+r} \frac{z^k}{(a)_{1+k}} \right) \text{ for } r \in \mathbb{Z} \wedge$$

$$\Gamma(a,0,z) = \Gamma(a) - e^{-z} z^a \left(\frac{(-1)^r z^{-r-1} (1-a)_r}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}\left(1+(-1)^k\right)k + \frac{1}{2}\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}+r\right)}{\frac{z}{1}} + 1} - \sum_{k=1}^{r} (-z)^{-k} (1-a)_{-1+k} \right) \text{ for } r \in \mathbb{Z}$$

$$\Gamma(a,0,z) = \frac{e^{-z} z^a}{a \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)\right)}{2(-1+a+k)(a+k)} + \frac{\left(1+(-1)^k\right)k}{4(-1+a+k)(a+k)} \right) z}{1} + 1 \right)} \text{ for } (a,z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z < 0)$$

$$\Gamma(a,0,z) = e^{-z} z^a \left(\frac{z^r}{(a)_{r+1} \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1+(-1)^k\right)k}{4(-1+a+k+r)(a+k+r)} - \frac{\left(1-(-1)^k\right)\left(a+\frac{1+k}{2}\right)}{2(-1+a+k+r)(a+k+r)} \right) z}{1} + 1 \right)} + \sum_{k=0}^{-1+r} \frac{z^k}{(a)_{1+k}} \right) \text{ for } r \in$$

$$\Gamma(a,0,z) = e^{-z} z^a \left(\sum_{k=1}^{r} (-z)^{-k} (1-a)_{-1+k} - \frac{(-1)^r z^{-r} (1-a)_{r-1}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1+(-1)^k\right)k}{4(-1+a+k-r)(a+k-r)} - \frac{\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)-r\right)}{2(-1+a+k-r)(a+k-r)} \right) z}{1} + 1} \right) \text{ for}$$

$$\Gamma(a,0,z) = \Gamma(a) - \frac{e^{-z} z^a}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2^{\frac{1}{2}\left(-1-(-1)^k\right)} k^{\frac{1}{2}\left(1+(-1)^k\right)} \left(-a+\frac{1+k}{2}\right)^{\frac{1}{2}\left(1-(-1)^k\right)}}{z^{\frac{1}{2}\left(1+(-1)^k\right)}} + z} \text{ for } (a,z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z < 0)$$

$$\Gamma(a,0,z) = \Gamma(a) - \frac{e^{-z} z^a}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k(-a+k)}{1-a+2k+z} - a + z + 1} \text{ for } (a,z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z < 0)$$

$$\Gamma(a,0,z) = \Gamma(a) - e^{-z} z^{a-1} \left(\frac{a-1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k(1-a+k)}{2-a+2k+z} - a + z + 2} + 1 \right) \text{ for } (a,z) \in \mathbb{C}^2 \wedge \neg (z \in \mathbb{R} \wedge z < 0)$$

$$\Gamma(a,0,z)=\frac{e^{-z}z^a}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-z}+a-z} \text{ for } (a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge z<0)$$

$$\Gamma(a,0,z)=\frac{e^{-z}z^a}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(1-a-k)z}{a+k+z}+a} \text{ for } (a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge z<0)$$

$$\Gamma(a,0,z)=e^{-z}z^a\left(\frac{z^r}{(a)_r\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k+r-z}+a+r-z\right)}+\sum_{k=0}^{-1+r}\frac{z^k}{(a)_{1+k}}\right) \text{ for } r\in\mathbb{Z}\wedge(a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge$$

$$\Gamma(a,0,z)=e^{-z}z^a\left(\frac{(-1)^r z^{-r}(1-a)_r}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-r-z}+a-r-z}+\sum_{k=1}^{r}(-z)^{-k}(1-a)_{-1+k}\right) \text{ for } r\in\mathbb{Z}\wedge(a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{$$

$$\frac{1}{\Gamma(a,0,z)}=e^z z^{-a}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-z}+a-z\right) \text{ for } (a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge z<0)$$

$$\frac{\Gamma(z)\Gamma(z+1)}{\Gamma\left(z+\frac{1}{2}\right)^2}=\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1+2k)(1+2k)}{8z}+8z-1}+1 \text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$$\frac{\Gamma\left(\frac{z+1}{4}\right)^2}{\Gamma\left(\frac{z+3}{4}\right)^2}=\frac{4}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1+2k)^2}{2z}+z} \text{ for } z\in\mathbb{R}\wedge z>4$$

$$\frac{\Gamma\left(\frac{z+1}{4}\right)^4}{\Gamma\left(\frac{z+3}{4}\right)^4}=\frac{8}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)^2}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2)}+\frac{1}{2}\left(z^2-1\right)} \text{ for } z\in\mathbb{C}\wedge-\frac{\pi}{2}<\arg(z)<\frac{\pi}{2}$$

$$\frac{\Gamma\left(\frac{z}{2}\right)^2}{\Gamma\left(\frac{z+1}{2}\right)^2}=\frac{2\left(\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-1+4k^2}{4z}+4z-1}+1\right)}{z} \text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$$\frac{(2z-1)\Gamma\left(z-\frac{1}{2}\right)^2}{\Gamma(z)^2}=\frac{4}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1+2k)(1+2k)}{-4+8z}+8z-5}+2 \text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$$\frac{\Gamma\left(\frac{z}{4}\right)^2}{\Gamma\left(\frac{z+2}{4}\right)^2}=\frac{4}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{4}+k^2}{z}+z-\frac{1}{2}\right)}+\frac{4}{z} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\frac{\Gamma\left(\frac{z}{4}\right)^2}{\Gamma\left(\frac{z+2}{4}\right)^2} = \frac{4\left(\frac{2}{\mathrm{K}_{k=1}^{\infty}\frac{(-1+2k)(1+2k)}{2z}+2z-1}+1\right)}{z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\frac{\Gamma\left(\frac{z}{4}\right)^2}{\Gamma\left(\frac{z+2}{4}\right)^2} = \frac{4}{\mathrm{K}_{k=1}^{\infty}\frac{(-1+2k)^2}{2(-1+z)}+z-1} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\frac{\Gamma\left(\frac{z+3}{4}\right)^2}{\Gamma\left(\frac{z+1}{4}\right)^2} = \frac{1}{8\left(\mathrm{K}_{k=1}^{\infty}\frac{k\left(\frac{1}{2}+k\right)^2(1+k)}{(1+k)z}+z\right)} + \frac{z}{4} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\frac{\Gamma\left(\frac{z+3}{4}\right)^2}{\Gamma\left(\frac{z+1}{4}\right)^2} = \frac{1}{4}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1+2k)^2}{2z}+z\right) \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\frac{\Gamma\left(z+\frac{1}{2}\right)^2}{\Gamma(z)^2} = \frac{1}{4}\left(\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{(1+2k)^2}{-2+8z}+8z-2}+4z-1\right) \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\frac{\Gamma(z+1)^2}{\Gamma\left(z+\frac{1}{2}\right)^2} = \frac{1}{4}\left(\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{(1+2k)^2}{2+8z}+8z+2}+4z+1\right) \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\frac{\Gamma\left(\frac{1}{4}(-a+z+1)\right)\Gamma\left(\frac{1}{4}(a+z+1)\right)}{\Gamma\left(\frac{1}{4}(-a+z+3)\right)\Gamma\left(\frac{1}{4}(a+z+3)\right)} = \frac{4}{\mathrm{K}_{k=1}^{\infty}\frac{-a^2+(-1+2k)^2}{2z}+z} \text{ for } (z,a) \in \mathbb{C}^2 \wedge \Re(a) > 0 \wedge \Re(z) > 1$$

$$\frac{\Gamma\left(\frac{z+1}{8}\right)\Gamma\left(\frac{z+3}{8}\right)}{\Gamma\left(\frac{z+5}{8}\right)\Gamma\left(\frac{z+7}{8}\right)} = \frac{8}{\mathrm{K}_{k=1}^{\infty}\frac{(1+4(-1+k))(3+4(-1+k))}{2z}+z} \text{ for } (z,a) \in \mathbb{C}^2 \wedge \Re(a) > 0 \wedge \Re(z) > 1$$

$$\frac{\Gamma\left(\frac{1}{8}(-2a+z+6)\right)\Gamma\left(\frac{1}{8}(2a+z+6)\right)}{\Gamma\left(\frac{1}{8}(-2a+z+2)\right)\Gamma\left(\frac{1}{8}(2a+z+2)\right)} = \frac{1}{8}z\left(\frac{2\left(1-a^2\right)}{\mathrm{K}_{k=1}^{\infty}\frac{-a^2+(1+2k)^2}{z^{1+(-1)^k}}+z^2}+1\right) \text{ for } (z,a) \in \mathbb{C}^2 \wedge |a| < 1 \wedge \Re(z)$$

$$\frac{\Gamma\left(\frac{1}{4}(-2a+z+6)\right)\Gamma\left(\frac{1}{4}(2a+z)\right)}{\Gamma\left(\frac{1}{4}(-2a+z+4)\right)\Gamma\left(\frac{1}{4}(2a+z-2)\right)} = \frac{z}{4} - \frac{(a-2)(a-1)}{2\left(\mathrm{K}_{k=1}^{\infty}\frac{k(1+k)(2-a+k)(-1+a+k)}{(1+k)z}+z\right)} \text{ for } (z,a) \in \mathbb{C}^2 \wedge |a| > 1/$$

$$\frac{\Gamma\left(\frac{1}{4}(-a+z+3)\right)\Gamma\left(\frac{1}{4}(a+z+3)\right)}{\Gamma\left(\frac{1}{4}(-a+z+1)\right)\Gamma\left(\frac{1}{4}(a+z+1)\right)} = \frac{1}{4}\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-a^2+(-1+2k)^2}{2z}+z\right) \text{ for } (z,a)\in\mathbb{C}^2\wedge\Re(z)>0\wedge|a|<1$$

$$\frac{\Gamma\left(\frac{1}{4}(-a+z+1)\right)^2\Gamma\left(\frac{1}{4}(a+z+1)\right)^2}{\Gamma\left(\frac{1}{4}(-a+z+3)\right)^2\Gamma\left(\frac{1}{4}(a+z+3)\right)^2} = \frac{8}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)(-1+k)^2+\frac{1}{2}(1-(-1)^k)(-a^2+k^2)}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2)}+\frac{1}{2}\left(a^2+z^2-1\right)} \text{ for}$$

$$\frac{(4a^2+(z+1)^2)\,\Gamma\left(\frac{1}{2}(-2a+z+1)\right)\Gamma\left(\frac{1}{2}(2a+z+1)\right)-4\Gamma\left(\frac{1}{2}(-2ia+z+3)\right)\Gamma\left(\frac{1}{2}(2ia+z+3)\right)}{(4a^2+(z+1)^2)\,\Gamma\left(\frac{1}{2}(-2a+z+1)\right)\Gamma\left(\frac{1}{2}(2a+z+1)\right)+4\Gamma\left(\frac{1}{2}(-2ia+z+3)\right)\Gamma\left(\frac{1}{2}(2ia+z+3)\right)} = \frac{}{\mathrm{K}_{k=}^{\infty}}$$

$$\frac{\Gamma(-a+z+1)\Gamma\left(\frac{1}{2}\left(1-i\sqrt{3}\right)a+z+1\right)\Gamma\left(\frac{1}{2}\left(1+i\sqrt{3}\right)a+z+1\right)}{\Gamma(a+z+1)\Gamma\left(\frac{1}{2}\left(-1+i\sqrt{3}\right)a+z+1\right)\Gamma\left(-\frac{1}{2}\left(1+i\sqrt{3}\right)a+z+1\right)} = \frac{2a^3}{\mathrm{K}_{k=1}^{\infty}\frac{a^6-k^6}{(1+2k)(1+k+k^2+2z+2z^2)}-a^3+}$$

$$\frac{\Gamma\left(\frac{1}{2}(-a-b+z+1)\right)\Gamma\left(\frac{1}{2}(a+b+z+1)\right)-\Gamma\left(\frac{1}{2}(a-b+z+1)\right)\Gamma\left(\frac{1}{2}(-a+b+z+1)\right)}{\Gamma\left(\frac{1}{2}(a-b+z+1)\right)\Gamma\left(\frac{1}{2}(-a+b+z+1)\right)+\Gamma\left(\frac{1}{2}(-a-b+z+1)\right)\Gamma\left(\frac{1}{2}(a+b+z+1)\right)} = \frac{ab}{\mathrm{K}_{k=1}^{\infty}\frac{(a^2-k^2)(}{(1+2}}$$

$$\frac{\Gamma\left(\frac{1}{4}(-a-b+z+1)\right)\Gamma\left(\frac{1}{4}(a-b+z+1)\right)\Gamma\left(\frac{1}{4}(-a+b+z+1)\right)\Gamma\left(\frac{1}{4}(a+b+z+1)\right)}{\Gamma\left(\frac{1}{4}(-a-b+z+3)\right)\Gamma\left(\frac{1}{4}(a-b+z+3)\right)\Gamma\left(\frac{1}{4}(-a+b+z+3)\right)\Gamma\left(\frac{1}{4}(a+b+z+3)\right)} = \frac{}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)}{\frac{1}{2}(1}}$$

$$\frac{\Gamma\left(\frac{1}{4}(-a-b+z+1)\right)\Gamma\left(\frac{1}{4}(a-b+z+3)\right)\Gamma\left(\frac{1}{4}(-a+b+z+1)\right)\Gamma\left(\frac{1}{4}(a+b+z+3)\right)}{\Gamma\left(\frac{1}{4}(-a-b+z+3)\right)\Gamma\left(\frac{1}{4}(a-b+z+1)\right)\Gamma\left(\frac{1}{4}(-a+b+z+3)\right)\Gamma\left(\frac{1}{4}(a+b+z+1)\right)} = \frac{}{\mathrm{K}_{k=1}^{\infty}\frac{-\frac{1}{2}(1+(-1)}{}}$$

$$\frac{1-\frac{\Gamma\left(\frac{1}{4}(-a-b+z+3)\right)\Gamma\left(\frac{1}{4}(a-b+z+1)\right)\Gamma\left(\frac{1}{4}(-a+b+z+3)\right)\Gamma\left(\frac{1}{4}(a+b+z+1)\right)}{\Gamma\left(\frac{1}{4}(-a-b+z+1)\right)\Gamma\left(\frac{1}{4}(a-b+z+3)\right)\Gamma\left(\frac{1}{4}(-a+b+z+1)\right)\Gamma\left(\frac{1}{4}(a+b+z+3)\right)}}{\frac{\Gamma\left(\frac{1}{4}(-a-b+z+3)\right)\Gamma\left(\frac{1}{4}(a-b+z+1)\right)\Gamma\left(\frac{1}{4}(-a+b+z+3)\right)\Gamma\left(\frac{1}{4}(a+b+z+1)\right)}{\Gamma\left(\frac{1}{4}(-a-b+z+1)\right)\Gamma\left(\frac{1}{4}(a-b+z+3)\right)\Gamma\left(\frac{1}{4}(-a+b+z+1)\right)\Gamma\left(\frac{1}{4}(a+b+z+3)\right)}+1} = \frac{a}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)(-a^2+k^2)+\frac{1}{2}(1-(-1)^k)(-b^2}{z}}$$

$$\frac{1-\frac{\Gamma\left(\frac{1}{4}(-a-b+z+3)\right)\Gamma\left(\frac{1}{4}(a-b+z+1)\right)\Gamma\left(\frac{1}{4}(-a+b+z+1)\right)\Gamma\left(\frac{1}{4}(a+b+z+3)\right)}{\Gamma\left(\frac{1}{4}(-a-b+z+1)\right)\Gamma\left(\frac{1}{4}(a-b+z+3)\right)\Gamma\left(\frac{1}{4}(-a+b+z+3)\right)\Gamma\left(\frac{1}{4}(a+b+z+1)\right)}}{\frac{\Gamma\left(\frac{1}{4}(-a-b+z+3)\right)\Gamma\left(\frac{1}{4}(a-b+z+1)\right)\Gamma\left(\frac{1}{4}(-a+b+z+1)\right)\Gamma\left(\frac{1}{4}(a+b+z+3)\right)}{\Gamma\left(\frac{1}{4}(-a-b+z+1)\right)\Gamma\left(\frac{1}{4}(a-b+z+3)\right)\Gamma\left(\frac{1}{4}(-a+b+z+3)\right)\Gamma\left(\frac{1}{4}(a+b+z+1)\right)}+1} = \frac{ab}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)(-a^2+k^2)+\frac{1}{2}(1-(-1)^k)(-b^2}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2}}$$

$$\frac{\Gamma\left(\frac{1}{2}(a-b-c-d-h+1)\right)\Gamma\left(\frac{1}{2}(a+b+c-d-h+1)\right)\Gamma\left(\frac{1}{2}(a+b-c+d-h+1)\right)\Gamma\left(\frac{1}{2}(a-b+c+d-\right.}{\Gamma\left(\frac{1}{2}(a+b-c-d-h+1)\right)\Gamma\left(\frac{1}{2}(a-b+c-d-h+1)\right)\Gamma\left(\frac{1}{2}(a-b-c+d-h+1)\right)\Gamma\left(\frac{1}{2}(a+b+c+d-\right.}$$

$$1-\frac{\frac{\Gamma\left(\frac{1}{2}(a-b-c+z+1)\right)\Gamma\left(\frac{1}{2}(-a+b-c+z+1)\right)\Gamma\left(\frac{1}{2}(-a-b+c+z+1)\right)\Gamma\left(\frac{1}{2}(a+b+c+z+1)\right)}{\Gamma\left(\frac{1}{2}(-a-b-c+z+1)\right)\Gamma\left(\frac{1}{2}(a+b-c+z+1)\right)\Gamma\left(\frac{1}{2}(a-b+c+z+1)\right)\Gamma\left(\frac{1}{2}(-a+b+c+z+1)\right)}}{\frac{\Gamma\left(\frac{1}{2}(a-b-c+z+1)\right)\Gamma\left(\frac{1}{2}(-a+b-c+z+1)\right)\Gamma\left(\frac{1}{2}(-a-b+c+z+1)\right)\Gamma\left(\frac{1}{2}(a+b+c+z+1)\right)}{\Gamma\left(\frac{1}{2}(-a-b-c+z+1)\right)\Gamma\left(\frac{1}{2}(a+b-c+z+1)\right)\Gamma\left(\frac{1}{2}(a-b+c+z+1)\right)\Gamma\left(\frac{1}{2}(-a+b+c+z+1)\right)}+1}=\frac{2abc}{\mathrm{K}_{k=1}^{\infty}\frac{4(a^2-k^2)(b^2-k^2)(c^2-k^2}{(1+2k)(1-a^2-b^2-c^2+2k(1+k}}$$

$$\frac{\frac{\Gamma\left(\frac{1}{4}(-2a+z+1)\right)\Gamma\left(\frac{1}{4}(2a+z+3)\right)}{\Gamma\left(\frac{1}{4}(-2a+z+3)\right)\Gamma\left(\frac{1}{4}(2a+z+1)\right)}-1}{\frac{\Gamma\left(\frac{1}{4}(-2a+z+1)\right)\Gamma\left(\frac{1}{4}(2a+z+3)\right)}{\Gamma\left(\frac{1}{4}(-2a+z+3)\right)\Gamma\left(\frac{1}{4}(2a+z+1)\right)}+1}=\frac{a}{\mathrm{K}_{k=1}^{\infty}\frac{-a^2+k^2}{z}+z}\text{ for }(z,a)\in\mathbb{C}^2\wedge|a|<1\wedge\Re(z)>\max(0,2\Re(a)-1)$$

$$\frac{\frac{\Gamma\left(\frac{1}{4}(-a+z+1)\right)^2\Gamma\left(\frac{1}{4}(a+z+3)\right)^2}{\Gamma\left(\frac{1}{4}(-a+z+3)\right)^2\Gamma\left(\frac{1}{4}(a+z+1)\right)^2}-1}{\frac{\Gamma\left(\frac{1}{4}(-a+z+1)\right)^2\Gamma\left(\frac{1}{4}(a+z+3)\right)^2}{\Gamma\left(\frac{1}{4}(-a+z+3)\right)^2\Gamma\left(\frac{1}{4}(a+z+1)\right)^2}+1}=\frac{a}{\mathrm{K}_{k=1}^{\infty}\frac{-\frac{1}{2}(1+(-1)^k)a^2+k^2}{z}+z}\text{ for }(z,a)\in\mathbb{C}^2\wedge|a|<2\wedge\Re(z)>\max(0,\Re(a$$

$$Q(a,z)=1-\frac{e^{-z}z^a}{\Gamma(a)\left(\mathrm{K}_{k=1}^{\infty}\frac{(-1)^k z\left(a^{\frac{1}{2}(1-(-1)^k)}+\left\lfloor\frac{1}{2}(-1+k)\right\rfloor\right)}{a+k}+a\right)}\text{ for }(a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge z<0)$$

$$Q(a,z)=\frac{e^{-z}z^{a-1}}{\Gamma(a)\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{\frac{1}{4}(1+(-1)^k)k+\frac{1}{2}(1-(-1)^k)\left(-a+\frac{1+k}{2}\right)}{z}}{1}+1\right)}\text{ for }(a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge z<0)$$

$$Q(a,z)=\frac{e^{-z}z^a\left(\frac{z^{r-1}}{(a)_r\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{\frac{1}{4}(1+(-1)^k)k+\frac{1}{2}(1-(-1)^k)\left(-a+\frac{1+k}{2}-r\right)}{z}}{1}+1\right)}-\sum_{k=0}^{-1+r}\frac{z^k}{(a)_{1+k}}\right)}{\Gamma(a)}\text{ for }r\in\mathbb{Z}\wedge(a,z)\in\mathbb{C}^$$

$$Q(a,z)=\frac{e^{-z}z^a\left(\frac{(-1)^r z^{-r-1}(1-a)_r}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{\frac{1}{4}(1+(-1)^k)k+\frac{1}{2}(1-(-1)^k)\left(-a+\frac{1+k}{2}+r\right)}{z}}{1}+1}-\sum_{k=1}^{r}(-z)^{-k}(1-a)_{-1+k}\right)}{\Gamma(a)}\text{ for }r\in\mathbb{Z}\wedge(a,z)$$

$$Q(a,z)=1-\frac{e^{-z}z^a}{\Gamma(a+1)\left(\mathrm{K}_{k=1}^{\infty}\frac{\left(-\frac{(1-(-1)^k)\left(a+\frac{1}{2}(-1+k)\right)}{2(-1+a+k)(a+k)}+\frac{(1+(-1)^k)k}{4(-1+a+k)(a+k)}\right)z}{1}+1\right)}\text{ for }(a,z)\in\mathbb{C}^2\wedge\neg(z\in\mathbb{R}\wedge z<$$

$$Q(a,z)=1-\frac{e^{-z}z^{a}\left(\frac{z^{r}}{(a)_{r+1}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(\frac{\left(1+(-1)^{k}\right)k}{4(-1+a+k+r)(a+k+r)}-\frac{\left(1-(-1)^{k}\right)\left(a+\frac{1+k}{2}\right)}{2(-1+a+k+r)(a+k+r)}\right)z}{1}+1\right)}+\sum_{k=0}^{-1+r}\frac{z^{k}}{(a)_{1+k}}\right)}{\Gamma(a)}\text{ for }r\in\mathbb{Z}$$

$$Q(a,z)=1-\frac{e^{-z}z^{a}\left(\sum_{k=1}^{r}(-z)^{-k}(1-a)_{-1+k}-\frac{(-1)^{r}z^{-r}(1-a)_{r-1}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(\frac{\left(1+(-1)^{k}\right)k}{4(-1+a+k-r)(a+k-r)}-\frac{\left(1-(-1)^{k}\right)\left(a+\frac{1}{2}(-1+k)-r\right)}{2(-1+a+k-r)(a+k-r)}\right)z}{1}+1}\right)}{\Gamma(a)}\text{ fo}$$

$$Q(a,z)=\frac{e^{-z}z^{a}}{\Gamma(a)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{2^{\frac{1}{2}\left(-1-(-1)^{k}\right)}k^{\frac{1}{2}\left(1+(-1)^{k}\right)}\left(-a+\frac{1+k}{2}\right)^{\frac{1}{2}\left(1-(-1)^{k}\right)}}{z^{\frac{1}{2}\left(1+(-1)^{k}\right)}}+z\right)}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$Q(a,z)=\frac{e^{-z}z^{a}}{\Gamma(a)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k(-a+k)}{1-a+2k+z}-a+z+1\right)}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$Q(a,z)=\frac{e^{-z}z^{a-1}\left(\frac{a-1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k(1-a+k)}{2-a+2k+z}-a+z+2}+1\right)}{\Gamma(a)}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$Q(a,z)=1-\frac{e^{-z}z^{a}}{\Gamma(a)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k-z}+a-z\right)}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$Q(a,z)=1-\frac{e^{-z}z^{a}}{\Gamma(a)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(1-a-k)z}{a+k+z}+a\right)}\text{ for }(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0)$$

$$Q(a,z)=1-\frac{e^{-z}z^{a}\left(\frac{z^{r}}{(a)_{r}\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{kz}{a+k+r-z}+a+r-z\right)}+\sum_{k=0}^{-1+r}\frac{z^{k}}{(a)_{1+k}}\right)}{\Gamma(a)}\text{ for }r\in\mathbb{Z}\land(a,z)\in\mathbb{C}^{2}\land\neg(z\in\mathbb{R}\land z<0$$

$$Q(a,z) = 1 - \frac{e^{-z} z^a \left(\frac{(-1)^r z^{-r} (1-a)_r}{\mathrm{K}_{k=1}^{\infty} \frac{kz}{a+k-r-z} + a - r - z} + \sum_{k=1}^{r} (-z)^{-k} (1-a)_{-1+k} \right)}{\Gamma(a)} \text{ for } r \in \mathbb{Z} \wedge (a,z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge$$

$$Q(z+1,z) = \frac{e^{-z} z^z \left(\frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{k(-k+z)}{1+2k} + 1} + 1 \right)}{\Gamma(z+1)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$Q(z+1,z) = \frac{e^{-z} z^z \left(\frac{z-1}{\mathrm{K}_{k=1}^{\infty} \frac{k(-1-k+z)}{2+2k} + 2} + 2 \right)}{\Gamma(z+1)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$Q(z+1,z) = 1 - \frac{2e^{-z} z^{z+1}}{\Gamma(z+1) \left(\mathrm{K}_{k=1}^{\infty} \frac{(2+k)z}{2+k} + 2 \right)} \text{ for } z \in \mathbb{C} \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$Q(a,z) = 1 - \frac{z^a}{\Gamma(a+1) \left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{(-1+a+k)z}{k(a+k)}}{1 - \frac{(-1+a+k)z}{k(a+k)}} + 1 \right)} \text{ for } (a,z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge z < 0)$$

$$C_{\nu}^{(\lambda)}(1-2z) = \frac{\sqrt{\pi} 2^{1-2\lambda} \Gamma(2\lambda+\nu)}{\Gamma(\lambda) \Gamma\left(\lambda + \frac{1}{2}\right) \Gamma(\nu+1) \left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{2z(-1+k-\nu)(-1+k+2\lambda+\nu)}{k(-1+2k+2\lambda)}}{1 + \frac{2z(-1+k-\nu)(-1+k+2\lambda+\nu)}{k(-1+2k+2\lambda)}} + 1 \right)} \text{ for } (\nu,\lambda,z) \in \mathbb{C}^3 \wedge |z| < 1$$

$$C_{\nu}^{\left(-m-\frac{1}{2}\right)}(1-2z) = -\frac{\sqrt{\pi} (-1)^m 4^{m+1} z^{m+1}}{\Gamma\left(-m-\frac{1}{2}\right) \Gamma(m+2) \left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z(k+m-\nu)(-1+k-m+\nu)}{k(1+k+m)}}{1 + \frac{z(k+m-\nu)(-1+k-m+\nu)}{k(1+k+m)}} + 1 \right)} \text{ for } m \in \mathbb{Z} \wedge (\nu,z) \in \mathbb{C}^2 \wedge m$$

$$C_{\nu}^{(\lambda)}(2z-1) = \frac{\sec(\pi\lambda) \cos(\pi(\lambda+\nu)) \Gamma(2\lambda+\nu)}{\Gamma(2\lambda) \Gamma(\nu+1) \left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{2z(-1+k-\nu)(-1+k+2\lambda+\nu)}{k(-1+2k+2\lambda)}}{1 + \frac{2z(-1+k-\nu)(-1+k+2\lambda+\nu)}{k(-1+2k+2\lambda)}} + 1 \right)} - \frac{2^{1-2\lambda} \Gamma\left(\lambda - \frac{1}{2}\right) \sin(\pi\nu) z^{\frac{1}{2}-\lambda}}{\sqrt{\pi} \Gamma(\lambda) \left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z\left(-1-4(-1+k)k+4(\lambda+\nu\right.}{2k(1+2k-2\lambda)}}{1 - \frac{z\left(-1-4(-1+k)k+4(\lambda+\nu\right.}{2k(1+2k-2\lambda)}} \right.}$$

$$C_{\nu}^{\left(-m-\frac{1}{2}\right)}(2z-1) = -\frac{(-1)^m 4^{m+1} z^{m+1} \sin(\pi\nu) \log(z)}{\sqrt{\pi} (m+1)! \Gamma\left(-m-\frac{1}{2}\right) \left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z(k+m-\nu)(-1+k-m+\nu)}{k(1+k+m)}}{1 + \frac{z(k+m-\nu)(-1+k-m+\nu)}{k(1+k+m)}} + 1 \right)} + \frac{(-1)^m 4^{m}}{\sqrt{\pi} (m+1)! \Gamma\left(-m-\frac{1}{2}\right.}$$

$$C_\nu^{(\lambda)}(z) = \frac{2^\nu z^\nu \Gamma(\lambda+\nu)}{\Gamma(\lambda)\Gamma(\nu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(k-\lambda-\nu)}}{1+\frac{(-2+2k-\nu)(-1+2k-\nu)}{4kz^2(k-\lambda-\nu)}}+1\right)} - \frac{2^{-2\lambda-\nu}\sin(\pi\nu)\Gamma(-\lambda-\nu)\Gamma(2\lambda+\nu)z^{-2\lambda-\nu}}{\pi\Gamma(\lambda)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-1+2k+2\lambda+\nu)(2(-1+k+\lambda)+\nu)}{4kz^2(k+\lambda+\nu)}}{1+\frac{(-1+2k+2\lambda+\nu)(2(-1+k+\lambda)+\nu)}{4kz^2(k+\lambda+\nu)}}+1 \right.}$$

$$C_\nu^{(m-\nu)}(z) = \frac{2^\nu (m-1)!\left(z^2\right)^{\nu/2}}{\Gamma(\nu+1)\Gamma(m-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{-1+m} \frac{\frac{(-2+2k-\nu)(-1+2k-\nu)}{4k(-k+m)z^2}}{1-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4k(-k+m)z^2}}+1\right)} - \frac{(-1)^m 2^{\nu-2m} z^{-2m}\sin(\pi\nu)\left(z^2\right)^{\nu/2}}{\pi m!\Gamma(m-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-1+2k+2m-}{4k(}}{1+\frac{(-1+2k+2m-}{4k(}} \right.}$$

$$C_\nu^{(-m-\nu)}(z) = \frac{(-1)^m 2^\nu \left(z^2\right)^{\nu/2}\log\left(z^2\right)}{m!\Gamma(\nu+1)\Gamma(-m-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(-2+2k-\nu)(-1+2k-\nu)}{4k(k+m)z^2}}{1+\frac{(-2+2k-\nu)(-1+2k-\nu)}{4k(k+m)z^2}}+1\right)} + \frac{(-1)^m 2^\nu \left(z^2\right.}{m!\Gamma(\nu+1)\Gamma(-m-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\right.}$$

$$\phi = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{1} + 1$$

$$\frac{1}{\phi} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{1}$$

$$\sqrt{\phi} = 1 + \Re\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{2+2i}\right)$$

$$-e\mathrm{Ei}(-1) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2}{2(1+k)} + 2}$$

$$-e\mathrm{Ei}(-1) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor}{1} + 1}$$

$$H_\nu^{(1)}(z) = \frac{2^{-\nu}(1+i\cot(\pi\nu))z^\nu}{\Gamma(\nu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+\nu)}}{1-\frac{z^2}{4k(k+\nu)}}+1\right)} - \frac{i2^\nu \csc(\pi\nu)z^{-\nu}}{\Gamma(1-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k-\nu)}}{1-\frac{z^2}{4k(k-\nu)}}+1\right)} \text{ for } (\nu,z)\in\mathbb{C}^2 \wedge \nu \notin \mathbb{Z}$$

$$H_0^{(1)}(z) = \frac{\pi + 2i\log\left(\frac{z}{2}\right)}{\pi\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{4k^2}}{1-\frac{z^2}{4k^2}}+1\right)} + \frac{2i\gamma}{\pi\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2\psi^{(0)}(1+k)}{4k^2\psi^{(0)}(k)}}{1-\frac{z^2\psi^{(0)}(1+k)}{4k^2\psi^{(0)}(k)}}+1\right)} \text{ for } z\in\mathbb{C}$$

$$H_m^{(1)}(z) = -\frac{i 2^m (m-1)! z^{-m}}{\pi \left(\operatorname*{K}_{k=1}^{-1+m} \frac{\frac{z^2}{4k(k-m)}}{1-\frac{z^2}{4k(k-m)}} + 1 \right)} + \frac{2^{-m} z^m \left(\pi + 2i \log\left(\frac{z}{2}\right) \right)}{\pi m! \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+m)}}{1-\frac{z^2}{4k(k+m)}} + 1 \right)} - \frac{i 2^{-m} z^m (\psi^{(0)}(m+1) - \cdots}{\pi m! \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2(\psi^{(0)}(1+k)+\psi^{(0)}(1+\cdots}{4k(k+m)(\psi^{(0)}(k)+\psi^{(0)}\cdots}}{1-\frac{z^2(\psi^{(0)}(1+k)+\psi^{(0)}(\cdots}{4k(k+m)(\psi^{(0)}(k)+\psi^{(\cdots}}} \cdots \right.}$$

$$\frac{H_{\nu+1}^{(1)}(z)}{H_\nu^{(1)}(z)} = \frac{2\nu - 2iz + 1}{2z} - \frac{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{1}{4}(-1+2k)^2+\nu^2}{2(-k+iz)}}{z} \text{ for } (\nu, z) \in \mathbb{C}^2 \land -\frac{\pi}{2} < \arg(z) \leq \pi$$

$$\frac{\frac{\partial H_\nu^{(1)}(z)}{\partial z}}{H_\nu^{(1)}(z)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{1}{4}(-1+2k)^2+\nu^2}{2(-k+iz)}}{z} - \frac{1}{2z} + i \text{ for } (\nu, z) \in \mathbb{C}^2 \land -\frac{\pi}{2} < \arg(z) \leq \pi$$

$$H_\nu^{(2)}(z) = \frac{i 2^\nu \csc(\pi\nu) z^{-\nu}}{\Gamma(1-\nu) \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k-\nu)}}{1-\frac{z^2}{4k(k-\nu)}} + 1 \right)} + \frac{2^{-\nu}(1 - i\cot(\pi\nu)) z^\nu}{\Gamma(\nu+1) \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+\nu)}}{1-\frac{z^2}{4k(k+\nu)}} + 1 \right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \land \nu \notin \mathbb{Z}$$

$$H_0^{(2)}(z) = \frac{\pi - 2i \log\left(\frac{z}{2}\right)}{\pi \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k^2}}{1-\frac{z^2}{4k^2}} + 1 \right)} - \frac{2i\gamma}{\pi \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2 \psi^{(0)}(1+k)}{4k^2 \psi^{(0)}(k)}}{1-\frac{z^2 \psi^{(0)}(1+k)}{4k^2 \psi^{(0)}(k)}} + 1 \right)} \text{ for } z \in \mathbb{C}$$

$$H_m^{(2)}(z) = \frac{i 2^m (m-1)! z^{-m}}{\pi \left(\operatorname*{K}_{k=1}^{-1+m} \frac{\frac{z^2}{4k(k-m)}}{1-\frac{z^2}{4k(k-m)}} + 1 \right)} + \frac{2^{-m} z^m \left(\pi - 2i \log\left(\frac{z}{2}\right) \right)}{\pi m! \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2}{4k(k+m)}}{1-\frac{z^2}{4k(k+m)}} + 1 \right)} + \frac{i 2^{-m} z^m (\psi^{(0)}(m+1) - \cdots}{\pi m! \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z^2(\psi^{(0)}(1+k)+\psi^{(0)}(1+k\cdots}{4k(k+m)(\psi^{(0)}(k)+\psi^{(0)}(k\cdots}}{1-\frac{z^2(\psi^{(0)}(1+k)+\psi^{(0)}(1+\cdots}{4k(k+m)(\psi^{(0)}(k)+\psi^{(0)}(\cdots}} \cdots \right.}$$

$$\frac{H_{\nu+1}^{(2)}(z)}{H_\nu^{(2)}(z)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{1}{4}(-1+2k)^2+\nu^2}{2(k+iz)}}{z} + \frac{2\nu + 2iz + 1}{2z} \text{ for } (\nu, z) \in \mathbb{C}^2 \land -\pi < \arg(z) \leq \frac{\pi}{2}$$

$$\frac{\frac{\partial H_\nu^{(2)}(z)}{\partial z}}{H_\nu^{(2)}(z)} = -\frac{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{1}{4}(-1+2k)^2+\nu^2}{2(k+iz)}}{z} - \frac{1}{2z} - i \text{ for } (\nu, z) \in \mathbb{C}^2 \land -\pi < \arg(z) \leq \frac{\pi}{2}$$

$$H_z = \frac{\pi^2 z}{6 \left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z\zeta(2+k)}{\zeta(1+k)}}{1-\frac{z\zeta(2+k)}{\zeta(1+k)}} + 1 \right)} \text{ for } z \in \mathbb{C}$$

$$H_z^{(r)} = \frac{rz\zeta(r+1)}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(k+r)z\zeta(1+k+r)}{(1+k)\zeta(k+r)}}{1-\frac{(k+r)z\zeta(1+k+r)}{(1+k)\zeta(k+r)}} + 1} \text{ for } (z, r) \in \mathbb{C}^2 \land |z| < 1$$

$$H_z - H_{z-\frac{1}{2}} = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{1+4z} + 4z + 1} \text{ for } z \in \mathbb{C}$$

$$H_z - H_{z-\frac{1}{2}} = \frac{1 - \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2(1+k)^2}{4(1+k)z} + 4z}}{2z} \text{ for } z \in \mathbb{C}$$

$$H_\nu(z) = \frac{\sqrt{\pi}2^\nu}{\Gamma\left(\frac{1-\nu}{2}\right)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{2z\Gamma\left(\frac{k-\nu}{2}\right)}{k\Gamma\left(\frac{1}{2}(-1+k-\nu)\right)}}{1 - \frac{2z\Gamma\left(\frac{k-\nu}{2}\right)}{k\Gamma\left(\frac{1}{2}(-1+k-\nu)\right)}} + 1\right)} \text{ for } (\nu, z) \in \mathbb{C}^2$$

$$\Phi(-1,1,z) = \frac{\frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(1+k)}{2z} + 2z} + 1}{2z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\Phi\left(-1,1,\frac{z+1}{2}\right) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{z} + z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 2$$

$$\Phi(-1,1,z+1) = \frac{z + \frac{1}{2}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor \left(-1 + 2\left\lfloor \frac{1+k}{2} \right\rfloor\right)}{\frac{1}{2}(1-(-1)^k) + \frac{1}{2}(1+(-1)^k)(2z+2z^2)} + 2z^2 + 2z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > -\frac{1}{2}$$

$$\Phi(z,s,c) = \frac{c^{-s}}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\left(1-\frac{1}{c+k}\right)^s z}{1+\left(1-\frac{1}{c+k}\right)^s z} + 1} \text{ for } (z,s,c) \in \mathbb{C}^3 \wedge |z| < 1$$

$$\Phi\left(-1,1,\frac{1}{2}(-a+z+1)\right) + \Phi\left(-1,1,\frac{1}{2}(a+z+1)\right) = \frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2 + \frac{1}{2}(1-(-1)^k)(-a^2+k^2)}{z} + z} \text{ for } (a,$$

$$\Phi\left(-1,1,\frac{1}{2}(-a+z+1)\right) - \Phi\left(-1,1,\frac{1}{2}(a+z+1)\right) = \frac{2a}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2 + \frac{1}{2}(1-(-1)^k)(-a^2+(1+k)^2)}{-(-1)^k + \frac{1}{2}(1+(-1)^k)z^2} + z^2 - 1$$

$$\Phi(-1,1,p) - \Phi(-1,1,q) = \frac{q-p}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+k+p)^2(-1+k+q)^2}{-1+2k+p+q} + pq} \text{ for } (p,q) \in \mathbb{C}^2 \wedge -\frac{\pi}{2} < \arg(p-q) \le \frac{\pi}{2}$$

$$\zeta(3,z) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor^3}{\frac{1}{2}(1-(-1)^k) + (1+(-1)^k)(1+k)z(1+z)} + 2z(z+1)} + \frac{1}{z^3} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(2z+1) \le \frac{\pi}{2}$$

$$\zeta(3,z)=\frac{1}{\operatorname*{K}_{k=1}^{\infty}\frac{-k^6}{(1+2k)(1+k+k^2+2z+2z^2)}+2z^2+2z+1}+\frac{1}{z^3}\text{ for } z\in\mathbb{C}\wedge-\frac{\pi}{2}<\arg(2z+1)\leq\frac{\pi}{2}$$

$$\zeta(3,z)=\frac{\frac{1}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{\left(1+(-1)^k\right)k(2+k)^2}{32(1+k)}+\frac{\left(1-(-1)^k\right)(1+k)^2(3+k)}{32(2+k)}}{z}+z}+2z+2}{4z^3}\text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$$\zeta(s,c)=\frac{c^{-s}}{\operatorname*{K}_{k=1}^{\infty}\frac{-\left(1-\frac{1}{c+k}\right)^s}{1+\left(1-\frac{1}{c+k}\right)^s}+1}\text{ for } (s,c)\in\mathbb{C}^2\wedge\Re(s)>1$$

$$\zeta\left(2,\frac{z+1}{4}\right)-\zeta\left(2,\frac{z+3}{4}\right)=\frac{8}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)(1+k)^2}{-(-1)^k+\frac{1}{2}(1+(-1)^k)z^2}+z^2-1}\text{ for } z\in\mathbb{C}\wedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

$$\zeta(2,z)-\zeta\left(2,z+\frac{1}{2}\right)=\frac{\frac{1}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{1}{8}\left(1-(-1)^k\right)(1+k)^2+\frac{1}{4}\left(1+(-1)^k\right)(2+k)\left(-1+\frac{2+k}{2}\right)}{2z}+2z}+1}{2z^2}\text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$$\zeta\left(2,\frac{z}{2}\right)-\zeta\left(2,\frac{z+1}{2}\right)=\frac{2\left(\frac{1}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{1}{4}\left(1+(-1)^k\right)\left(1+\frac{k}{2}\right)k+\frac{1}{8}\left(1-(-1)^k\right)(1+k)^2}{z}+z}+1\right)}{z^2}\text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$${}_0F_1(;b;z)=\frac{z}{b\left(\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{z}{(1+k)(b+k)}}{1+\frac{z}{(1+k)(b+k)}}+1\right)}+1\text{ for } (b,z)\in\mathbb{C}^2\wedge\neg(b\in\mathbb{Z}\wedge b\leq 0)$$

$${}_0F_1(;b;z)=\frac{1}{\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{z}{k(-1+b+k)}}{1+\frac{z}{k(-1+b+k)}}+1}\text{ for } (b,z)\in\mathbb{C}^2\wedge\neg(b\in\mathbb{Z}\wedge b\leq 0)$$

$$\frac{{}_0F_1(;b+1;z)}{{}_0F_1(;b;z)}=\frac{1}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{z}{(-1+b+k)(b+k)}}{1}+1}\text{ for } (b,z)\in\mathbb{C}^2\wedge\neg(b\in\mathbb{Z}\wedge b\leq 0)$$

$$\frac{{}_0F_1(;b;z)}{{}_0F_1(;b+1;z)}=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{z}{(-1+b+k)(b+k)}}{1}+1\text{ for } (b,z)\in\mathbb{C}^2\wedge\neg(b\in\mathbb{Z}\wedge b\leq 0)$$

$$\frac{{}_0F_1(;b+1;z)}{{}_0F_1(;b;z)} = \frac{b}{\mathrm{K}_{k=1}^{\infty} \frac{z}{b+k} + b} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$\frac{{}_0F_1(;b;z)}{{}_0F_1(;b+1;z)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{z}{b+k} + b}{b} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$\frac{{}_0F_1(;b;z)}{{}_0F_1(;b+1;z)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{-2(-1+2b+2k)\sqrt{z}}{2b+k+4\sqrt{z}}}{2b} + \frac{\sqrt{z}}{b} + 1 \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$${}_0\tilde{F}_1(;b;z) = \frac{z}{\Gamma(b+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z}{(1+k)(b+k)}}{1+\frac{z}{(1+k)(b+k)}} + 1\right)} + \frac{1}{\Gamma(b)} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$${}_0\tilde{F}_1(;b;z) = \frac{1}{\Gamma(b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z}{k(-1+b+k)}}{1+\frac{z}{k(-1+b+k)}} + 1\right)} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$${}_0\tilde{F}_1(;-m;z) = \frac{z^{m+1}}{(m+1)!\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z}{k+k^2+km}}{1+\frac{z}{k+k^2+km}} + 1\right)} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m \geq 0$$

$$\frac{{}_0\tilde{F}_1(;b+1;z)}{{}_0\tilde{F}_1(;b;z)} = \frac{1}{b\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z}{(-1+b+k)(b+k)}}{1} + 1\right)} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$\frac{{}_0\tilde{F}_1(;b;z)}{{}_0\tilde{F}_1(;b+1;z)} = b\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{z}{(-1+b+k)(b+k)}}{1} + 1\right) \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$\frac{{}_0\tilde{F}_1(;b+1;z)}{{}_0\tilde{F}_1(;b;z)} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{z}{b+k} + b} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$\frac{{}_0\tilde{F}_1(;b;z)}{{}_0\tilde{F}_1(;b+1;z)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{z}{b+k} + b \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$\frac{{}_0\tilde{F}_1(;b;z)}{{}_0\tilde{F}_1(;b+1;z)} = \frac{1}{2}\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{-2(-1+2b+2k)\sqrt{z}}{2b+k+4\sqrt{z}} + 2b + 2\sqrt{z}\right) \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$ {}_1F_1(a;b;z) = \frac{az}{b\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(a+k)z}{(1+k)(b+k)}}{1+\frac{(a+k)z}{(1+k)(b+k)}} + 1\right)} + 1 \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0) $$

$$ {}_1F_1(a;b;z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z}{k(-1+b+k)}}{1+\frac{(-1+a+k)z}{k(-1+b+k)}} + 1} \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0) $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = \frac{z}{b\left(\mathrm{K}_{k=1}^{\infty} \frac{\left(\frac{(1+(-1)^k)k}{4(-1+b+k)(b+k)} - \frac{(1-(-1)^k)\left(-1+b+\frac{1+k}{2}\right)}{2(-1+b+k)(b+k)}\right)z}{1} + 1\right)} + 1 \text{ for } (b,z) \in \mathbb{C} $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = \frac{1}{1 - \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}(1+(-1)^k)\left(b+\frac{1}{2}(-2+k)\right)+\frac{1}{4}(1-(-1)^k)(1+k)\right)z}{b+k} + b}} \text{ for } (b,z) \in \mathbb{C}^2 $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = \frac{b-1}{\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}(1-(-1)^k)\left(b+\frac{1}{2}(-3+k)\right)+\frac{1}{4}(1+(-1)^k)k\right)z}{-1+b+k} + b - 1} \text{ for } (b,z) \in \mathbb{C}^2 $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}(1-(-1)^k)\left(b+\frac{1}{2}(-1+k)\right)+\frac{1}{4}(1+(-1)^k)k\right)z}{b+k} + b} + 1 \text{ for } (b,z) \in \mathbb{C}^2 $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{\left(\frac{1}{2}(1+(-1)^k)\left(1-b-\frac{k}{2}\right)+\frac{1}{4}(1-(-1)^k)(1+k)\right)z}{b+k} + b - z} + 1 \text{ for } (b,z) \in \mathbb{C}^2 $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = e^z z^{1-b}\Gamma(b) - \frac{b-1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{4}(3-3(-1)^k+2(-1+(-1)^k)b+2k)}{\frac{1}{2}(1-(-1)^k+z+(-1)^k z)} + z} \text{ for } (b,z) \in \mathbb{C}^2 $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = \frac{b-1}{\mathrm{K}_{k=1}^{\infty} \frac{kz}{-1+b+k-z} + b - z - 1} \text{ for } (b,z) \in \mathbb{C}^2 $$

$$ (b-1)e^z z^{1-b}(\Gamma(b-1)-\Gamma(b-1,z)) = e^z z^{1-b}\Gamma(b) - \frac{b-1}{\mathrm{K}_{k=1}^{\infty} \frac{-k(1-b+k)}{2-b+2k+z} - b + z + 2} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq $$

$$e^z z^{1-z}(\Gamma(z)-\Gamma(z,z)) = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(1+k)z}{1+k} + 1 \text{ for } z \in \mathbb{C}$$

$$\frac{{}_1F_1(a+1;b+1;z)}{{}_1F_1(a;b;z)} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{\left(1-(-1)^k\right)\left(-a+b+\frac{1}{2}(-1+k)\right)}{2(-1+b+k)(b+k)} + \frac{\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)}{2(-1+b+k)(b+k)}\right)z}{1} + 1} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b$$

$$\frac{{}_1F_1(a;b;z)}{{}_1F_1(a+1;b+1;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{\left(1-(-1)^k\right)\left(-a+b+\frac{1}{2}(-1+k)\right)}{2(-1+b+k)(b+k)} + \frac{\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)}{2(-1+b+k)(b+k)}\right)z}{1} + 1 \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z}$$

$$\frac{{}_1F_1(a;2a+1;z)}{{}_1F_1(a+1;2a+2;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(-1)^k z}{2+4a+4\left\lfloor\frac{k}{2}\right\rfloor}}{1} + 1 \text{ for } (a,z) \in \mathbb{C}^2$$

$$\frac{{}_1F_1(a;b+1;z)}{{}_1F_1(a;b;z)} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^{-1+k}\left(\frac{\left(1+(-1)^k\right)\left(-a+b+\frac{k}{2}\right)}{2(-1+b+k)(b+k)} + \frac{\left(1-(-1)^k\right)(-1+a+k)}{2(-1+b+k)(b+k)}\right)z}{1} + 1} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b$$

$$\frac{{}_1F_1(a;b;z)}{{}_1F_1(a;b+1;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1)^{-1+k}\left(\frac{\left(1+(-1)^k\right)\left(-a+b+\frac{k}{2}\right)}{2(-1+b+k)(b+k)} + \frac{\left(1-(-1)^k\right)(-1+a+k)}{2(-1+b+k)(b+k)}\right)z}{1} + 1 \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z}$$

$$\frac{{}_1F_1(a+1;b+1;z)}{{}_1F_1(a;b;z)} = \frac{b}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{1}{2}\left(1-(-1)^k\right)\left(a-b+\frac{1-k}{2}\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)\right)z}{b+k} + b} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b \leq$$

$$\frac{{}_1F_1(a+1;b+1;z)}{{}_1F_1(a;b;z)} = \frac{b \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+a+k)z}{-1+b+k-z}}{az} \text{ for } (a,b,z) \in \mathbb{C}^3$$

$$\frac{{}_1F_1(a;b;z)}{{}_1F_1(a+1;b+1;z)} = \frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(a+k)z}{b+k-z} + b - z}{b} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b \leq 0)$$

$$\frac{{}_1F_1(-m;b;z)}{{}_1F_1(1-m;b+1;z)} = \frac{\mathop{\mathrm{K}}_{k=1}^{-1+m} \frac{(k-m)z}{b+k-z}}{b} - \frac{z}{b} + 1 \text{ for } m \in \mathbb{Z} \land (b,z) \in \mathbb{C}^2 \land m > 0$$

$$\frac{{}_1F_1(a;b+1;z)}{{}_1F_1(a;b;z)} = \frac{b}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{1}{2}\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(a-b-\frac{k}{2}\right)\right)z}{b+k} + b} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b \leq$$

$$\frac{{}_1F_1(a;b;z)}{{}_1F_1(a+1;b+1;z)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{\left(\frac{1}{2}\left(1-(-1)^k\right)\left(a-b+\frac{1-k}{2}\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)\right)z}{b+k}}{b} + 1 \text{ for } (a,b,z) \in \mathbb{C}^3 \land b \notin \mathbb{Z}$$

$$\frac{{}_1F_1(-ia+r+1;2r+2;2iz)}{{}_1F_1(r-ia;2r;2iz)} = \frac{r(r+1)(2r+1)}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{(1-k-r)(1+k+r)(a^2+(k+r)^2)}{(1+2k+2r)\left(a+\frac{(k+r)(1+k+r)}{z}\right)} + (2r+1)\left(a+\frac{r(r+1)}{z}\right)\right)} \text{ for } (a,r,z)$$

$${}_1\tilde{F}_1(a;b;z) = \frac{\frac{az}{b\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(a+k)z}{(1+k)(b+k)}}{1+\frac{(a+k)z}{(1+k)(b+k)}}+1\right)} + 1}{\Gamma(b)} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b \leq 0)$$

$${}_1\tilde{F}_1(a;b;z) = \frac{1}{\Gamma(b)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z}{k(-1+b+k)}}{1+\frac{(-1+a+k)z}{k(-1+b+k)}} + 1\right)} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b \leq 0)$$

$${}_1\tilde{F}_1(a;-m;z) = \frac{z^{m+1}(a)_{m+1}}{(m+1)!\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(a+k+m)z}{k(1+k+m)}}{1+\frac{(a+k+m)z}{k(1+k+m)}} + 1\right)} \text{ for } m \in \mathbb{Z} \land (a,z) \in \mathbb{C}^2 \land m \geq 0$$

$$e^z z^{1-b}(1-Q(b-1,z)) = \frac{z}{\Gamma(b+1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\left(\frac{\left(1+(-1)^k\right)k}{4(-1+b+k)(b+k)} - \frac{\left(1-(-1)^k\right)\left(-1+b+\frac{1+k}{2}\right)}{2(-1+b+k)(b+k)}\right)z}{1} + 1\right)} + \frac{1}{\Gamma(b)} \text{ for } (b,z) \in \mathbb{C}^2$$

$$e^z z^{1-b}(1-Q(b-1,z)) = \frac{1}{\Gamma(b)\left(1 - \frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\left(1+(-1)^k\right)\left(b+\frac{1}{2}(-2+k)\right)+\frac{1}{4}\left(1-(-1)^k\right)(1+k)\right)z}{b+k} + b}\right)} \text{ for } (b,z) \in \mathbb{C}^2$$

$$e^z z^{1-b}(1-Q(b-1,z)) = \frac{1}{\Gamma(b-1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\left(1-(-1)^k\right)\left(b+\frac{1}{2}(-3+k)\right)+\frac{1}{4}\left(1+(-1)^k\right)k\right)z}{-1+b+k} + b - 1\right)} \text{ for } (b,z) \in \mathbb{C}^2$$

$$e^z z^{1-b}(1-Q(b-1,z)) = \frac{\frac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\left(1-(-1)^k\right)\left(b+\frac{1}{2}(-1+k)\right)+\frac{1}{4}\left(1+(-1)^k\right)k\right)z}{b+k} + b} + 1}{\Gamma(b)} \text{ for } (b,z) \in \mathbb{C}^2$$

$$e^z z^{1-b}(1-Q(b-1,z)) = \frac{\frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{\left(\frac{1}{2}(1+(-1)^k)\left(1-b-\frac{k}{2}\right)+\frac{1}{4}(1-(-1)^k)(1+k)\right)z}{b+k}+b-z}+1}{\Gamma(b)} \text{ for } (b,z) \in \mathbb{C}^2 \wedge \neg(b \in \mathbb{Z} \wedge b \leq 0)$$

$$e^z z^{1-b}(1-Q(b-1,z)) = e^z z^{1-b} - \frac{1}{\Gamma(b-1)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{4}(3-3(-1)^k+2(-1+(-1)^k)b+2k)}{\frac{1}{2}(1-(-1)^k+z+(-1)^k z)}+z\right)} \text{ for } (b,z) \in \mathbb{C}^2$$

$$e^z z^{1-b}(1-Q(b-1,z)) = \frac{1}{\Gamma(b-1)\left(\mathrm{K}_{k=1}^{\infty} \frac{kz}{-1+b+k-z}+b-z-1\right)} \text{ for } (b,z) \in \mathbb{C}^2$$

$$e^z z^{1-m}(1-Q(m-1,z)) = e^z z^{1-m} - \frac{1}{\Gamma(m-1)\left(\mathrm{K}_{k=1}^{\infty} \frac{-k(1+k-m)}{2+2k-m+z}-m+z+2\right)} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m > 1$$

$$e^z z^{-z}(1-Q(z,z)) = \frac{\mathrm{K}_{k=1}^{\infty} \frac{(1+k)z}{1+k}+1}{\Gamma(z+1)} \text{ for } z \in \mathbb{C}$$

$$\frac{{}_1\tilde{F}_1(a+1;b+1;z)}{{}_1\tilde{F}_1(a;b;z)} = \frac{1}{b\left(\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{(1-(-1)^k)\left(-a+b+\frac{1}{2}(-1+k)\right)}{2(-1+b+k)(b+k)}+\frac{(1+(-1)^k)\left(a+\frac{k}{2}\right)}{2(-1+b+k)(b+k)}\right)z}{1}+1\right)} \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge \neg(b \in$$

$$\frac{{}_1\tilde{F}_1(a;b;z)}{{}_1\tilde{F}_1(a+1;b+1;z)} = b\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\left(-\frac{(1-(-1)^k)\left(-a+b+\frac{1}{2}(-1+k)\right)}{2(-1+b+k)(b+k)}+\frac{(1+(-1)^k)\left(a+\frac{k}{2}\right)}{2(-1+b+k)(b+k)}\right)z}{1}+1\right) \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge \neg$$

$$\frac{{}_1\tilde{F}_1(a;b+1;z)}{{}_1\tilde{F}_1(a;b;z)} = \frac{1}{b\left(\mathrm{K}_{k=1}^{\infty} \frac{(-1)^{-1+k}\left(\frac{(1+(-1)^k)\left(-a+b+\frac{k}{2}\right)}{2(-1+b+k)(b+k)}+\frac{(1-(-1)^k)(-1+a+k)}{2(-1+b+k)(b+k)}\right)z}{1}+1\right)} \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge \neg(b \in$$

$$\frac{{}_1\tilde{F}_1(a;b;z)}{{}_1\tilde{F}_1(a;b+1;z)} = b\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{(-1)^{-1+k}\left(\frac{(1+(-1)^k)\left(-a+b+\frac{k}{2}\right)}{2(-1+b+k)(b+k)}+\frac{(1-(-1)^k)(-1+a+k)}{2(-1+b+k)(b+k)}\right)z}{1}+1\right) \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge$$

$$\frac{{}_1\tilde{F}_1(a+1;b+1;z)}{{}_1\tilde{F}_1(a;b;z)} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\left(\frac{1}{2}(1-(-1)^k)\left(a-b+\frac{1-k}{2}\right)+\frac{1}{2}(1+(-1)^k)\left(a+\frac{k}{2}\right)\right)z}{b+k} + b} \text{ for } (a,b,z) \in \mathbb{C}^3 \land \neg(b \in \mathbb{Z} \land b \leq$$

$$\frac{{}_1\tilde{F}_1(a+1;b+1;z)}{{}_1\tilde{F}_1(a;b;z)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{(-1+a+k)z}{-1+b+k-z}}{az} \text{ for } (a,b,z) \in \mathbb{C}^3$$

$$\frac{{}_1\tilde{F}_1(a;b;z)}{{}_1\tilde{F}_1(a+1;b+1;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(a+k)z}{b+k-z} + b - z \text{ for } (a,b,z) \in \mathbb{C}^3$$

$$\frac{{}_1\tilde{F}_1(-m;b;z)}{{}_1\tilde{F}_1(1-m;b+1;z)} = \mathop{\mathrm{K}}_{k=1}^{-1+m} \frac{(k-m)z}{b+k-z} + b - z \text{ for } m \in \mathbb{Z} \land (b,z) \in \mathbb{C}^2 \land m > 0$$

$$\frac{{}_1\tilde{F}_1(a;b+1;z)}{{}_1\tilde{F}_1(a;b;z)} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\left(\frac{1}{2}(1-(-1)^k)\left(a+\frac{1}{2}(-1+k)\right)+\frac{1}{2}(1+(-1)^k)\left(a-b-\frac{k}{2}\right)\right)z}{b+k} + b} \text{ for } (a,b,z) \in \mathbb{C}^3$$

$${}_2F_1(a,b;c;z) = \frac{abz}{c\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(a+k)(b+k)z}{(1+k)(c+k)}}{1+\frac{(a+k)(b+k)z}{(1+k)(c+k)}} + 1\right)} + 1 \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |z| < 1$$

$${}_2F_1(a,b;c;z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)(-1+b+k)z}{k(-1+c+k)}}{1+\frac{(-1+a+k)(-1+b+k)z}{k(-1+c+k)}} + 1} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |z| < 1$$

$${}_2F_1(a,b;c;1-z) = \frac{\Gamma(c)z^{-a-b+c}\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1-a+c+k)(-1-b+c+k)z}{k(-a-b+c+k)}}{1+\frac{(-1-a+c+k)(-1-b+c+k)z}{k(-a-b+c+k)}} + 1\right)} + \frac{\Gamma(c)\Gamma(-a-b+c)}{\Gamma(c-a)\Gamma(c-b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)(-1}{k(a+b-c}}{1+\frac{(-1+a+k)(-1}{k(a+b-c}}\right.}$$

$${}_2F_1(a,b;a+b;1-z) = -\frac{\log(z)\Gamma(a+b)}{\Gamma(a)\Gamma(b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)(-1+b+k)z}{k^2}}{1+\frac{(-1+a+k)(-1+b+k)z}{k^2}} + 1\right)} - \frac{\Gamma(a+b)(\psi^{(0)}(a)+}{\Gamma(a)\Gamma(b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)(-1+b+k)z(-}{k^2(-2\psi^{(0)}(k)+\psi}}{1+\frac{(-1+a+k)(-1+b+k)z(}{k^2(-2\psi^{(0)}(k)+\psi}}\right.}$$

$${}_2F_1(a,b;a+b-m;1-z) = \frac{(m-1)!z^{-m}\Gamma(a+b-m)}{\Gamma(a)\Gamma(b)\left(\mathrm{K}_{k=1}^{-1+m} \frac{-\frac{(-1+a+k-m)(-1+b+k-m)z}{k(k-m)}}{1+\frac{(-1+a+k-m)(-1+b+k-m)z}{k(k-m)}} + 1\right)} - \frac{(-1)^m\log(z)\Gamma}{m!\Gamma(a-m)\Gamma(b-m)\left(\mathrm{K}^{\infty}\right.}$$

$$ {}_2F_1(a,b;a+b+m;1-z) = -\frac{(-z)^m \log(z)\Gamma(a+b+m)}{m!\Gamma(a)\Gamma(b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k+m)(-1+b+k+m)z}{k(k+m)}}{1+\frac{(-1+a+k+m)(-1+b+k+m)z}{k(k+m)}} + 1\right)} + \frac{(-z)^m\Gamma(a+}{m!\Gamma(a)\Gamma(b)\left(\mathrm{K}_{k=1}^{\infty} \frac{(-1}{1-(-}\right.} $$

$$ {}_2F_1(a,b;c;z) = \frac{(-z)^{-a}\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)(a-c+k)}{k(a-b+k)z}}{1+\frac{(-1+a+k)(a-c+k)}{k(a-b+k)z}} + 1\right)} + \frac{(-z)^{-b}\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+b+k)(b-c+k)}{k(-a+b+k)z}}{1+\frac{(-1+b+k)(b-c+k)}{k(-a+b+k)z}} + 1\right)} $$

$$ {}_2F_1(a,a+m;c;z) = \frac{\Gamma(c)(a)_m(-z)^{-a-m}(a-c+1)_m(-\psi^{(0)}(-a+c-m) - \psi^{(0)}(a+m) + \psi^{(0)}(m}{m!\Gamma(c-a)\Gamma(a+m)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{(-1+a+k+m)(a-c+k+m)(-\psi^{(0)}(1+k)+\psi^{(0)}(-a+c-k-m)-\psi^{(0)}(1+k+m)+}{k(k+m)z(\psi^{(0)}(k)-\psi^{(0)}(1-a+c-k-m)+\psi^{(0)}(k+m)-\psi^{(0)}(-1+a+k}}{1-\frac{(-1+a+k+m)(a-c+k+m)(-\psi^{(0)}(1+k)+\psi^{(0)}(-a+c-k-m)-\psi^{(0)}(1+k+m)}{k(k+m)z(\psi^{(0)}(k)-\psi^{(0)}(1-a+c-k-m)+\psi^{(0)}(k+m)-\psi^{(0)}(-1+a-}}\right.} $$

$$ {}_2F_1(a,a+m;a-p;z) = \frac{(-1)^p(m+p)!(-z)^{-a-m}\Gamma(a-p)}{m!\Gamma(a)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k+m)(k+m+p)}{k(k+m)z}}{1+\frac{(-1+a+k+m)(k+m+p)}{k(k+m)z}} + 1\right)} \text{ for } m \in \mathbb{Z} \land p \in \mathbb{Z} \land (a,z) \in \mathbb{C}^2 \land m \geq 0 \land p $$

$$ {}_2F_1(a,a+m;a+p;z) = \frac{(-1)^m(-z)^{-a-p}\Gamma(a+p)^2}{p!\Gamma(a)(p-m)!\Gamma(a+m)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{k(-1+a+k+p)}{(k+p)(k-m+p)z}}{1+\frac{k(-1+a+k+p)}{(k+p)(k-m+p)z}} + 1\right)} + \frac{(-1)^m \log(-z)}{m!\Gamma(a)(-m+p-1)!\left(\mathrm{K}\right.} $$

$$ {}_2F_1(a,a+m;a+p;z) = \frac{(-1)^p(m-p)!(-z)^{-a-m}(a)_p}{m!\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k+m)(k+m-p)}{k(k+m)z}}{1+\frac{(-1+a+k+m)(k+m-p)}{k(k+m)z}} + 1\right)} + \frac{(-z)^{-a}\Gamma(m)\Gamma(a+p)}{\Gamma(p)\Gamma(a+m)\left(\mathrm{K}_{k=1}^{-1+p} \frac{-\frac{(-1+a+k)(k-p)}{k(k-m)z}}{1+\frac{(-1+a+k)(k-p)}{k(k-m)z}} + \right.} $$

$$ {}_2F_1(1,b;c;z) = \frac{bz}{c\left(\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{(1+(-1)^k)\left(-1-b+c+\frac{k}{2}\right)k}{4(-1+c+k)(c+k)} - \frac{(1-(-1)^k)\left(b+\frac{1+k}{2}\right)\left(-1+c+\frac{1+k}{2}\right)}{2(-1+c+k)(c+k)}\right)z}{1} + 1\right)} + 1 \text{ for } (b,c,z) \in \mathbb{C}^3 \land $$

$$ {}_2F_1(1,b;c;z) = \frac{bz}{c\left(\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{(1+(-1)^k)\left(-1-b+c+\frac{k}{2}\right)k}{4(-1+c+k)(c+k)} - \frac{(1-(-1)^k)\left(b+\frac{1+k}{2}\right)\left(-1+c+\frac{1+k}{2}\right)}{2(-1+c+k)(c+k)}\right)z}{1} + 1\right)} + 1 \text{ for } (b,c,z) \in \mathbb{C}^3 \land $$

$$ {}_2F_1(1,b;c;z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{(1-(-1)^k)\left(b+\frac{1}{2}(-1+k)\right)\left(-1+c+\frac{1}{2}(-1+k)\right)}{2(-2+c+k)(-1+c+k)} - \frac{(1+(-1)^k)\left(-1-b+c+\frac{k}{2}\right)k}{4(-2+c+k)(-1+c+k)}\right)z}{1} + 1} \text{ for } (b,c,z) \in \mathbb{C}^3 \land |a $$

$$ {}_2F_1(1,b;c;z) = \frac{1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left(-3+6b+2c-4bc+6k-4ck-2k^2+(-1)^k(-1+2b)(-3+2c+2k)\right)z}{8(-2+c+k)(-1+c+k)}}{1} + 1} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge |\arg(1-z)| < \pi $$

$$ {}_2F_1(1,b;c;z) = \frac{1}{1 - \frac{bz}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{z\left(-\frac{1}{2}\left(1+(-1)^k\right)\left(c+\left\lfloor \frac{1}{2}(-1+k)\right\rfloor\right)\left(b+\left\lfloor \frac{k}{2}\right\rfloor\right)+\frac{1}{2}\left(1-(-1)^k\right)\left(b-c-\left\lfloor \frac{1}{2}(-1+k)\right\rfloor\right)\left\lfloor \frac{1+k}{2}\right\rfloor\right)}{c+k} + c}} \text{ for } (b,c,z) \in $$

$$ {}_2F_1(1,b;c;z) = \frac{c-1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{1}{2}(1-(-1)^k)\left(c+\frac{1}{2}(-3+k)\right)\left(b+\frac{1}{2}(-1+k)\right)-\frac{1}{4}(1+(-1)^k)\left(-1-b+c+\frac{k}{2}\right)k\right)z}{-1+c+k} + c - 1} \text{ for } (b,c,z) \in $$

$$ {}_2F_1(1,b;c;z) = \frac{c-1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}(1+(-1)^k)k(1-z)-\frac{1}{2}(1-(-1)^k)\left(-1+b+\frac{1+k}{2}\right)z}{-(-1)^k+\frac{1}{2}(1+(-1)^k)c} + c - 1} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge \Re(c) > 1 \wedge |\arg(1 $$

$$ {}_2F_1(1,b;c;z) = \frac{c-1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{k(-1+b+k)(1-z)z}{-1+c+k-(b+2k)z} - bz + c - 1} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge |\arg(1-z)| < \pi \wedge \Re(z) < \frac{1}{2} $$

$$ {}_2F_1(1,b;c;z) = \frac{\Gamma(1-b)(-z)^{1-c}\Gamma(c)(1-z)^{-b+c-1}}{\Gamma(c-b)} - \frac{c-1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{-k(1-c+k)(1-z)}{2-c+2k+(-1+b-k)z} + (b-1)z - c + 2} \text{ for } (b,c, $$

$$ {}_2F_1(1,b;c;z) = \frac{c-1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{-k(-1-b+c+k)z}{-1+c+k+(1-b+k)z} + (1-b)z + c - 1} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge |z| < 1 $$

$$ {}_2F_1(1,b;c;z) = \frac{c-1}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{k(-1+b+k)(z-z^2)}{-1+c+k-(b+2k)z} - bz + c - 1} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge |\arg(1-z)| < \pi $$

$$ {}_2F_1(1,b;c;z) = \frac{c-1}{z\left(\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{k(-1-b+c+k)}{z}}{1-b+k+\frac{-1+c+k}{z}} - b + \frac{c-1}{z} + 1\right)} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge |z| < 1 $$

$$ {}_2F_1\left(1, \frac{1}{m}; \frac{1}{m}+1; -z^m\right) = \frac{1}{\frac{z^m}{\mathop{\Large\mathrm{K}}_{k=1}^{\infty} \frac{z^m\left(\frac{1}{2}\left(1+(-1)^k\right)+m\left\lfloor \frac{1+k}{2}\right\rfloor\right)^2}{1+(1+k)m} + m + 1} + 1} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m > 0 \wedge |\arg(1-z) $$

$$ {}_2F_1\left(1, \frac{z}{t-1}; \frac{z}{t-1}+1; \frac{1}{t}\right) = \frac{tz}{(t-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{t^{\frac{1}{2}(1+(-1)^k)\left\lfloor\frac{1+k}{2}\right\rfloor}}{z^{\frac{1}{2}(1+(-1)^k)}} + z\right)} \text{ for } (t, z) \in \mathbb{C}^2 \wedge \neg (t, z) \in \mathbb{R}^2 $$

$$ {}_2F_1\left(m, \frac{m+z}{2}; \frac{m+z}{2}+1; \frac{a-1}{a+1}\right) = \frac{2^{-m}(a+1)^m(m+z)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{4}(1+(-1)^k)(1+a)k + \frac{1}{2}(1-(-1)^k)(-1+a)\left(-1+\frac{1+k}{2}+m\right)}{(m+z)^{\frac{1}{2}(1+(-1)^k)}} + m + z} \text{ for } ($$

$$ {}_2F_1\left(m, \frac{m+z}{2}; \frac{m+z}{2}+1; \frac{a-1}{a+1}\right) = \frac{2^{-m}(a+1)^m(m+z)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(1-a^2)k(-1+k+m)}{a(2k+m)+z} + am + z} \text{ for } (a, m, z) \in \mathbb{C}^3 $$

$$ {}_2F_1\left(m, \frac{m+z}{2}; \frac{m+z}{2}+1; -1\right) = \frac{2^{-m}(m+z)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(-1+k+m)}{z} + z} \text{ for } (m, z) \in \mathbb{C}^2 $$

$$ {}_2F_1\left(1, \frac{p+1}{q}; \frac{p+q+1}{q}; -z^q\right) = \frac{p+1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{8}(1+(-1)^k)k^2q^2z^q + \frac{1}{2}(1-(-1)^k)\left(1+p+\frac{1}{2}(-1+k)q\right)^2 z^q}{1+p+kq} + p + 1} \text{ for } (p, q, z) $$

$$ \frac{{}_2F_1(a, b+1; c+1; z)}{{}_2F_1(a, b; c; z)} = \frac{1}{1 - \frac{az(c-b)}{c(c+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(\frac{1}{2}(1-(-1)^k)\left(-b-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-a+c+\left\lfloor\frac{1+k}{2}\right\rfloor\right) - \frac{1}{2}(1+(-1)^k)\left(a+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-b+c+\left\lfloor\frac{1+k}{2}\right\rfloor\right.\right.}{(c+k)(1+c+k)}}{1}\right.}} $$

$$ \frac{{}_2F_1(a, b+1; c+1; z)}{{}_2F_1(a, b; c; z)} = \frac{c}{\frac{az(b-c)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z\left(\frac{1}{2}(1+(-1)^k)\left(b-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(a+\left\lfloor\frac{1+k}{2}\right\rfloor\right) + \frac{1}{2}(1-(-1)^k)\left(a-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(b+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\right)}{1+c+k} + c + 1} + c} \text{ fo} $$

$$ \frac{{}_2F_1(a, b+1; c+1; z)}{{}_2F_1(a, b; c; z)} = \frac{c}{(z(-a+b+1)+c)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{(b+k)(-a+c+k)z}{(c+z-az+bz)^2}}{\frac{c+k+z-az+bz+kz}{c+z-az+bz}} + 1\right)} \text{ for } (a, b, c, z) \in \mathbb{C}^4 \wedge |z| < 1 $$

$$ \frac{{}_2F_1(a, b+1; c+1; z)}{{}_2F_1(a, b; c; z)} = \frac{c}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-b-k)(-a+c+k)z}{c+k+(1-a+b+k)z} + z(-a+b+1) + c} \text{ for } (a, b, c, z) \in \mathbb{C}^4 \wedge |z| < 1 $$

$$ \frac{{}_2F_1(a+1, b; c+1; z)}{{}_2F_1(a, b; c; z)} = \frac{1}{1 - \frac{bz(c-a)}{c(c+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(-\frac{1}{2}(1+(-1)^k)\left(b+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-a+c+\left\lfloor\frac{1+k}{2}\right\rfloor\right) + \frac{1}{2}(1-(-1)^k)\left(-a-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-b+c+\left\lfloor\frac{1-}{}\right.\right.\right.}{(c+k)(1+c+k)}}{1}\right.}} $$

$$\frac{{}_2F_1(a+1,b;c+1;z)}{{}_2F_1(a,b;c;z)} = \frac{c}{\frac{bz(a-c)}{\mathrm{K}_{k=1}^{\infty} \frac{z\left(\frac{1}{2}\left(1-(-1)^k\right)\left(b-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(a+\left\lfloor\frac{1+k}{2}\right\rfloor\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(a-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(b+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\right)}{1+c+k}+c+1}+c} \text{ fo}$$

$$\frac{{}_2F_1(a+1,b;c+1;z)}{{}_2F_1(a,b;c;z)} = \frac{c}{(z(a-b+1)+c)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(a+k)(-b+c+k)z}{(c+z+az-bz)^2}}{\frac{c+k+z+az-bz+kz}{c+z+az-bz}}+1\right)} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |z| < 1$$

$$\frac{{}_2F_1(a+1,b;c+1;z)}{{}_2F_1(a,b;c;z)} = \frac{c}{\mathrm{K}_{k=1}^{\infty} \frac{(-a-k)(-b+c+k)z}{c+k+(1+a-b+k)z}+z(a-b+1)+c} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |z| < 1$$

$$\frac{{}_2F_1(a+1,b;c+1;-1)}{{}_2F_1(a,b;c;-1)} = \frac{c}{\mathrm{K}_{k=1}^{\infty} \frac{(a+k)(-b+c+k)}{-1-a+b+c}-a+b+c-1} \text{ for } (a,b,c) \in \mathbb{C}^3 \land \Re(a) > 0 \land \Re(c-b) > 0 \land \Re$$

$$\frac{{}_2F_1(a+1,b+1;c+1;z)}{{}_2F_1(a,b;c;z)} = \frac{c\,\mathrm{K}_{k=1}^{\infty} \frac{(-1+a+k)(-1+b+k)\left(z-z^2\right)}{-1+c+k-(-1+a+b+2k)z}}{ab\left(z-z^2\right)} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |\arg(1-z)| < \pi \land \Re(z) <$$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a+1,b;c+1;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1-(-1)^k\right)\left(-b+\frac{1-k}{2}\right)\left(-a+c+\frac{1}{2}(-1+k)\right)}{2(-1+c+k)(c+k)}+\frac{\left(1+(-1)^k\right)\left(-a-\frac{k}{2}\right)\left(-b+c+\frac{k}{2}\right)}{2(-1+c+k)(c+k)}\right)z}{1}+1 \text{ for } ((\Re$$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a+1,b;c+1;z)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{(-a-k)(-b+c+k)z}{c+k+(1+a-b+k)z}}{c}+\frac{z(a-b+1)}{c}+1 \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |\arg(1-z)| < \pi \land \Re($$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a+1,b;c+1;z)} = \frac{z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{(b-c-k)(a+k)}{z}}{1+a-b+k+\frac{c+k}{z}}+a-b+1\right)}{c}+1 \text{ for } (z=-1 \land \Re(-a+b+c) > |\Im(a-b+c$$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a,b+1;c+1;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1-(-1)^k\right)\left(-a+\frac{1-k}{2}\right)\left(-b+c+\frac{1}{2}(-1+k)\right)}{2(-1+c+k)(c+k)}+\frac{\left(1+(-1)^k\right)\left(-b-\frac{k}{2}\right)\left(-a+c+\frac{k}{2}\right)}{2(-1+c+k)(c+k)}\right)z}{1}+1 \text{ for } ((\Re$$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a,b+1;c+1;z)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{(a-c-k)(b+k)z}{c+k+(1-a+b+k)z}}{c}+\frac{z(-a+b+1)+c}{c} \text{ for } (z=-1 \land \Re(a+b+c)-1 > |\Im(-a+$$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a,b+1;c+1;z)} = \frac{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(a-c-k)(b+k)}{z}}{1-a+b+k+\frac{c+k}{z}} - a + b + 1\right)}{c} + 1 \text{ for } (z = -1 \wedge \Re(a-b+c) > |\Im(-a+b+c$$

$$\frac{{}_2F_1(a,b;c;z)}{{}_2F_1(a+1,b+1;c+1;z)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{(a+k)(b+k)\left(z-z^2\right)}{c+k-(1+a+b+2k)z}}{c} - \frac{z(a+b+1)}{c} + 1 \text{ for } \left(z = \frac{1}{2} \wedge \Re(-a-b+2c) - 1 >\right.$$

$$\frac{{}_2F_1(b,-m;c;z)}{{}_2F_1(b+1,1-m;c+1;z)} = \frac{\operatorname*{K}_{k=1}^{-1+m} \frac{(b+k)(k-m)\left(z-z^2\right)}{c+k-(1+b+2k-m)z}}{c} - \frac{z(b-m+1)}{c} + 1 \text{ for } m \in \mathbb{Z} \wedge (b,c,z) \in \mathbb{C}^3 \wedge m \geq 1$$

$$\frac{{}_2F_1(a,b+1;a+b+2;-1)}{{}_2F_1(a,b;a+b+1;-1)} = \frac{a+b+1}{\operatorname*{K}_{k=1}^{\infty} \frac{(b+k)(1+b+k)}{2a} + 2a} \text{ for } (a,b) \in \mathbb{C}^2 \wedge \Re(a) > 0 \wedge \Re(b) > 0$$

$$\frac{-2c\,{}_2F_1\left(1, \frac{c+1}{2}; \frac{c+5}{2}; -1\right) + c + 3}{\psi^{(0)}\left(\frac{c+3}{4}\right) - \psi^{(0)}\left(\frac{c+1}{4}\right)} = \frac{(c+1)(c+3)}{2\left(\operatorname*{K}_{k=1}^{\infty} \frac{(1+k)^2}{c} + c\right)} \text{ for } c \in \mathbb{C} \wedge \Re(c) > 0$$

$${}_2\tilde{F}_1(a,b;c;z) = \frac{\frac{abz}{c\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(a+k)(b+k)z}{(1+k)(c+k)}}{1+\frac{(a+k)(b+k)z}{(1+k)(c+k)}} + 1\right)} + 1}{\Gamma(c)} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \wedge |z| < 1$$

$${}_2\tilde{F}_1(a,b;c;z) = \frac{1}{\Gamma(c)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)(-1+b+k)z}{k(-1+c+k)}}{1+\frac{(-1+a+k)(-1+b+k)z}{k(-1+c+k)}} + 1\right)} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \wedge |z| < 1$$

$${}_2\tilde{F}_1(a,b;-m;z) = \frac{z^{m+1}(a)_{m+1}(b)_{m+1}}{(m+1)!\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(a+k+m)(b+k+m)z}{k(1+k+m)}}{1+\frac{(a+k+m)(b+k+m)z}{k(1+k+m)}} + 1\right)} \text{ for } m \in \mathbb{Z} \wedge (a,b,z) \in \mathbb{C}^3 \wedge m \geq 0 \wedge |z| < 1$$

$${}_2\tilde{F}_1(1,b;c;z) = \frac{\frac{bz}{c\left(\operatorname*{K}_{k=1}^{\infty} \frac{\left(-\frac{\left(1+(-1)^k\right)\left(-1-b+c+\frac{k}{2}\right)k}{4(-1+c+k)(c+k)} - \frac{\left(1-(-1)^k\right)\left(b+\frac{1+k}{2}\right)\left(-1+c+\frac{1+k}{2}\right)}{2(-1+c+k)(c+k)}\right)z}{1} + 1\right)} + 1}{\Gamma(c)} \text{ for } (b,c,z) \in \mathbb{C}^3 \wedge \neg($$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(-\frac{\left(1-(-1)^k\right)\left(b+\frac{1}{2}(-1+k)\right)\left(-1+c+\frac{1}{2}(-1+k)\right)}{2(-2+c+k)(-1+c+k)}-\frac{\left(1+(-1)^k\right)\left(-1-b+c+\frac{k}{2}\right)k}{4(-2+c+k)(-1+c+k)}\right)z}{1}+1\right)} \text{ for } (b,c,z)$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left(-11+6b+6c-4bc+10k-4ck-2k^2-(-1)^k(-1+2b)(-5+2c+2k)\right)z}{8(-3+c+k)(-2+c+k)}}{1}+1\right)} \text{ for } (b,c,z)\in\mathbb{C}^3\wedge\neg(c\in$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c)\left(1-\frac{bz}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{z\left(\frac{1}{2}\left(1-(-1)^k\right)\left(b-c-\left\lfloor\frac{k}{2}\right\rfloor\right)\left(1+\left\lfloor\frac{k}{2}\right\rfloor\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(1-c-\left\lfloor\frac{k}{2}\right\rfloor\right)\left(b+\left\lfloor\frac{k}{2}\right\rfloor\right)\right)}{c+k}+c}\right)} \text{ for } (b,c,z)$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(-\frac{1}{2}(1-(-1)^k)\left(c+\frac{1}{2}(-3+k)\right)\left(b+\frac{1}{2}(-1+k)\right)-\frac{1}{4}(1+(-1)^k)\left(-1-b+c+\frac{k}{2}\right)k\right)z}{-1+c+k}+c-1\right)} \text{ f}$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{4}(1+(-1)^k)k(1-z)-\frac{1}{2}(1-(-1)^k)\left(-1+b+\frac{1+k}{2}\right)z}{-(-1)^k+\frac{1}{2}(1+(-1)^k)c}+c-1\right)} \text{ for } (b,c,z)\in\mathbb{C}^3\wedge\neg(c$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k(-1+b+k)(1-z)z}{-1+c+k-(b+2k)z}-bz+c-1\right)} \text{ for } (b,c,z)\in\mathbb{C}^3\wedge\neg(c\in\mathbb{Z}\wedge c\leq 1)\wedge\Re(z)<$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{\Gamma(1-b)(-z)^{1-c}(1-z)^{-b+c-1}}{\Gamma(c-b)}-\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k(1-c+k)(1-z)}{2-c+2k+(-1+b-k)z}+(b-1)z-c+2\right)} \text{ fo}$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k(-1-b+c+k)z}{-1+c+k+(1-b+k)z}+(1-b)z+c-1\right)} \text{ for } (b,c,z)\in\mathbb{C}^3\wedge\neg(c\in\mathbb{Z}\wedge c\leq 1)\wedge|$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k(-1+b+k)(z-z^2)}{-1+c+k-(b+2k)z}-bz+c-1\right)} \text{ for } (b,c,z)\in\mathbb{C}^3\wedge\neg(c\in\mathbb{Z}\wedge c\leq 1)\wedge\Re(z)<$$

$${}_2\tilde{F}_1(1,b;c;z)=\frac{1}{\Gamma(c-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k(-1-b+c+k)}{z}}{1-b+k+\frac{-1+c+k}{z}}-b+\frac{c-1}{z}+1\right)} \text{ for } (b,c,z)\in\mathbb{C}^3\wedge\neg(c\in\mathbb{Z}\wedge c\leq 1)\wedge|z|<1$$

$$ {}_2\tilde{F}_1\left(1, \frac{1}{m}; \frac{1}{m}+1; -z^m\right) = \frac{1}{\Gamma\left(1+\frac{1}{m}\right)\left(\frac{z^m}{\mathrm{K}_{k=1}^{\infty} \frac{z^m\left(\frac{1}{2}\left(1+(-1)^k\right)+m\left\lfloor\frac{1+k}{2}\right\rfloor\right)^2}{1+(1+k)m}+m+1}+1\right)} \text{ for } m \in \mathbb{Z} \land z \in \mathbb{C} \land m > $$

$$ {}_2\tilde{F}_1\left(1, \frac{z}{t-1}; \frac{z}{t-1}+1; \frac{1}{t}\right) = \frac{t}{\Gamma\left(\frac{z}{t-1}\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{t^{\frac{1}{2}\left(1+(-1)^k\right)}\left\lfloor\frac{1+k}{2}\right\rfloor}{z^{\frac{1}{2}\left(1+(-1)^k\right)}}+z\right)} \text{ for } (t,z) \in \mathbb{C}^2 \land \neg (t,z) \in \mathbb{R}^2 $$

$$ {}_2\tilde{F}_1\left(m, \frac{m+z}{2}; \frac{m+z}{2}+1; \frac{a-1}{a+1}\right) = \frac{2^{1-m}(a+1)^m}{\Gamma\left(\frac{m+z}{2}\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{4}(1+(-1)^k)(1+a)k+\frac{1}{2}(1-(-1)^k)(-1+a)\left(-1+\frac{1+k}{2}+m\right)}{(m+z)^{\frac{1}{2}\left(1+(-1)^k\right)}} + m $$

$$ {}_2\tilde{F}_1\left(m, \frac{m+z}{2}; \frac{m+z}{2}+1; \frac{a-1}{a+1}\right) = \frac{2^{1-m}(a+1)^m}{\Gamma\left(\frac{m+z}{2}\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{(1-a^2)k(-1+k+m)}{a(2k+m)+z}+am+z\right)} \text{ for } (a,m,z) \in \mathbb{C}^3 $$

$$ {}_2\tilde{F}_1\left(m, \frac{m+z}{2}; \frac{m+z}{2}+1; -1\right) = \frac{2^{1-m}}{\Gamma\left(\frac{m+z}{2}\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{k(-1+k+m)}{z}+z\right)} \text{ for } (m,z) \in \mathbb{C}^2 $$

$$ \frac{{}_2\tilde{F}_1(a,b+1;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{c\left(1-\frac{az(c-b)}{c(c+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z\left(\frac{1}{2}\left(1-(-1)^k\right)\left(-b-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-a+c+\left\lfloor\frac{1+k}{2}\right\rfloor\right)-\frac{1}{2}\left(1+(-1)^k\right)\left(a+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-b+c+ \right.\right.}{(c+k)(1+c+k)}}{1} $$

$$ \frac{{}_2\tilde{F}_1(a,b+1;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{\frac{az(b-c)}{\mathrm{K}_{k=1}^{\infty} \frac{z\left(\frac{1}{2}\left(1+(-1)^k\right)\left(b-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(a+\left\lfloor\frac{1+k}{2}\right\rfloor\right)+\frac{1}{2}\left(1-(-1)^k\right)\left(a-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(b+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\right)}{1+c+k}+c+1}+c} \text{ fo} $$

$$ \frac{{}_2\tilde{F}_1(a,b+1;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{(z(-a+b+1)+c)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(b+k)(-a+c+k)z}{(c+z-az+bz)^2}}{\frac{c+k+z-az+bz+kz}{c+z-az+bz}}+1\right)} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |z| < 1 $$

$$ \frac{{}_2\tilde{F}_1(a,b+1;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{(-b-k)(-a+c+k)z}{c+k+(1-a+b+k)z}+z(-a+b+1)+c} \text{ for } (a,b,c,z) \in \mathbb{C}^4 \land |z| < 1 $$

$$\frac{{}_2\tilde{F}_1(a+1,b;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{c\left(1-\frac{bz(c-a)}{c(c+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z\left(-\frac{1}{2}\left(1+(-1)^k\right)\left(b+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-a+c+\left\lfloor\frac{1+k}{2}\right\rfloor\right)+\frac{1}{2}\left(1-(-1)^k\right)\left(-a-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(-b+c\cdots\right)\right)}{(c+k)(1+c+k)}}{1}\cdots\right)}\right)\cdots}$$

$$\frac{{}_2\tilde{F}_1(a+1,b;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{\frac{bz(a-c)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{z\left(\frac{1}{2}\left(1-(-1)^k\right)\left(b-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(a+\left\lfloor\frac{1+k}{2}\right\rfloor\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(a-c-\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(b+\left\lfloor\frac{1+k}{2}\right\rfloor\right)\right)}{1+c+k}+c+1}+c} \text{ fo}\cdots$$

$$\frac{{}_2\tilde{F}_1(a+1,b;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{(z(a-b+1)+c)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(a+k)(-b+c+k)z}{(c+z+az-bz)^2}}{\frac{c+k+z+az-bz+kz}{c+z+az-bz}}+1\right)} \text{ for } (a,b,c,z)\in\mathbb{C}^4\wedge|z|<1$$

$$\frac{{}_2\tilde{F}_1(a+1,b;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-a-k)(-b+c+k)z}{c+k+(1+a-b+k)z}+z(a-b+1)+c} \text{ for } (a,b,c,z)\in\mathbb{C}^4\wedge|z|<1$$

$$\frac{{}_2\tilde{F}_1(a+1,b;c+1;-1)}{{}_2\tilde{F}_1(a,b;c;-1)} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(a+k)(-b+c+k)}{-1-a+b+c}-a+b+c-1} \text{ for } (a,b,c)\in\mathbb{C}^3\wedge\Re(a)>0\wedge\Re(c-b)>0\wedge\Re\cdots$$

$$\frac{{}_2\tilde{F}_1(a+1,b+1;c+1;z)}{{}_2\tilde{F}_1(a,b;c;z)} = \frac{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-1+a+k)(-1+b+k)\left(z-z^2\right)}{-1+c+k-(-1+a+b+2k)z}}{ab\left(z-z^2\right)} \text{ for } (a,b,c,z)\in\mathbb{C}^4\wedge\Re(z)<\frac{1}{2}$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a+1,b;c+1;z)} = c\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\left(\frac{\left(1-(-1)^k\right)\left(-b+\frac{1-k}{2}\right)\left(-a+c+\frac{1}{2}(-1+k)\right)}{2(-1+c+k)(c+k)}+\frac{\left(1+(-1)^k\right)\left(-a-\frac{k}{2}\right)\left(-b+c+\frac{k}{2}\right)}{2(-1+c+k)(c+k)}\right)z}{1}\right)+c \text{ fo}\cdots$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a+1,b;c+1;z)} = \mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(-a-k)(-b+c+k)z}{c+k+(1+a-b+k)z}+z(a-b+1)+c \text{ for } (a,b,c,z)\in\mathbb{C}^4\wedge\Re(c-a)>0\wedge\Re(a)\cdots$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a+1,b;c+1;z)} = z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(b-c-k)(a+k)}{z}}{1+a-b+k+\frac{c+k}{z}}+a-b+1\right)+c \text{ for } (z=-1\wedge\Re(-a+b+c)>|\Im(a-b\cdots$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a,b+1;c+1;z)} = c\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\left(\frac{\left(1-(-1)^k\right)\left(-a+\frac{1-k}{2}\right)\left(-b+c+\frac{1}{2}(-1+k)\right)}{2(-1+c+k)(c+k)} + \frac{\left(1+(-1)^k\right)\left(-b-\frac{k}{2}\right)\left(-a+c+\frac{k}{2}\right)}{2(-1+c+k)(c+k)}\right) z}{1}\right) + c \text{ fo}$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a,b+1;c+1;z)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{(a-c-k)(b+k)z}{c+k+(1-a+b+k)z} + z(-a+b+1) + c \text{ for } (z=-1 \wedge \Re(a+b+c)-1 > |\Im(-a$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a,b+1;c+1;z)} = z\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{(a-c-k)(b+k)}{z}}{1-a+b+k+\frac{c+k}{z}} - a + b + 1\right) + c \text{ for } (z=-1 \wedge \Re(a-b+c) > |\Im(-a+b$$

$$\frac{{}_2\tilde{F}_1(a,b;c;z)}{{}_2\tilde{F}_1(a+1,b+1;c+1;z)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{(a+k)(b+k)\left(z-z^2\right)}{c+k-(1+a+b+2k)z} - z(a+b+1) + c \text{ for } \left(z = \frac{1}{2} \wedge \Re(-a-b+2c) - 1\right.$$

$$\frac{{}_2\tilde{F}_1(b,-m;c;z)}{{}_2\tilde{F}_1(b+1,1-m;c+1;z)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{(b+k)(k-m)\left(z-z^2\right)}{c+k-(1+b+2k-m)z} - z(b-m+1) + c \text{ for } m \in \mathbb{Z} \wedge (b,c,z) \in \mathbb{C}^3 \wedge m \geq$$

$$\frac{{}_2\tilde{F}_1(a,b+1;a+b+2;-1)}{{}_2\tilde{F}_1(a,b;a+b+1;-1)} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{(b+k)(1+b+k)}{2a} + 2a} \text{ for } (a,b) \in \mathbb{C}^2 \wedge \Re(a) > 0 \wedge \Re(b) > 0$$

$$(1-z)^{-a} = \frac{az}{\mathrm{K}_{k=1}^{\infty} \frac{-\frac{(a+k)z}{1+k}}{1+\frac{(a+k)z}{1+k}} + 1} + 1 \text{ for } (a,z) \in \mathbb{C}^2 \wedge |\arg(1-z)| < \pi$$

$${}_2F_0(1-nz) = \frac{1}{\mathrm{K}_{k=1}^{2n} \frac{\left(-\frac{1}{4}(1+(-1)^k)k - \frac{1}{2}(1-(-1)^k)\left(\frac{1}{2}(-1+k)-n\right)\right)z}{1} + 1} \text{ for } n \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge n \geq 0$$

$$\frac{{}_2F_3\left(a,a+\frac{1}{2};2a,2a-b+1,b;z\right)}{{}_2F_3\left(a-\frac{1}{2},a;2a-1,2a-b,b;z\right)} = \frac{2a-b}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{z}{4}}{2a-b+k} + 2a - b} \text{ for } (a,b,z) \in \mathbb{C}^3$$

$${}_3F_2\left(1,a,b;-a+\frac{z}{2}+2,-b+\frac{z}{2}+2;1\right) = \frac{(-2a+z+2)(-2b+z+2)}{(-2a-2b+z+3)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{k(2-2a-2b+k+z)(2-4a+2k+z)(2-4b+2k+z)}{(1-2a-2b+2k+z)(3-2a-2b+2k+z)}}{z} + \right.}$$

$$ {}_3F_2\left(1,a,a+\frac{1}{2};b,b+\frac{1}{2};1\right) = \frac{(b-1)(2b-1)}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{4}k(-2-2a+2b+k)(-3-2a+2b+2k)(-1-2a+2b+2k)}{\frac{1}{2}(-3+2a+2b)(-1-2a+2b+2k)} - \frac{1}{2}(2a-2b+1)(2a+2 $$

$$ \frac{{}_3F_2\left(a,b,c;1,\frac{3}{2};1\right)}{{}_3F_2\left(a,b,c;\frac{1}{2},1;1\right)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{(2a-k)(2b-k)(2c-k)(1-2a-2b-2c+2k)}{1-2a-2b+4ab-2c+4ac+4bc+3k-4ak-4bk-4ck+3k^2}}{(2a-1)(1-2b)(1-2c)} \text{ for } (a,b,c)\in\mathbb{C}^3\wedge\Re(a+b+c) < \frac{3}{2} $$

$$ \frac{{}_3F_2(a,b,c;2,d;1)}{{}_3F_2(a,b,c;1,d;1)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{(a-k)(-b+k)(-c+k)(-a-b-c+d+k)}{-1+a+b-ab+c-ac-bc+(-2+2a+2b+2c-d)k-2k^2}}{(1-a)(1-b)(1-c)} \text{ for } (a,b,c,d)\in\mathbb{C}^4\wedge\Re(a+b+c-d) < $$

$$ \frac{{}_3F_2(a,b,c;1,d;1)}{{}_3F_2(a+1,b+1,c+1;2,d+1;1)} = -\frac{\operatorname*{K}_{k=1}^{\infty} \frac{(1+a+b+c-d-k)(-a+k)(-b+k)(-c+k)}{b+c-bc-d-a(-1+b+c-2k)+(-1+2b+2c-d)k-2k^2}}{d} - \frac{a+b+c}{d} + 1 \text{ for} $$

$$ \frac{{}_2F_0(a-nz)}{{}_2F_0(a1-nz)} = \operatorname*{K}_{k=1}^{2n} \frac{\left(-\frac{1}{2}\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)\right) - \frac{1}{2}\left(1+(-1)^k\right)\left(\frac{k}{2}-n\right)\right)z}{1} + 1 \text{ for } n\in\mathbb{Z}\wedge(a,z)\in\mathbb{C} $$

$$ \frac{{}_2F_0(a-nz)}{{}_2F_0(a1-nz)} = \frac{az}{\operatorname*{K}_{k=1}^{-1+n} \frac{(-a-k)(k-n)z^2}{-1+(1+a+2k-n)z} + (1-n)z - 1} + 1 \text{ for } n\in\mathbb{Z}\wedge(a,z)\in\mathbb{C}^2\wedge n\geq 0 $$

$$ U(a,b,z) = \frac{z^{1-b}\Gamma(b-1)}{\Gamma(a)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(a-b+k)z}{(-1+b-k)k}}{1-\frac{(a-b+k)z}{(-1+b-k)k}} + 1\right)} + \frac{\Gamma(1-b)}{\Gamma(a-b+1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z}{k(-1+b+k)}}{1+\frac{(-1+a+k)z}{k(-1+b+k)}} + 1\right)} \text{ for } (a,b,z)\in\mathbb{C}^3\wedge b \notin $$

$$ U(a,1,z) = -\frac{\frac{\log(z)}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z}{k^2}}{1+\frac{(-1+a+k)z}{k^2}} + 1} + \frac{\psi^{(0)}(a)+2\gamma}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z(2\psi^{(0)}(1+k)-\psi^{(0)}(a+k))}{k^2(2\psi^{(0)}(k)-\psi^{(0)}(-1+a+k))}}{1+\frac{(-1+a+k)z(2\psi^{(0)}(1+k)-\psi^{(0)}(a+k))}{k^2(2\psi^{(0)}(k)-\psi^{(0)}(-1+a+k))}} + 1}}{\Gamma(a)} \text{ for } (a,z)\in\mathbb{C}^2 $$

$$ U(a,m,z) = \frac{(-1)^m \left(\frac{\log(z)}{(m-1)!\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z}{k(-1+k+m)}}{1+\frac{(-1+a+k)z}{k(-1+k+m)}} + 1\right)} + \frac{\psi^{(0)}(a)-\psi^{(0)}(m)+\gamma}{(m-1)!\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(-1+a+k)z(\psi^{(0)}(1+k)-\psi^{(0)}(a+k)+\psi^{(0)}(k+}{k(-1+k+m)(\psi^{(0)}(k)-\psi^{(0)}(-1+a+k)+\psi^{(0)}(-1-}}{1+\frac{(-1+a+k)z(\psi^{(0)}(1+k)-\psi^{(0)}(a+k)+\psi^{(0)}(k+}{k(-1+k+m)(\psi^{(0)}(k)-\psi^{(0)}(-1+a+k)+\psi^{(0)}(-1-}} \right.}}{\Gamma(a-m+1)} $$

$$U(a,-m,z)=\frac{(-1)^m\left(\frac{(-1)^m m!}{(a)_{m+1}\left(\mathrm{K}_{k=1}^{m}\frac{\frac{(-1+a+k)z}{k(1-k+m)}}{1-\frac{(-1+a+k)z}{k(1-k+m)}}+1\right)}+\frac{z^{m+1}\log(z)}{(m+1)!\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{(a+k+m)z}{k(1+k+m)}}{1+\frac{(a+k+m)z}{k(1+k+m)}}+1\right)}+\cdots\right.}{\Gamma(a)}$$

$$\frac{U(a,b,z)}{U(a,b-1,z)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{\frac{1}{2}\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1+k)\right)+\frac{1}{2}\left(1+(-1)^k\right)\left(1+a-b+\frac{k}{2}\right)}{z}}{1}+1 \text{ for } (a,b,z)\in\mathbb{C}^3\wedge b\notin\mathbb{Z}$$

$$\frac{U(a,b,z)}{U(a,b-1,z)}=\frac{a}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{\frac{1}{2}\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)+\frac{1}{2}\left(1-(-1)^k\right)\left(1+a-b+\frac{1+k}{2}\right)}{z}}{1}+1\right)}+1 \text{ for } (a,b,z)\in\mathbb{C}^3\wedge b\notin\mathbb{Z}$$

$$\frac{U(a,b,z)}{U(a+1,b,z)}=-\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1-a+b-k)(a+k)}{-2-2a+b-2k-z}+2a-b+z+2 \text{ for } (a,b,z)\in\mathbb{C}^3\wedge b\notin\mathbb{Z}$$

$$\frac{U(a,b,z)}{U(a+1,b,z)}=z\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{(a+k)(1+a-b+k)}{z^2}}{1+\frac{2+2a-b+2k}{z}}+\frac{2a-b+2}{z}+1\right) \text{ for } (a,b,z)\in\mathbb{C}^3\wedge b\notin\mathbb{Z}$$

$$\frac{\frac{\partial U(a,b,z)}{\partial z}}{U(a,b,z)}=-\frac{a(a-b+1)}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{(-1-a+b-k)(a+k)}{-2-2a+b-2k-z}-2a+b-z-2\right)}-\frac{a}{z} \text{ for } (a,b,z)\in\mathbb{C}^3\wedge b\notin\mathbb{Z}$$

$$\frac{U\left(a,\frac{1}{2},z^2\right)}{U\left(a+1,\frac{3}{2},z^2\right)}=\frac{z\left(\mathrm{K}_{k=1}^{\infty}\frac{2a+k}{\sqrt{2}z}+\sqrt{2}z\right)}{\sqrt{2}} \text{ for } (a,z)\in\mathbb{C}^2$$

$$\frac{U\left(a+1,\frac{3}{2},z^2\right)}{U\left(a,\frac{1}{2},z^2\right)}=\frac{\mathrm{K}_{k=1}^{\infty}\frac{-1+2a+k}{\sqrt{2}z}}{\sqrt{2}az} \text{ for } (a,z)\in\mathbb{C}^2$$

$$\frac{U\left(\frac{a+1}{2},\frac{1}{2},z^2\right)}{U\left(\frac{a}{2},\frac{1}{2},z^2\right)}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{a+k}{2}}{z}+z} \text{ for } (a,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^1\frac{t^z}{1+t^2}\,dt=\frac{1}{2\left(\mathrm{K}_{k=1}^{\infty}\frac{k^2}{z}+z\right)} \text{ for } z\in\mathbb{C}\wedge\Re(z)>-1$$

$$\int_0^\infty\frac{e^{-t}}{t+z}\,dt=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-k^2}{1+2k+z}+z+1} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\int_0^\infty e^{-\frac{t}{z}}(1+t)^{-a}\,dt=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1-(-1)^k)\left(a+\frac{1}{2}(-1+k)\right)z+\frac{1}{4}(1+(-1)^k)kz}{1}+1}\text{ for }(a,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-\frac{t}{z}}(1+t)^{-a}\,dt=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{-k(-1+a+k)z^2}{1+(a+2k)z}+az+1}\text{ for }(a,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\left(\frac{1-c}{-c^b+e^{(1-c)t}}\right)^a\,dt=\frac{\left(\frac{1-c}{1-c^b}\right)^a}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{(1-(-1)^k)(1-c)\left(a+\frac{1}{2}(-1+k)\right)}{2(1-c^b)}+\frac{(1+(-1)^k)(1-c)c^b k}{4(1-c^b)}}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)z}+z}\text{ for }(a,b,c,z)\in\mathbb{C}^4\wedge\Re$$

$$\int_0^\infty\frac{e^{-t}t^{-1+a}}{t+z}\,dt=\frac{\Gamma(a)}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1-(-1)^k)\left(a+\frac{1}{2}(-1+k)\right)+\frac{1}{4}(1+(-1)^k)k}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)z}+z}\text{ for }(a,z)\in\mathbb{C}^2\wedge\Re(a)>0\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\operatorname{sech}^2(t)\,dt=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{k(1+k)}{z}+z}\text{ for }z\in\mathbb{C}\wedge\Re(z)>2$$

$$\int_0^\infty e^{-tz}t\operatorname{sech}(t)\,dt=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)(1+k)^2}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2)}+z^2-1}\text{ for }z\in\mathbb{C}\wedge\Re(z)>1$$

$$\int_0^\infty 4e^{-\sqrt{5}t}t\operatorname{sech}(t)\,dt=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{8}(1+(-1)^k)k^2+\frac{1}{8}(1-(-1)^k)(1+k)^2}{1}+1}$$

$$\int_0^\infty e^{-tz}\cosh(qt)\operatorname{sech}(t)\,dt=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)(k^2-q^2)}{z}+z}\text{ for }(q,z)\in\mathbb{C}^2\wedge\Re(z)>|\Re(q)|-1$$

$$\int_0^\infty e^{-tz}t\operatorname{csch}(t)\,dt=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{k^4}{(1+2k)z}+z}\text{ for }z\in\mathbb{C}\wedge\Re(z)>1$$

$$\int_0^\infty e^{-tz}\operatorname{csch}(ct)\sinh(at)\sinh(bt)\,dt=\frac{ab}{c\left(\mathrm{K}_{k=1}^{\infty}\frac{-4k^2(-a^2+c^2k^2)(-b^2+c^2k^2)}{(1+2k)(-a^2-b^2+c^2(1+2k+2k^2)+z^2)}-a^2-b^2+c^2+z^2\right)}\text{ for (}$$

$$\int_0^\infty e^{-tz}\operatorname{csch}(ct)\sinh(at)\,dt=\frac{a}{c\left(\mathrm{K}_{k=1}^{\infty}\frac{k^2(-a^2+c^2k^2)}{(1+2k)z}+z\right)}\text{ for }(a,c,z)\in\mathbb{C}^3\wedge\Re(z)>|\Re(a)|-|\Re(c)|$$

$$\int_0^\infty e^{-tz}(\cosh(t)+a\sinh(t))^{-b}\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(1-a^2)k(-1+b+k)}{a(b+2k)+z} + ab + z} \text{ for } (a,b,z)\in\mathbb{C}^3\wedge\Re(b+z)>0$$

$$\int_0^\infty e^{-tz}\operatorname{sech}(t)\sinh(bt)\,dt = \frac{b}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)(-b^2+(1+k)^2)}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2)} + z^2 - 1} \text{ for } (b,z)\in\mathbb{C}^2\wedge\Re(z)>|\Re$$

$$\int_0^\infty e^{-t}\operatorname{sn}(tz|m)\,dt = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{4(1-2k)k^2(1+2k)mz^4}{1+(1+2k)^2(1+m)z^2} + (m+1)z^2 + 1} \text{ for } (m,z)\in\mathbb{C}^2$$

$$\int_0^\infty e^{-tz}\operatorname{sn}\left(t\,\middle|m^2\right)\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{4(1-2k)k^2(1+2k)m^2}{(1+2k)^2(1+m^2)+z^2} + m^2 + z^2 + 1} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-t}\operatorname{cn}(tz|m)\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-4k^2(-1+2k)^2mz^4}{1+((1+2k)^2+4k^2m)z^2} + z^2 + 1} \text{ for } (m,z)\in\mathbb{C}^2$$

$$\int_0^\infty e^{-tz}\operatorname{sn}\left(t\,\middle|m^2\right)^2\,dt = \frac{2}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-2k(1+2k)^2(2+2k)m^2}{4(1+k)^2(1+m^2)+z^2} + 4\left(m^2+1\right) + z^2\right)} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\operatorname{cn}\left(t\,\middle|m^2\right)\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1-(-1)^k)k^2+\frac{1}{2}(1+(-1)^k)k^2m^2}{z} + z} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\operatorname{dn}\left(t\,\middle|m^2\right)\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)k^2m^2}{z} + z} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\operatorname{dn}\left(t\,\middle|m^2\right)\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)k^2m^2}{z} + z} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\operatorname{dn}\left(t\,\middle|m^2\right)\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)k^2m^2}{z} + z} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty \frac{e^{-tz}\operatorname{cn}\left(t\,\middle|m^2\right)\operatorname{sn}\left(t\,\middle|m^2\right)}{\operatorname{dn}\left(t\,\middle|m^2\right)}\,dt = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{4(1-2k)k^2(1+2k)m^4}{2(1+2k)^2(2-m^2)+z^2} + 2\left(2-m^2\right) + z^2} \text{ for } (m,z)\in\mathbb{C}^2\wedge\Re(z)>0$$

$$\int_0^\infty e^{-tz}\,{}_2F_1\left(a,b;\frac{1}{2}(1+a+b);-\sinh^2(t)\right)\,dt=\frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{4k(-1+a+k)(-1+b+k)(-2+a+b+k)}{(-3+a+b+2k)(-1+a+b+2k)}}{z}+z}\text{ for }(a,b,z)\in\mathbb{C}^3\wedge$$

$$\exp\left(\int_0^\infty\frac{e^{-tz}(1-\cosh(2at)\operatorname{sech}(2t))}{t}\,dt\right)=2\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-a^2+(-1+2k)^2}{\frac{1}{2}\left(1+(-1)^k\right)+\frac{1}{2}\left(1-(-1)^k\right)z^2}+1\text{ for }(a,z)\in\mathbb{C}^2\wedge\Re(z$$

$$\tanh\left(\int_0^\infty\frac{e^{-tz}\operatorname{sech}(t)\sinh(at)}{t}\,dt\right)=\frac{a}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{2}(1-(-1)^k)k^2+\frac{1}{2}(1+(-1)^k)(-a^2+k^2)}{z}+z}\text{ for }(a,z)\in\mathbb{C}^2\wedge\Re(z)>|$$

$$\tanh\left(\frac{1}{2}\int_0^\infty\frac{e^{-tz}\operatorname{sech}(t)\sinh(2at)}{t}\,dt\right)=\frac{a}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-a^2+k^2}{z}+z}\text{ for }(a,z)\in\mathbb{C}^2\wedge\Re(z)>|\Re(a)|-1$$

$$\frac{\int_0^1 t^a\left(\frac{1-t}{1+t}\right)^b\,dt}{\int_0^1 t^{-1+a}\left(\frac{1-t}{1+t}\right)^b\,dt}=\frac{a}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(a+k)(1+a+k)}{2b}+2b}\text{ for }(a,b)\in\mathbb{C}^2\wedge\Re(a)>0\wedge\Re(b)>-1$$

$$\frac{\int_0^1\frac{t^a\left(\frac{1-t}{1+t}\right)^b}{1-t}\,dt}{\int_0^1\frac{t^a\left(\frac{1-t}{1+t}\right)^b}{1-t^2}\,dt}=\frac{a+1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{(a+k)(1+a+k)}{2b}+2b}+1\text{ for }(a,b)\in\mathbb{C}^2\wedge\Re(a)>-1\wedge\Re(b)>0$$

$$P_\nu^{(a,b)}(1-2z)=\frac{\Gamma(a+\nu+1)}{\Gamma(a+1)\Gamma(\nu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z(-1+k-\nu)(a+b+k+\nu)}{k(a+k)}}{1+\frac{z(-1+k-\nu)(a+b+k+\nu)}{k(a+k)}}+1\right)}\text{ for }(\nu,a,b,z)\in\mathbb{C}^4\wedge|z|<1$$

$$\operatorname{bei}_0(z)=\frac{z^2}{4\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k^2(1+2k)^2}}{1-\frac{z^4}{64k^2(1+2k)^2}}+1\right)}\text{ for }z\in\mathbb{C}$$

$$\operatorname{bei}_\nu(z)=\frac{2^{-\nu}\sin\left(\frac{3\pi\nu}{4}\right)z^\nu}{\Gamma(\nu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^2\tan\left(\frac{1}{4}\pi(2k+3\nu)\right)}{4k(k+\nu)}}{1-\frac{z^2\tan\left(\frac{1}{4}\pi(2k+3\nu)\right)}{4k(k+\nu)}}+1\right)}\text{ for }(\nu,z)\in\mathbb{C}^2$$

$$\operatorname{bei}_{-2m}(z)=\frac{ii^m2^{-2m-1}\left(1-(-1)^m\right)z^{2m}}{(2m)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(-1+2k)(k+m)(-1+2k+2m)}}{1-\frac{z^4}{64k(-1+2k)(k+m)(-1+2k+2m)}}+1\right)}+\frac{i^m2^{-2m-3}\left((-1)^m+1\right)z^{2m+2}}{(2m+1)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(1+2k)(k+m)(1+2k+2m)}}{1-\frac{z^4}{64k(1+2k)(k+m)(1+2k+2m)}}\right.}$$

$$\mathrm{bei}_{-2m-1}(z) = \frac{2^{-2m-\frac{3}{2}}(-1)^{\left\lfloor \frac{m-1}{2} \right\rfloor + m} z^{2m+1}}{(2m+1)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k(-1+2k)(k+m)(1+2k+2m)}}{1-\frac{z^4}{64k(-1+2k)(k+m)(1+2k+2m)}}+1\right)} + \frac{2^{-2m-\frac{7}{2}}(-1)^{\left\lfloor \frac{m}{2} \right\rfloor + m} z^{2m+3}}{(2m+2)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k(1+2k)(1+k+m)(1+2k\cdots}}{1-\frac{z^4}{64k(1+2k)(1+k+m)(1+2\cdots}}\cdots\right.}$$

$$\mathrm{ber}_0(z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64(1-2k)^2k^2}}{1-\frac{z^4}{64(1-2k)^2k^2}}+1} \text{ for } z \in \mathbb{C}$$

$$\mathrm{ber}_\nu(z) = \frac{2^{-\nu}\cos\left(\frac{3\pi\nu}{4}\right) z^\nu}{\Gamma(\nu+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z^2\cot\left(\frac{1}{4}\pi(2k+3\nu)\right)}{4k(k+\nu)}}{1+\frac{z^2\cot\left(\frac{1}{4}\pi(2k+3\nu)\right)}{4k(k+\nu)}}+1\right)} \text{ for } (\nu,z) \in \mathbb{C}^2$$

$$\mathrm{ber}_{-2m}(z) = \frac{2^{-2m-2}z^{2m+2}\sin\left(\frac{\pi m}{2}\right)}{(2m+1)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k(1+2k)(k+m)(1+2k+2m)}}{1-\frac{z^4}{64k(1+2k)(k+m)(1+2k+2m)}}+1\right)} + \frac{2^{-2m}z^{2m}\cos\left(\frac{\pi m}{2}\right)}{(2m)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k(-1+2k)(k+m)(-1+2k+2m)}}{1-\frac{z^4}{64k(-1+2k)(k+m)(-1+2k+2m)}}\cdots\right.}$$

$$\mathrm{ber}_{-2m-1}(z) = \frac{2^{-2m-\frac{3}{2}}(-1)^{\left\lfloor \frac{m+1}{2} \right\rfloor} z^{2m+1}}{(2m+1)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k(-1+2k)(k+m)(1+2k+2m)}}{1-\frac{z^4}{64k(-1+2k)(k+m)(1+2k+2m)}}+1\right)} + \frac{2^{-2m-\frac{7}{2}}(-1)^{\left\lfloor \frac{m}{2} \right\rfloor} z^{2m+3}}{(2m+2)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k(1+2k)(1+k+m)(1+2k\cdots}}{1-\frac{z^4}{64k(1+2k)(1+k+m)(1+\cdots}}\cdots\right.}$$

$$\mathrm{kei}_0(z) = -\frac{\pi}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64(1-2k)^2k^2}}{1-\frac{z^4}{64(1-2k)^2k^2}}+1\right)} - \frac{z^2\log\left(\frac{z}{2}\right)}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k^2(1+2k)^2}}{1-\frac{z^4}{64k^2(1+2k)^2}}+1\right)} + \frac{(1-\gamma)z^2}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4\psi^{(0)}(2+2k)}{64\left(k+2k^2\right)^2\psi^{(0)}(2k)}}{1-\frac{z^4\psi^{(0)}(2+2k)}{64\left(k+2k^2\right)^2\psi^{(0)}(2k)}}+1\cdots\right.}$$

$$\mathrm{kei}_\nu(z) = -\frac{2^{\nu-1}\sin\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)z^{-\nu}}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2\tan\left(\frac{k\pi}{2}-\frac{3\pi\nu}{4}\right)}{4k^2-4k\nu}}{1-\frac{z^2\tan\left(\frac{k\pi}{2}-\frac{3\pi\nu}{4}\right)}{4k^2-4k\nu}}+1} - \frac{2^{-\nu-1}\sin\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)z^{\nu}}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2\cot\left(\frac{1}{4}\pi(2-2k+\nu)\right)}{4k(k+\nu)}}{1-\frac{z^2\cot\left(\frac{1}{4}\pi(2-2k+\nu)\right)}{4k(k+\nu)}}+1} \text{ for } (\nu,z) \in \mathbb{C}^2$$

$$\mathrm{kei}_{2m+1}(z) = -\frac{2^{-2(m+1)}\left((-1)^m+i\right)e^{-\frac{1}{4}i\pi(2m+1)}z^{2m+1}\log\left(\frac{z}{2}\right)}{(2m+1)!\left(1+\mathrm{K}_{k=1}^{\infty} \frac{\frac{i\left(i+(-1)^{k+m}\right)z^2}{4\left(-i+(-1)^{k+m}\right)k(1+k+2m)}}{1+\frac{\left(1-i(-1)^{k+m}\right)z^2}{4\left(-i+(-1)^{k+m}\right)k(1+k+2m)}}\right)} - \frac{2^{-2m-3}\left((-1)^m+i\right)e^{-\frac{1}{4}i\pi(2m+1)}z^{2\cdots}}{(2m+1)!\left(1+\mathrm{K}_{k=1}^{\infty} \frac{\frac{i\left(i+(-1)^{k+m}\right)z\cdots}{4\left(-i+(-1)^{k+m}\right)k(1\cdots}}{1-\frac{i\left(i+(-1)^{k+m}\right)\cdots}{4\left(-i+(-1)^{k+m}\right)k(\cdots}}\right.}$$

$$\mathrm{kei}_0(z) = -\frac{\pi}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k^2(-1+2k)^2}}{1-\frac{z^4}{64k^2(-1+2k)^2}}+1\right)} - \frac{z^2\log\left(\frac{z}{2}\right)}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4}{64k^2(1+2k)^2}}{1-\frac{z^4}{64k^2(1+2k)^2}}+1\right)} - \frac{(\gamma-1)z^2}{4\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^4\psi^{(0)}(2+2k)}{64k^2(1+2k)^2\psi^{(0)}(2k)}}{1-\frac{z^4\psi^{(0)}(2+2k)}{64k^2(1+2k)^2\psi^{(0)}(2k)}}+\cdots\right.}$$

$$\mathrm{kei}_{4m}(z)=-\frac{\pi(-1)^m 2^{-4m-2}z^{4m}}{(4m)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(-1+2k)(k+2m)(-1+2k+4m)}}{1-\frac{z^4}{64k(-1+2k)(k+2m)(-1+2k+4m)}}+1\right)}-\frac{(-1)^m 2^{-4m-2}z^{4m+2}\log\left(\frac{z}{2}\right)}{(4m+1)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(1+2k)(k+2m)(1+2k+4m)}}{1-\frac{z^4}{64k(1+2k)(k+2m)(1+2k+4m)}}\cdots\right.}\cdots$$

$$\mathrm{kei}_{4m+2}(z)=-\frac{\pi(-1)^m 4^{-2m-3}z^{4m+4}}{(4m+3)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(1+2k)(1+k+2m)(3+2k+4m)}}{1-\frac{z^4}{64k(1+2k)(1+k+2m)(3+2k+4m)}}+1\right)}+\frac{(-1)^m 4^{-2m-1}z^{4m+2}\log\left(\cdots\right.}{(4m+2)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(-1+2k)(1+k+2m)(\cdots}}{1-\frac{z^4}{64k(-1+2k)(1+k+2m\cdots}}\cdots\right.}\cdots$$

$$\mathrm{ker}_0(z)=-\frac{\log\left(\frac{z}{2}\right)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64(1-2k)^2k^2}}{1-\frac{z^4}{64(1-2k)^2k^2}}+1}-\frac{\gamma}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4\psi^{(0)}(1+2k)}{64(1-2k)^2k^2\psi^{(0)}(-1+2k)}}{1-\frac{z^4\psi^{(0)}(1+2k)}{64(1-2k)^2k^2\psi^{(0)}(-1+2k)}}+1}+\frac{\pi z^2}{16\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k^2(1+2k)^2}}{1-\frac{z^4}{64k^2(1+2k)^2}}+1\right)}\ \text{fo}\cdots$$

$$\mathrm{ker}_{\nu}(z)=\frac{2^{\nu-1}\cos\left(\frac{3\pi\nu}{4}\right)\Gamma(\nu)z^{-\nu}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^2\cot\left(\frac{1}{4}\pi(-2k+3\nu)\right)}{4k^2-4k\nu}}{1-\frac{z^2\cot\left(\frac{1}{4}\pi(-2k+3\nu)\right)}{4k^2-4k\nu}}+1}+\frac{2^{-\nu-1}\cos\left(\frac{\pi\nu}{4}\right)\Gamma(-\nu)z^{\nu}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2\tan\left(\frac{1}{4}\pi(2-2k+\nu)\right)}{4k(k+\nu)}}{1+\frac{z^2\tan\left(\frac{1}{4}\pi(2-2k+\nu)\right)}{4k(k+\nu)}}+1}\ \text{for }(\nu,z)\in\mathbb{C}^2$$

$$\mathrm{ker}_{2m+1}(z)=\frac{4^{-m-1}\left(1+i(-1)^m\right)e^{-\frac{1}{4}i\pi(2m+1)}z^{2m+1}\log\left(\frac{z}{2}\right)}{(2m+1)!\left(1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left(1+i(-1)^{k+m}\right)z^2}{4\left(i+(-1)^{k+m}\right)k(1+k+2m)}}{1-\frac{\left(1+i(-1)^{k+m}\right)z^2}{4\left(i+(-1)^{k+m}\right)k(1+k+2m)}}\right)}+\frac{2^{-2m-3}\left(1+i(-1)^m\right)e^{-\frac{1}{4}i\pi(2m+1)}z^{2m+\cdots}}{(2m+1)!\left(1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left(1+i(-1)^{k+m}\right)z^2(\psi^{(\cdots}}{4\left(i+(-1)^{k+m}\right)k(1+k+2\cdots}}{1-\frac{\left(1+i(-1)^{k+m}\right)z^2(\psi\cdots}{4\left(i+(-1)^{k+m}\right)k(1+k+\cdots}}\cdots\right.}\cdots$$

$$\mathrm{ker}_0(z)=-\frac{\log\left(\frac{z}{2}\right)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k^2(-1+2k)^2}}{1-\frac{z^4}{64k^2(-1+2k)^2}}+1}-\frac{\gamma}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4\psi^{(0)}(1+2k)}{64k^2(-1+2k)^2\psi^{(0)}(-1+2k)}}{1-\frac{z^4\psi^{(0)}(1+2k)}{64k^2(-1+2k)^2\psi^{(0)}(-1+2k)}}+1}+\frac{\pi z^2}{16\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k^2(1+2k)^2}}{1-\frac{z^4}{64k^2(1+2k)^2}}+1\right)}\cdots$$

$$\mathrm{ker}_{4m}(z)=\frac{\pi(-1)^m 4^{-2(m+1)}z^{4m+2}}{(4m+1)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(1+2k)(k+2m)(1+2k+4m)}}{1-\frac{z^4}{64k(1+2k)(k+2m)(1+2k+4m)}}+1\right)}-\frac{\left(-\frac{1}{16}\right)^m z^{4m}\log\left(\frac{z}{2}\right)}{(4m)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(-1+2k)(k+2m)(-1+2k+4m)}}{1-\frac{z^4}{64k(-1+2k)(k+2m)(-1+2k+4m)}}\cdots\right.}\cdots$$

$$\mathrm{ker}_{4m+2}(z)=-\frac{\pi(-1)^m 2^{-4(m+1)}z^{4m+2}}{(2(2m+1))!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^4}{64k(-1+2k)(1+k+2m)(1+2k+4m)}}{1-\frac{z^4}{64k(-1+2k)(1+k+2m)(1+2k+4m)}}+1\right)}-\frac{(-1)^m 2^{-4(m+1)}z^{4m+4\cdots}}{(4m+3)!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^{4}}{64k(1+2k)(1+k+\cdots}}{1-\frac{z^4}{64k(1+2k)(1+k\cdots}}\cdots\right.}\cdots$$

$$L_\nu(z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{z(1-k+\nu)}{k^2}}{1-\frac{z(1-k+\nu)}{k^2}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2$$

$$L_\nu^\lambda(z) = \frac{\Gamma(\lambda+\nu+1)}{\Gamma(\lambda+1)\Gamma(\nu+1)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z(1-k+\nu)}{k(k+\lambda)}}{1-\frac{z(1-k+\nu)}{k(k+\lambda)}} + 1\right)} \text{ for } (\nu, \lambda, z) \in \mathbb{C}^3$$

$$P_\nu(1-2z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k^2}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k^2}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge |z| < 1$$

$$P_\nu^\mu(1-2z) = \frac{(1-z)^{\mu/2} z^{-\mu/2}}{\Gamma(1-\mu)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}} + 1\right)} \text{ for } (\nu, \mu, z) \in \mathbb{C}^3 \wedge |z| < 1$$

$$\frac{P_\nu^\mu(z)}{P_\nu^{\mu-1}(z)} = \frac{2\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{1}{4}\left(-1+z^2\right)(k-\mu-\nu)(1+k-\mu+\nu)}{z(1+k-\mu)} + (1-\mu)z\right)}{\sqrt{1-z^2}} \text{ for } (\nu, \mu, z) \in \mathbb{C}^3 \wedge |1-z| < 1$$

$$\frac{P_\nu^m(z)}{P_\nu^{m-1}(z)} = \frac{\mathrm{K}_{k=1}^{\infty} \frac{\left(1-z^2\right)(-2+k+m-\nu)(-1+k+m+\nu)}{2(-1+k+m)z}}{\sqrt{1-z^2}} \text{ for } m \in \mathbb{Z} \wedge (\nu, z) \in \mathbb{C}^2 \wedge m \geq 0 \wedge |1-z| < 1$$

$$\frac{P_\nu^\mu(z)}{P_\nu^{\mu-1}(z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(k-\mu-\nu)(1+k-\mu+\nu)}{\frac{2z(1+k-\mu)}{\sqrt{1-z^2}}} + \frac{2(1-\mu)z}{\sqrt{1-z^2}} \text{ for } (\nu, \mu, z) \in \mathbb{C}^3$$

$$\frac{P_\nu^m(z)}{P_\nu^{m-1}(z)} = -\frac{\sqrt{z-1}\,\mathrm{K}_{k=1}^{\infty} \frac{(1-k-m-\nu)(-2+k+m-\nu)}{-\frac{2(-1+k+m)z}{\sqrt{-1+z^2}}}}{\sqrt{1-z}} \text{ for } m \in \mathbb{Z} \wedge (\nu, z) \in \mathbb{C}^2 \wedge m \geq 0 \wedge |1-z| < 1$$

$$P_\nu^\mu(1-2z) = \frac{(1-z)^{\mu/2}(-z)^{-\mu/2}}{\Gamma(1-\mu)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}} + 1\right)} \text{ for } (\nu, \mu, z) \in \mathbb{C}^3 \wedge |z| < 1$$

$$\frac{P_\nu^\mu(z)}{P_\nu^{\mu-1}(z)} = \frac{2\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{1}{4}\left(-1+z^2\right)(k-\mu-\nu)(1+k-\mu+\nu)}{z(1+k-\mu)} + (1-\mu)z\right)}{\sqrt{z-1}\sqrt{z+1}} \text{ for } (\nu, \mu, z) \in \mathbb{C}^3 \wedge |1-z| < 1$$

$$\frac{P_\nu^m(z)}{P_\nu^{m-1}(z)} = \frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(1-z^2\right)(-2+k+m-\nu)(-1+k+m+\nu)}{2(-1+k+m)z}}{\sqrt{z-1}\sqrt{z+1}} \text{ for } m \in \mathbb{Z} \land (\nu, z) \in \mathbb{C}^2 \land m \geq 0 \land |1-z| < 1$$

$$\frac{P_\nu^\mu(z)}{P_\nu^{\mu-1}(z)} = \frac{\sqrt{1-z}\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(k-\mu-\nu)(1+k-\mu+\nu)}{\frac{2z(1+k-\mu)}{\sqrt{1-z^2}}} + \frac{2(1-\mu)z}{\sqrt{1-z^2}}\right)}{\sqrt{z-1}} \text{ for } (\nu, \mu, z) \in \mathbb{C}^3 \land |1-z| < 1$$

$$\frac{P_\nu^m(z)}{P_\nu^{m-1}(z)} = -\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(1-k-m-\nu)(-2+k+m-\nu)}{-\frac{2(-1+k+m)z}{\sqrt{-1+z^2}}} \text{ for } m \in \mathbb{Z} \land (\nu, z) \in \mathbb{C}^2 \land m \geq 0 \land |1-z| < 1$$

$$Q_\nu(1-2z) = \frac{\frac{1}{2}(\log(1-z)-\log(z)) - \psi^{(0)}(\nu+1)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k^2}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k^2}} + 1} - \frac{\gamma}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z(-1+k-\nu)(k+\nu)\psi^{(0)}(1+k)}{k^2\psi^{(0)}(k)}}{1+\frac{z(-1+k-\nu)(k+\nu)\psi^{(0)}(1+k)}{k^2\psi^{(0)}(k)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \land |z| <$$

$$Q_\nu^\mu(1-2z) = \frac{1}{2}\pi\csc(\pi\mu)\left(\frac{\cos(\pi\mu)(1-z)^{\mu/2}z^{-\mu/2}}{\Gamma(1-\mu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}} + 1\right)} - \frac{(1-z)^{-\mu/2}z^{\mu/2}(-\mu+\nu+1)_{2\mu}}{\Gamma(\mu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k+\mu)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k+\mu)}} + 1\right)}\right.$$

$$Q_\nu^0(1-2z) = \frac{\frac{1}{2}(\log(1-z)-\log(z)) - \psi^{(0)}(\nu+1)}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k^2}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k^2}} + 1} - \frac{\gamma}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z(-1+k-\nu)(k+\nu)\psi^{(0)}(1+k)}{k^2\psi^{(0)}(k)}}{1+\frac{z(-1+k-\nu)(k+\nu)\psi^{(0)}(1+k)}{k^2\psi^{(0)}(k)}} + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \land |z| <$$

$$Q_\nu^m(1-2z) = \frac{1}{2}\left(\frac{(-1)^m z^{m/2}(1-z)^{-m/2}(\log(1-z)-\log(z))\Gamma(m+\nu+1)}{2m!\Gamma(-m+\nu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k+m)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k+m)}} + 1\right)} + \frac{((1-z)z)^{m/2}(\log(1-z)-\log\ldots}{2m!\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \ldots\right.}\right.$$

$$Q_\nu^\mu(1-2z) = \frac{1}{2}\pi e^{i\pi\mu}\csc(\pi\mu)\left(\frac{(1-z)^{\mu/2}(-z)^{-\mu/2}}{\Gamma(1-\mu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k-\mu)}} + 1\right)} - \frac{(1-z)^{-\mu/2}(-z)^{\mu/2}(-\mu+\nu+1\ldots}{\Gamma(\mu+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k+\mu)}}{1-\frac{z\left(k-k^2+\nu+\nu^2\right)}{k(k+\mu)}} \ldots\right.}\right.$$

$$Q_\nu^0(1-2z) = \frac{\log(1-z)-\log(z)}{4\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z(k-k^2+\nu+\nu^2)}{k^2}}{1-\frac{z(k-k^2+\nu+\nu^2)}{k^2}}+1\right)} - \frac{\psi^{(0)}(\nu+1)+\frac{\gamma}{2}}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z(-1+k-\nu)(k+\nu)(-\psi^{(0)}(1+k)+2\psi^{(0)}(1+\nu))}{k^2(\psi^{(0)}(k)-2\psi^{(0)}(1+\nu))}}{1-\frac{z(-1+k-\nu)(k+\nu)(-\psi^{(0)}(1+k)+2\psi^{(0)}(1+\nu))}{k^2(\psi^{(0)}(k)-2\psi^{(0)}(1+\nu))}}+1} - \frac{\cdots}{2\left(\operatorname*{K}_{k=1}^{\infty} \cdots\right.}$$

$$Q_\nu^m(1-2z) = (-z)^{\frac{1-m}{2}} z^{\frac{m-1}{2}} \left(\frac{\sqrt{z}\left(\left(\frac{(-1)^m z^{m/2}(1-z)^{-m/2}(\log(1-z)-\log(z))\Gamma(m+\nu+1)}{2m!\Gamma(-m+\nu+1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z(k-k^2+\nu+\nu^2)}{k(k+m)}}{1-\frac{z(k-k^2+\nu+\nu^2)}{k(k+m)}}+1\right)} - \frac{\gamma z^{m/2}(1-z)^{m/2}(-\cdots}{m!\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z(-1+k+m-\nu)(\cdots}{k(k+m\cdots}}{1+\frac{z(-1+k+m-\nu)(\cdots}{k(k+m\cdots}}\right.}\right.\right.}{\cdots}\right.$$

$$\frac{Q_\nu^\mu(z)}{Q_{\nu+1}^\mu(z)} = \frac{(2\nu+3)z\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{k^2-\mu^2+2k(1+\nu)+(1+\nu)^2}{z^2(1+2k+2\nu)(3+2k+2\nu)}}{1}+1\right)}{\mu+\nu+1} \text{ for } (\nu,\mu,z)\in\mathbb{C}^3 \wedge |z|>1$$

$$\frac{Q_\nu^\mu(z)}{Q_{\nu+1}^\mu(z)} = \frac{2z^2\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{(1+2k-\mu+\nu)(1+2k+\mu+\nu)}{4z^2}}{\frac{3}{2}+k\left(1+\frac{1}{z^2}\right)+\frac{1}{2z^2}+\nu}\right)+(2\nu+3)z^2+1}{z(\mu+\nu+1)} \text{ for } (\nu,\mu,z)\in\mathbb{C}^3 \wedge |z|>1$$

$$\frac{Q_\nu^\mu(z)}{Q_{\nu+1}^\mu(z)} = \frac{2\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{1}{4}z^2(1+2k-\mu+\nu)(1+2k+\mu+\nu)}{\frac{1}{2}+k(1+z^2)+z^2\left(\frac{3}{2}+\nu\right)}+(2\nu+3)z^2+1}{z(\mu+\nu+1)} \text{ for } (\nu,\mu,z)\in\mathbb{C}^3 \wedge |z|>1$$

$$\frac{Q_\nu^\mu(z)}{Q_{\nu+1}^{\mu+1}(z)} = -\frac{2z^2\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\left(-1+z^2\right)(1+2k+\mu+\nu)(2+2k+\mu+\nu)}{4z^4}}{\frac{3}{2}+k+\nu-\frac{5+4k+2\mu+2\nu}{2z^2}}\right)-2\mu-2\nu+(2\nu+3)z^2-5}{\sqrt{z-1}\sqrt{z+1}(\mu+\nu+1)(\mu+\nu+2)} \text{ for } (\nu,\mu,z)\in\mathbb{C}^3 \wedge |z|>\cdots$$

$$\Phi(z,s,a) = \frac{\left(a^2\right)^{-s/2}}{\operatorname*{K}_{k=1}^{\infty} \frac{-((-1+a+k)^2)^{s/2}((a+k)^2)^{-s/2}z}{1+((-1+a+k)^2)^{s/2}((a+k)^2)^{-s/2}z}+1} \text{ for } (z,s,a)\in\mathbb{C}^3 \wedge (|z|<1 \vee (|z|=1 \wedge \Re(s)>1)) \wedge \neg(a \in \cdots$$

$$a = \operatorname*{K}_{k=1}^{\infty} \frac{-(a+k)^2}{1+2a+2k} + 2a + 1 \text{ for } a \in \mathbb{C}$$

$$az = \cfrac{abz}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(a+k)(b+k)z}{b+k-(1+a+k)z} - (a+1)z + b} \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge |z| < 1$$

$$m + z = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{kz}{k-m-z} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m \geq 0$$

$$- \cfrac{\cdots}{b^3 + \beta b^2 - \beta^2 b + 3db + 2eb + \delta b - \beta^3 + 3d\beta + 2e\beta + \beta\delta - (b+\beta)\left(b^2 - \beta^2 + 2d + e + 2\delta + \epsilon\right) - \cfrac{(b+\beta)\Big(3d^3 \cdots}{\cdots}}$$

$$\cfrac{\cdots}{-b^3 - \beta b^2 + \beta^2 b - 3db - \delta b + \beta^3 - 3d\beta - \beta\delta + (b+\beta)\left(b^2 - \beta^2 + 2d + 2\delta + \epsilon\right) + \cfrac{(b+\beta)\Big(3d^3 + \sqrt{\frac{d^2}{\delta^2}}(b^2 - \beta^2 + \delta)d^2 \cdots}{\sqrt{\frac{d^2}{\delta^2}} \cdots}}$$

$$\cfrac{\cdots}{-b^3 + \beta^3 - \beta b^2 + (b+\beta)\left(b^2 - \beta^2 + 2\delta + e + \epsilon\right) + \cfrac{(b+\beta)\Big(b^2\sqrt{\delta^2(b+\beta)^2} - \beta^2\sqrt{\delta^2(b+\beta)^2} + b\delta(3\delta+2\epsilon) + \beta\delta(3\delta+2\epsilon) + (\delta+2e) \cdots}{\sqrt{\delta^2(b+\beta)^2}\,{}_2F_1\Big(\frac{\delta(5\delta+2\epsilon)b^2 + 2\beta\delta(5\delta+2 \cdots}{\cdots}}}$$

$$\cfrac{\cdots}{-b^3 + \beta^3 + (b+\beta)\left(b^2 - \beta^2 + 2\delta + \epsilon\right) - \beta b^2 + \cfrac{(b+\beta)\Big(b^2\sqrt{\delta^2(b+\beta)^2} - \beta^2\sqrt{\delta^2(b+\beta)^2} + \delta\sqrt{\delta^2(b+\beta)^2} + b\delta(3\delta+2\epsilon) + \beta\delta(3\delta+2\epsilon \cdots}{\sqrt{\delta^2(b+\beta)^2}\,{}_2F_1\Big(\frac{\delta(5\delta+2\epsilon)b^2 + 2\beta\delta(5\delta+2\epsilon)b + 5\beta^2 \cdots}{4(b+\beta)^2\delta \cdots}}}$$

$$- \cfrac{\cdots}{b^3 + \beta b^2 - \beta^2 b + 3db + 2eb + \delta b - \beta^3 + 3d\beta + 2e\beta + \beta\delta - (b+\beta)\left(b^2 - \beta^2 + 2d + e + 2\delta\right) - \cfrac{(b+\beta)\Big(3d^3 + \Big(2 \cdots}{\cdots}}$$

$$\frac{d(b+\beta)(d-\delta)\,{}_2F_1\left(\frac{5\left(d^2-\delta^2\right)b^2 + 10\beta\left(d^2-\delta^2\right)b + 5d^2\beta^2 - 5\beta^2\delta^2 - \sqrt{(b+\beta)^4\left(\delta^2-d^2\right)^2}}{4(b+\beta)^2\left(d^2-\delta^2\right)}, \frac{5\left(d^2-\delta^2\right)b^2 + 10\beta\left(d^2-\delta^2\right)b + \cdots}{4(b+\cdots} \cdots\right.}{d(\delta-d)\,{}_2F_1\left(\frac{5(d^2-\delta^2)b^2 + 10\beta(d^2-\delta^2)b + 5d^2\beta^2 - 5\beta^2\delta^2 - \sqrt{(b+\beta)^4(d^2-\delta^2)^2}}{4(b+\beta)^2(d^2-\delta^2)}, \frac{5(d^2-\delta^2)b^2 + 10\beta(d^2-\delta^2)b + 5d^2\beta^2 - 5\beta^2\delta^2 + \sqrt{(b+\beta \cdots}}{4(b+\beta)^2(d^2-\delta^2)} \cdots\right.}$$

$$-b^3+\beta^3-\beta b^2+(b+\beta)\left(b^2-\beta^2+2\delta+e\right)+\frac{(b+\beta)\left(b^2\sqrt{\delta^2(b+\beta)^2}-\beta^2\sqrt{\delta^2(b+\beta)^2}+3b\delta^2+3\beta\delta^2+(\delta+2e)\sqrt{\delta^2(b+\beta)^2}\right){}_2F \ldots}{\sqrt{\delta^2(b+\beta)^2}\,{}_2F_1\left(\frac{5b^2\delta^2+5\beta^2\delta^2+10b\beta\delta^2-\sqrt{(b+\beta)^4\delta^2(\delta-2e)^2}\ldots}{4(b+\beta)^2\delta^2}\ldots\right.}$$

$$-\frac{\delta(b+\beta)^2}{-b^3+\beta^3+(b+\beta)\left(b^2-\beta^2+2\delta\right)-\beta b^2+\frac{(b+\beta)\left(b^2\sqrt{\delta^2(b+\beta)^2}-\beta^2\sqrt{\delta^2(b+\beta)^2}+\delta\sqrt{\ldots}\right.}{\sqrt{\delta^2(b+\beta)^2}\,{}_2F_1\left(1,\frac{2b^2\delta^2+2\beta^2\delta^2+4b\beta\delta^2}{4b^2\delta^2+4\beta^2\delta^2+8b\beta\delta^2}+1;\frac{2b^2\delta^2+2\beta^2\delta^2+4b\beta\delta^2+3\left(4b^2\delta^2+4\beta^2\delta^2\ldots\right.}{2\ldots}\ldots\right.}}$$

$$\frac{\beta\ldots}{\frac{\left(3d^3+\delta d\left(2\epsilon\left(\sqrt{\frac{d^2}{\delta^2}}-1\right)-3\delta\right)+d^2\left((\delta-\beta^2)\sqrt{\frac{d^2}{\delta^2}}+2e\right)+\delta^2\sqrt{\frac{d^2}{\delta^2}}(\beta^2-\delta-2e)\right){}_2F_1\left(\frac{d^2\beta^2-\delta^2\beta^2+2de\beta^2-2\delta\epsilon\beta^2+\sqrt{\beta^4\left(d^2-2\epsilon d-\delta^2+2e\delta\right)^2}}{4\beta^2\left(d^2-\delta^2\right)},\ldots\right.}{\sqrt{\frac{d^2}{\delta^2}}\left(d^2-\delta^2\right){}_2F_1\left(\frac{5d^2\beta^2-5\delta^2\beta^2+2de\beta^2-2\delta\epsilon\beta^2+\sqrt{\beta^4\left(d^2-2\epsilon d-\delta^2+2e\delta\right)^2}}{4\beta^2\left(d^2-\delta^2\right)},-\frac{-5d^2\beta^2+5\delta^2\beta^2-2de\beta^2+2\delta\epsilon\beta^2+\ldots}{4\beta^2\left(d^2-\delta^2\ldots\right.}\ldots\right.}}$$

$$\frac{\beta(d-\delta-\epsilon)}{\frac{\left(d^3(\delta-\beta^2)+\delta d^2\left(3\delta\sqrt{\frac{d^2}{\delta^2}}+2\epsilon\right)-\delta^3\sqrt{\frac{d^2}{\delta^2}}(3\delta+2\epsilon)+\delta^2d(\beta^2-\delta)\right){}_2F_1\left(\frac{d^2\beta^2-\delta(\delta+2\epsilon)\beta^2+\sqrt{\beta^4\left(-d^2+2\epsilon d+\delta^2\right)^2}}{4\beta^2\left(d^2-\delta^2\right)},-\frac{-d^2\beta^2+\delta(\delta+2\epsilon)\beta^2+\sqrt{\beta^4(\ldots}}{4\beta^2\left(d^2-\delta^2\right)}\ldots\right.}{d\left(d^2-\delta^2\right){}_2F_1\left(\frac{5d^2\beta^2-\delta(5\delta+2\epsilon)\beta^2+\sqrt{\beta^4\left(-d^2+2\epsilon d+\delta^2\right)^2}}{4\beta^2\left(d^2-\delta^2\right)},-\frac{-5d^2\beta^2+\delta(5\delta+2\epsilon)\beta^2+\sqrt{\beta^4\left(-d^2+2\epsilon d+\delta^2\right)^2}}{4\beta^2\left(d^2-\delta^2\right)};\frac{7d^3+\sqrt{\frac{d^2}{\delta^2}}(\ldots}{\ldots}\ldots\right.}}$$

$$\frac{\beta(d-\delta+e)}{\frac{\left(3d^3+d^2\left((\delta-\beta^2)\sqrt{\frac{d^2}{\delta^2}}+2e\right)+\delta^2\sqrt{\frac{d^2}{\delta^2}}(\beta^2-\delta-2e)-3\delta^2d\right){}_2F_1\left(\frac{d^2\beta^2-\delta^2\beta^2+2de\beta^2+\sqrt{\beta^4\left(d^2+(2e-\delta)\delta\right)^2}}{4\beta^2\left(d^2-\delta^2\right)},-\frac{-d^2\beta^2+\delta^2\beta^2-2de\beta^2+\sqrt{\beta^4(d^2\ldots}}{4\beta^2\left(d^2-\delta^2\right)}\ldots\right.}{\sqrt{\frac{d^2}{\delta^2}}\left(d^2-\delta^2\right){}_2F_1\left(\frac{5d^2\beta^2-5\delta^2\beta^2+2de\beta^2+\sqrt{\beta^4\left(d^2+(2e-\delta)\delta\right)^2}}{4\beta^2\left(d^2-\delta^2\right)},-\frac{-5d^2\beta^2+5\delta^2\beta^2-2de\beta^2+\sqrt{\beta^4\left(d^2+(2e-\delta)\delta\right)^2}}{4\beta^2\left(d^2-\delta^2\right)};\frac{7d^3+\left(2e\ldots\right.}{\ldots}\ldots\right.}}$$

$$-\frac{\beta d(d-\delta)\,{}_2F_1\left(\frac{5d^2\beta^2-5\delta^2\beta^2+\sqrt{\beta^4(d^2-\delta^2)^2}}{4\beta^2(d^2-\delta^2)},-\frac{-5d^2\beta^2+5\delta^2\beta^2+\sqrt{\beta^4(d^2-\delta^2)^2}}{4\beta^2(d^2-\delta^2)};\frac{7d+\sqrt{\frac{d^2}{\delta^2}}\left(\delta-\beta^2\right)}{4d};\frac{1}{2}\left(1-\ldots\right.\right.}{-3\delta^2\sqrt{\frac{d^2}{\delta^2}}+d(d-\delta)\,{}_2F_1\left(\frac{5d^2\beta^2-5\delta^2\beta^2+\sqrt{\beta^4(\delta^2-d^2)^2}}{4\beta^2(d^2-\delta^2)},-\frac{-5d^2\beta^2+5\delta^2\beta^2+\sqrt{\beta^4(\delta^2-d^2)^2}}{4\beta^2(d^2-\delta^2)};\frac{7d+\sqrt{\frac{d^2}{\delta^2}}(\delta-\beta^2)}{4d};\frac{1}{2}\left(1-\sqrt{\ldots}\right.\right.}$$

$$\frac{\beta(-\delta+e-\epsilon)}{\delta-\frac{\left(\beta^2\sqrt{\beta^2\delta^2}-\beta\delta(3\delta+2\epsilon)-\sqrt{\beta^2\delta^2}(\delta+2e)\right){}_2F_1\left(\frac{\beta^2\delta(\delta+2\epsilon)-\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2},\frac{\delta(\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{3\beta\delta^2+2\beta\epsilon\delta+\sqrt{\beta^2\delta^2}\left(-\beta^2+2e+\delta\right)}{4\beta\delta^2}\ldots\right.}{\sqrt{\beta^2\delta^2}\,{}_2F_1\left(\frac{\beta^2\delta(5\delta+2\epsilon)-\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2},\frac{\delta(5\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{7\beta\delta^2+2\beta\epsilon\delta+\sqrt{\beta^2\delta^2}\left(-\beta^2+2e+\delta\right)}{4\beta\delta^2};\frac{1}{2}\right)}}$$

$$\frac{\beta(-\delta-\epsilon)}{\delta-\frac{\left(\beta^2\sqrt{\beta^2\delta^2}-\delta\sqrt{\beta^2\delta^2}-\beta\delta(3\delta+2\epsilon)\right){}_2F_1\left(\frac{\beta^2\delta(\delta+2\epsilon)-\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2},\frac{\delta(\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2};\frac{6\delta^2\beta^2+4\delta\epsilon\beta^2-2\sqrt{\beta^2\delta^2}(\beta^3-\beta\delta)}{8\beta^2\delta^2};\frac{1}{2}\right)}{\sqrt{\beta^2\delta^2}\,{}_2F_1\left(\frac{1}{4}\left(-\frac{\beta^2\delta^2}{\sqrt{\beta^4\delta^4}}+5+\frac{2\epsilon}{\delta}\right),\frac{\delta(5\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2};\frac{14\delta^2\beta^2+4\delta\epsilon\beta^2-2\sqrt{\beta^2\delta^2}(\beta^3-\beta\delta)}{8\beta^2\delta^2};\frac{1}{2}\right)}+\epsilon}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-\cdots}{\cdots}$$

$$\frac{\beta(e-\delta)}{\delta-\frac{\left(\beta^2\sqrt{\beta^2\delta^2}-3\beta\delta^2-\sqrt{\beta^2\delta^2}(\delta+2e)\right){}_2F_1\left(\frac{\beta^2\delta^2-\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2},\frac{\beta^2\delta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{3\beta\delta^2+\sqrt{\beta^2\delta^2}(-\beta^2+2e+\delta)}{4\beta\delta^2};\frac{1}{2}\right)}{\sqrt{\beta^2\delta^2}\,{}_2F_1\left(-\frac{\sqrt{\beta^4\delta^2(\delta-2e)^2}-5\beta^2\delta^2}{4\beta^2\delta^2},\frac{5\beta^2\delta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{7\beta\delta^2+\sqrt{\beta^2\delta^2}(-\beta^2+2e+\delta)}{4\beta\delta^2};\frac{1}{2}\right)}-e}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e\cdots}{\cdots}$$

$$-\frac{\beta^2\delta}{\beta\delta+\frac{-\beta^3+3\sqrt{\beta^2\delta^2}+\beta\delta}{2\,{}_2F_1\left(1,\frac{-\beta^3+\delta\beta+\sqrt{\beta^2\delta^2}}{4\sqrt{\beta^2\delta^2}};\frac{-\beta^3+\delta\beta+7\sqrt{\beta^2\delta^2}}{4\sqrt{\beta^2\delta^2}};-1\right)}}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1)^k k\delta}{(-1)^k\beta}\text{ for }(\beta,\delta)\in\mathbb{C}^2$$

$$\frac{(b+\beta)(d+e)U\left(\frac{d(5d+2e)b^2+2d(5d+2e)\beta b+5d^2\beta^2+2de\beta^2+\sqrt{d^4(b+\beta)^4}}{4d^2(b+\beta)^2},2b\cdots\right.}{2dU\left(\frac{d(d+2e)b^2+2d(d+2e)\beta b+d^2\beta^2+2de\beta^2+\sqrt{d^4(b+\beta)^4}}{4d^2(b+\beta)^2},\frac{2b^2d^2+2\beta^2d^2+4b\beta d^2+\sqrt{d^4(b+\beta)^4}}{2d^2(b+\beta)^2},\frac{b^2-\beta^2}{2d}\right)-(d+e)U\left(\frac{d(5d+2e\cdots}{\cdots}\right.}\cdots$$

$$\frac{\beta(d+e)U\left(\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+5\right),\frac{d^2\beta^2}{2\sqrt{d^4\beta^4}}+1,-\frac{\beta^2}{2d}\right)}{2dU\left(\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+1\right),\frac{d^2\beta^2}{2\sqrt{d^4\beta^4}}+1,-\frac{\beta^2}{2d}\right)-(d+e)U\left(\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+5\right),\frac{d^2\beta^2}{2\sqrt{d^4\beta^4}}+1,-\frac{\beta^2}{2d}\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\cdots$$

$$\frac{(b+\beta)e^{\frac{b^2}{2d}}E_{\frac{3}{2}}\left(\frac{b^2-\beta^2}{2d}\right)}{2e^{\frac{\beta^2}{2d}}-e^{\frac{b^2}{2d}}E_{\frac{3}{2}}\left(\frac{b^2-\beta^2}{2d}\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk}{b+(-1)^k\beta}\text{ for }(b,\beta,d)\in\mathbb{C}^3$$

$$\frac{\beta E_{\frac{3}{2}}\left(-\frac{\beta^2}{2d}\right)}{2e^{\frac{\beta^2}{2d}}-E_{\frac{3}{2}}\left(-\frac{\beta^2}{2d}\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk}{(-1)^k\beta}\text{ for }(\beta,d)\in\mathbb{C}^2$$

$$\frac{b(\cdots}{\frac{\left(d^2\left((b^2+\delta)\sqrt{\frac{d^2}{\delta^2}}+2e\right)-\delta^2\sqrt{\frac{d^2}{\delta^2}}(b^2+\delta+2e)+3d^3+\delta d\left(2\epsilon\left(\sqrt{\frac{d^2}{\delta^2}}-1\right)-3\delta\right)\right){}_2F_1\left(\frac{d^2b^2-\delta^2b^2+2deb^2-2\delta\epsilon b^2+\sqrt{b^4\left(d^2-2\epsilon d-\delta^2+2e\delta\right)^2}}{4b^2\left(d^2-\delta^2\right)},-\cdots\right.}{\sqrt{\frac{d^2}{\delta^2}}(d^2-\delta^2)\,{}_2F_1\left(\frac{5d^2b^2-5\delta^2b^2+2deb^2-2\delta\epsilon b^2+\sqrt{b^4\left(d^2-2\epsilon d-\delta^2+2e\delta\right)^2}}{4b^2\left(d^2-\delta^2\right)},-\frac{-5d^2b^2+5\delta^2b^2-2deb^2+2\delta\epsilon b^2+\sqrt{b^4\cdots}}{4b^2\left(d^2-\delta^2\right)}\cdots\right.}\cdots}$$

$$\frac{b(d-\delta-\epsilon)}{\frac{\left(d^2(b^2+\delta)\sqrt{\frac{d^2}{\delta^2}}-\delta^2(b^2+\delta)\sqrt{\frac{d^2}{\delta^2}}+3d^3+\delta d\left(2\epsilon\left(\sqrt{\frac{d^2}{\delta^2}}-1\right)-3\delta\right)\right){}_2F_1\left(\frac{b^2\left(d^2-\delta(\delta+2\epsilon)\right)-\sqrt{b^4\left(-d^2+2\epsilon d+\delta^2\right)^2}}{4b^2\left(d^2-\delta^2\right)},\frac{\left(d^2-\delta(\delta+2\epsilon)\right)b^2+\sqrt{b^4\left(-d^2-\right.}}{4b^2\left(d^2-\delta^2\right)}\right.}{\sqrt{\frac{d^2}{\delta^2}}(d^2-\delta^2)\,{}_2F_1\left(\frac{b^2\left(5d^2-\delta(5\delta+2\epsilon)\right)-\sqrt{b^4\left(-d^2+2\epsilon d+\delta^2\right)^2}}{4b^2\left(d^2-\delta^2\right)},\frac{\left(5d^2-\delta(5\delta+2\epsilon)\right)b^2+\sqrt{b^4\left(-d^2+2\epsilon d+\delta^2\right)^2}}{4b^2\left(d^2-\delta^2\right)};\frac{7d^3+\sqrt{\frac{d^2}{\delta^2}}\left(b^2-\right.}{}\right.}}$$

$$\frac{b(-\delta+e-\epsilon)}{\frac{\left(b^2\sqrt{b^2\delta^2}+\sqrt{b^2\delta^2}(\delta+2e)+b\delta(3\delta+2\epsilon)\right){}_2F_1\left(\frac{b^2\delta(\delta+2\epsilon)-\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2},\frac{\delta(\delta+2\epsilon)b^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{3b\delta^2+2b\epsilon\delta+\sqrt{b^2\delta^2}\left(b^2+2e+\delta\right)}{4b\delta^2};\frac{1}{2}\right)}{\sqrt{b^2\delta^2}\,{}_2F_1\left(\frac{b^2\delta(5\delta+2\epsilon)-\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2},\frac{\delta(5\delta+2\epsilon)b^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{7b\delta^2+2b\epsilon\delta+\sqrt{b^2\delta^2}\left(b^2+2e+\delta\right)}{4b\delta^2};\frac{1}{2}\right)}+\delta-}$$

$$\frac{b(d-\delta+e)}{\frac{\left(d^2\left((b^2+\delta)\sqrt{\frac{d^2}{\delta^2}}+2e\right)-\delta^2\sqrt{\frac{d^2}{\delta^2}}(b^2+\delta+2e)+3d^3-3\delta^2 d\right){}_2F_1\left(\frac{\left(d^2+2ed-\delta^2\right)b^2+\sqrt{b^4\left(d^2+(2e-\delta)\delta\right)^2}}{4b^2\left(d^2-\delta^2\right)},\frac{b^2\left(d^2+2ed-\delta^2\right)-\sqrt{b^4\left(d^2-\delta^2+2e\delta\right)^2}}{4b^2\left(d^2-\delta^2\right)};\right.}{\sqrt{\frac{d^2}{\delta^2}}(d^2-\delta^2)\,{}_2F_1\left(\frac{\left(5d^2+2ed-5\delta^2\right)b^2+\sqrt{b^4\left(d^2+(2e-\delta)\delta\right)^2}}{4b^2\left(d^2-\delta^2\right)},\frac{b^2\left(5d^2+2ed-5\delta^2\right)-\sqrt{b^4\left(d^2-\delta^2+2e\delta\right)^2}}{4b^2\left(d^2-\delta^2\right)};\frac{7d^3+\left(2e+\sqrt{\frac{d^2}{\delta^2}}\left(b^2+\delta\right.\right.}{4}\right.}}$$

$$\frac{b(-\delta-\epsilon)}{\frac{\left(b^2\sqrt{b^2\delta^2}+\delta\sqrt{b^2\delta^2}+b\delta(3\delta+2\epsilon)\right){}_2F_1\left(\frac{b^2\delta(\delta+2\epsilon)-\sqrt{b^4\delta^4}}{4b^2\delta^2},\frac{\delta(\delta+2\epsilon)b^2+\sqrt{b^4\delta^4}}{4b^2\delta^2};\frac{3b\delta^2+2b\epsilon\delta+\sqrt{b^2\delta^2}\left(b^2+\delta\right)}{4b\delta^2};\frac{1}{2}\right)}{\sqrt{b^2\delta^2}\,{}_2F_1\left(\frac{1}{4}\left(-\frac{b^2\delta^2}{\sqrt{b^4\delta^4}}+5+\frac{2\epsilon}{\delta}\right),\frac{\delta(5\delta+2\epsilon)b^2+\sqrt{b^4\delta^4}}{4b^2\delta^2};\frac{7b\delta^2+2b\epsilon\delta+\sqrt{b^2\delta^2}\left(b^2+\delta\right)}{4b\delta^2};\frac{1}{2}\right)}+\delta+\epsilon}=\operatorname*{K}_{k=1}^{\infty}\frac{(-1)^k k\delta+}{b}$$

$$\frac{bd(d-\delta)\,{}_2F_1\left(-\frac{\sqrt{b^4(d^2-\delta^2)^2}-5b^2\left(d^2-\delta^2\right)}{4b^2(d^2-\delta^2)},\frac{5\left(d^2-\delta^2\right)b^2+\sqrt{b^4(d^2-\delta^2)^2}}{4b^2(d^2-\delta^2)};\frac{7d+\sqrt{\frac{d^2}{\delta^2}}\left(b^2+\delta\right)}{4d};\frac{1}{2}\left(1-\sqrt{\frac{d^2}{\delta^2}}\right)\right)}{b^2 d+d(\delta-d)\,{}_2F_1\left(-\frac{\sqrt{b^4(d^2-\delta^2)^2}-5b^2(d^2-\delta^2)}{4b^2(d^2-\delta^2)},\frac{5(d^2-\delta^2)b^2+\sqrt{b^4(d^2-\delta^2)^2}}{4b^2(d^2-\delta^2)};\frac{7d+\sqrt{\frac{d^2}{\delta^2}}(b^2+\delta)}{4d};\frac{1}{2}\left(1-\sqrt{\frac{d^2}{\delta^2}}\right)\right)+\delta\left(3\delta\right.}$$

$$\frac{b(e-\delta)}{\frac{\left(b^2\sqrt{b^2\delta^2}+\sqrt{b^2\delta^2}(\delta+2e)+3b\delta^2\right){}_2F_1\left(\frac{b^2\delta^2-\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2},\frac{b^2\delta^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{3b\delta^2+\sqrt{b^2\delta^2}\left(b^2+2e+\delta\right)}{4b\delta^2};\frac{1}{2}\right)}{\sqrt{b^2\delta^2}\,{}_2F_1\left(-\frac{\sqrt{b^4\delta^2(\delta-2e)^2}-5b^2\delta^2}{4b^2\delta^2},\frac{5b^2\delta^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{7b\delta^2+\sqrt{b^2\delta^2}\left(b^2+2e+\delta\right)}{4b\delta^2};\frac{1}{2}\right)}+\delta-e}=\operatorname*{K}_{k=1}^{\infty}\frac{e+(-1}{b}$$

$$-\frac{b^2\delta}{\frac{b^3+3\sqrt{b^2\delta^2}+b\delta}{2\,{}_2F_1\left(1,\frac{\sqrt{b^2\delta^2}b^2+\delta^2 b+\delta\sqrt{b^2\delta^2}}{4b\delta^2};\frac{b^3+\delta b+7\sqrt{b^2\delta^2}}{4\sqrt{b^2\delta^2}};-1\right)}+b\delta}=\operatorname*{K}_{k=1}^{\infty}\frac{(-1)^k k\delta}{b}\text{ for }(b,\delta)\in\mathbb{C}^2$$

$$-\frac{\qquad}{b^3+2ab^2+\beta b^2-\beta^2 b+3db+2eb+4a\beta b+\delta b-\beta^3+2a\beta^2+3d\beta+2e\beta+\beta\delta-(b+\beta)\left(b^2-\beta^2+2d+e\right.}$$

$$-b^3+\beta b^2+\beta^2 b-db-2eb+\delta b-\beta^3+d\beta+2e\beta-\beta\delta+(b-\beta)(d+e-\delta-\epsilon)+\frac{(b-\beta)\left(-\sqrt{\frac{(d+a(b-\beta))^2}{a^2(b-\beta)^2+2ad(b-\beta)}}\right.}{\qquad}$$

$$-d-e+\delta+\epsilon+\frac{\left((\delta-\beta^2)d^3+\delta\left(3\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}\delta+2\epsilon\right)d^2+\delta^2\left(\beta^2-\delta+2e\left(\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}-1\right)\right)d-\delta^3\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}(3\delta+2\right.}{\qquad}$$

$$2\beta^3-2d\beta-4e\beta+2\delta\beta+2(d+e-\delta-\epsilon)\beta+\frac{2\left(-\left(\beta^2+\delta\right)d^3+\delta\left(\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}\delta+2\epsilon\right)d^2+\delta^2\left(\beta^2+\delta+2e\left(\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}\right.\right.\right.}{\qquad}$$

$$-d-e+\delta+\epsilon+\frac{\left((d^3-d\delta^2)b^2+a^2\left(\left(3\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}-1\right)d^2+2e\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}-1\right)d+\delta\left(-3\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}\delta+\delta-2\left(\sqrt{\right.\right.\right.\right.}{\qquad}$$

$$b^2+e+\epsilon-\frac{\left(\left(d^3-d\delta^2\right)b^2+a^2\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}-1\right)\left(d^2+2ed-\delta(\delta+2\epsilon)\right)b^2+a\left(\left(2\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}-1\right)d^3+\left(4\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}\right.\right.\right.}{\qquad}$$

$$-\overline{}$$

$$b^3 + 2ab^2 + \beta b^2 - \beta^2 b + 3db + 4a\beta b + \delta b - \beta^3 + 2a\beta^2 + 3d\beta + \beta\delta - (b+\beta)\left(b^2 - \beta^2 + 2d + 2a(b+\beta) + 2\right.$$

$$-2b^3 + 2\beta b^2 + 2\beta^2 b - 2db + 2\delta b - 2\beta^3 + 2d\beta - 2\beta\delta + 2(b-\beta)(d-\delta-\epsilon) + \frac{2(b-\beta)\left(-\sqrt{\frac{(d+a(b-\beta))^2}{a^2(b-\beta)^2+2ad(b-\beta)+\delta^2}}\,\delta^4\right.}{}$$

$$-d + \delta + \epsilon + \frac{\left((\delta-\beta^2)d^3 + \delta\left(3\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}\,\delta + 2\epsilon\right)d^2 + (\beta^2-\delta)\delta^2 d - \delta^3\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}\,(3\delta+2\epsilon) + a^2\beta^2\left(\left(3\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}} - 1\right)\right.\right.}{}$$

$$2\beta^3 - 2d\beta + 2\delta\beta + 2(d-\delta-\epsilon)\beta + \frac{2\left(-\left(\beta^2+\delta\right)d^3 + \delta\left(\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}\,\delta + 2\epsilon\right)d^2 + \delta^2\left(\beta^2+\delta\right)d - \delta^3\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}\,(\delta+2\epsilon)\right.}{}$$

$$-\overline{}$$

$$-d + \delta + \epsilon + \frac{\left((d^3 - d\delta^2)b^2 + a^2\left(\left(3\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)d^2 + \delta\left(-3\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}\,\delta + \delta - 2\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)\epsilon\right)\right)b^2 + a\left(\left(6\sqrt{}\right.\right.\right.}{}$$

$$b^2 + \epsilon - \frac{\left(\left(d^3 - d\delta^2\right)b^2 + a^2\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)\left(d^2 - \delta(\delta+2\epsilon)\right)b^2 + a\left(\left(2\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)d^3 - \delta d^2 - \delta\left(\delta\left(2\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}\right.\right.\right.\right.}{}$$

$$-\overline{}$$

$$- \overline{}$$

$$b^3 + 2ab^2 + \beta b^2 - \beta^2 b + 3db + 2eb + 4a\beta b + \delta b - \beta^3 + 2a\beta^2 + 3d\beta + 2e\beta + \beta\delta - (b+\beta)\left(b^2 - \beta^2 + 2d + e \cdots\right.$$

$$-2b^3 + 2\beta b^2 + 2\beta^2 b - 2db - 4eb + 2\delta b - 2\beta^3 + 2d\beta + 4e\beta + 2(b-\beta)(d+e-\delta) - 2\beta\delta + \frac{2(b-\beta)\left(-\sqrt{\frac{(d+\cdots}{a^2(b-\beta)^2 \cdots}}\cdots\right.}{\cdots}$$

$$-d - e + \delta + \frac{\left(-3\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}\delta^4 + d\left(\beta^2 - \delta + 2e\left(\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}} - 1\right)\right)\delta^2 + 3d^2\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}\delta^2 + d^3(\delta - \beta^2) + a^2\beta^2\left(\left(3\sqrt{\frac{\cdots}{a\cdots}}\cdots\right.\right.\right.}{\cdots}$$

$$2\beta^3 - 2d\beta - 4e\beta + 2(d+e-\delta)\beta + 2\delta\beta + \frac{2\left(-\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}\delta^4 + d\left(\beta^2 + \delta + 2e\left(\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}} - 1\right)\right)\delta^2 + d^2\sqrt{\frac{(d-\cdots}{a^2\beta^2-\cdots}}\cdots\right.}{\cdots}$$

$$- \overline{}$$

$$-d - e + \delta + \frac{\left((d^3 - d\delta^2)b^2 + a^2\left(\left(3\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)d^2 + 2e\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)d + \delta^2\left(1 - 3\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}\right)\right)b^2 + a\left(\left(6\sqrt{\cdots}\cdots\right.\right.\right.}{\cdots}$$

$$b^2 + e - \frac{\left(\left(d^3 - d\delta^2\right)b^2 + a^2\left(d^2 + 2ed - \delta^2\right)\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)b^2 + a\left(\left(2\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}} - 1\right)d^3 + \left(4\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}e - 2e - \delta\right)\cdots\right.\right.}{\cdots}$$

$$- \overline{}$$

$$-\frac{\quad}{b^3+2ab^2+\beta b^2-\beta^2 b+2eb+4a\beta b+\delta b-\beta^3+2a\beta^2+2e\beta+\beta\delta-(b+\beta)\left(b^2-\beta^2+e+2a(b+\beta)+2\delta\right.}$$

$$-b^3+\beta b^2+\beta^2 b-2eb+\delta b-\beta^3+2e\beta-\beta\delta+(b-\beta)(e-\delta-\epsilon)+\frac{\left(a^2\left(\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}-1\right)(\delta+2\epsilon)(b-\beta)^2+a\left(b^2-\right.\right.}{a\delta\,{}_2F_1\left(\frac{\delta(3\delta+}{}\right.}$$

$$\frac{\left(a^2\beta^2\left(2\epsilon\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+\delta\left(3\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)\right)+\delta^2(3\delta+2\epsilon)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta\delta(-\beta^2+\delta+2e)\right)\,{}_2F_1\left(\frac{\beta^2\delta(\delta+2\epsilon)-\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2},\frac{\delta(\delta+2\epsilon}{}\right.}{a\delta\,{}_2F_1\left(\frac{\beta^2\delta(5\delta+2\epsilon)-\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2},\frac{\delta(5\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{\delta\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}(-\beta^2+2e+\delta)+a}{}\right.}$$

$$\frac{\left(a^2\beta^2(\delta+2\epsilon)\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+\delta^2(\delta+2\epsilon)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta\delta\left(\beta^2+\delta-2e\right)\right)\,{}_2F_1\left(\frac{\sqrt{\beta^4\delta^2(2e+\delta)^2}-\beta^2\delta(\delta-2\epsilon)}{4\beta^2\delta^2},-\frac{\delta(\delta-2\epsilon)\beta^2+\sqrt{\beta^4\delta^2(2e+\delta)^2}}{4\beta^2\delta^2};\frac{\delta}{}\right.}{a\delta\,{}_2F_1\left(\frac{\beta^2\delta(3\delta+2\epsilon)-\sqrt{\beta^4\delta^2(2e+\delta)^2}}{4\beta^2\delta^2},\frac{\delta(3\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^2(2e+\delta)^2}}{4\beta^2\delta^2};\frac{\delta\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}(\beta^2-2e+\delta)+a\beta\left(-\delta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-5\right)\right.}{4a\beta\delta}\right.}$$

$$\beta^2$$

$$-\frac{\quad}{-\frac{\left(a^2b^2\left(2\epsilon\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+\delta\left(3\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)\right)+\delta^2(3\delta+2\epsilon)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab\delta(b^2+\delta+2e)\right)\,{}_2F_1\left(\frac{b^2\delta(\delta+2\epsilon)-\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2},\frac{\delta(\delta+2\epsilon)b^2}{}\right.}{a\delta\,{}_2F_1\left(\frac{b^2\delta(5\delta+2\epsilon)-\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2},\frac{\delta(5\delta+2\epsilon)b^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{\delta\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}(b^2+2e+\delta)+ab\left(\right.}{}\right.}}$$

$$\frac{\left(a^2b^2(\delta+2\epsilon)\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+\delta^2(\delta+2\epsilon)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab\delta\left(b^2-\delta+2e\right)\right)\,{}_2F_1\left(\frac{\sqrt{b^4\delta^2(2e+\delta)^2}-b^2\delta(\delta-2\epsilon)}{4b^2\delta^2},-\frac{\delta(\delta-2\epsilon)b^2+\sqrt{b^4\delta^2(2e+\delta)^2}}{4b^2\delta^2};\frac{\left(b^2+2\right.}{}\right.}{a\delta\,{}_2F_1\left(\frac{b^2\delta(3\delta+2\epsilon)-\sqrt{b^4\delta^2(2e+\delta)^2}}{4b^2\delta^2},\frac{\delta(3\delta+2\epsilon)b^2+\sqrt{b^4\delta^2(2e+\delta)^2}}{4b^2\delta^2};\frac{\left(b^2+2e-\delta\right)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}\delta+ab\left(-\delta\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-5\right)-2\left(\right.\right.}{4ab\delta}\right.}$$

$$b^2$$

$$-\frac{(b+\beta)\left(a(b+\beta)\left(6d\sqrt{\frac{(a(b+\beta)+d)^2}{a^2(b+\beta)^2+2ad(b+\beta)+\delta^2}}+b^2-\beta^2-d+\delta\right)+a^2(b+\beta)^2\left(3\sqrt{\frac{}{a^2(b}}\right.\right.}{(a(b+\beta)+d)\,{}_2F_1\left(\frac{5(d^2-\delta^2)b^2+10\beta(d^2-\delta^2)b+5d^2\beta^2-5\beta^2\delta^2-\sqrt{(b+\beta)^4(\delta^2-d^2)^2}}{4(b+\beta)^2(d^2-\delta^2)},\frac{5(d^2-\delta^2)b^2+10\beta(d^2-\delta^2)b+5d^2\beta^2-5\beta^2\delta^2+\sqrt{(b+\beta)^4(\delta^2}}{4(b+\beta)^2(d^2-\delta^2)}\right.}$$

$$\frac{2(b-\beta)\left(a(b-\beta)\left(2d\sqrt{\frac{(a(b-\beta)+d)^2}{a^2(b-\beta)^2+2ad(b-\beta)+\delta^2}}+b^2-\beta^2-d-\delta\right)+a^2(b-\beta)^2\left(\sqrt{\frac{(a(}{a^2(b-\beta)^2}}\right.\right.}{(a(b-\beta)+d)\,{}_2F_1\left(\frac{3(d^2-\delta^2)b^2+6\beta(\delta^2-d^2)b+3d^2\beta^2-3\beta^2\delta^2-\sqrt{(\beta-b)^4(\delta^2-d^2)^2}}{4(b-\beta)^2(d^2-\delta^2)},\frac{3(d^2-\delta^2)b^2+6\beta(\delta^2-d^2)b+3d^2\beta^2-3\beta^2\delta^2+\sqrt{(\beta-b)^4(\delta^2-d^2)^2}}{4(b-\beta)^2(d^2-\delta^2)};\right.}$$

$$\beta(d-\delta)$$

$$\frac{a^2\beta^2\left(3\sqrt{\frac{(a\beta+d)^2}{a^2\beta^2+2a\beta d+\delta^2}}-1\right)-a\beta\left(-6d\sqrt{\frac{(a\beta+d)^2}{a^2\beta^2+2a\beta d+\delta^2}}+\beta^2+d-\delta\right)+3\delta^2\sqrt{\frac{(a\beta+d)^2}{a^2\beta^2+2a\beta d+\delta^2}}+d(\delta-\beta^2)}{(a\beta+d)\,{}_2F_1\left(\frac{5d^2\beta^2-5\delta^2\beta^2+\sqrt{\beta^4(d^2-\delta^2)^2}}{4\beta^2(d^2-\delta^2)},-\frac{-5d^2\beta^2+5\delta^2\beta^2+\sqrt{\beta^4(d^2-\delta^2)^2}}{4\beta^2(d^2-\delta^2)};\frac{7d-a\beta\left(\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}-7\right)+(\delta-\beta^2)\sqrt{\frac{(d+a\beta)^2}{a^2\beta^2+2ad\beta+\delta^2}}}{4(d+a\beta)};\frac{1}{2}-\frac{1}{2}\right.}$$

$$-\frac{\dfrac{a^2\beta^2\left(\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2a\beta d+\delta^2}}-1\right)+a\beta\left(-2d\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2a\beta d+\delta^2}}+\beta^2+d+\delta\right)+\delta^2\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2a\beta d+\delta^2}}-d(\beta^2+\delta)}{(d-a\beta)\,{}_2F_1\left(\frac{3d^2\beta^2-3\delta^2\beta^2+\sqrt{\beta^4(d^2-\delta^2)^2}}{4\beta^2(d^2-\delta^2)},-\frac{-3d^2\beta^2+3\delta^2\beta^2+\sqrt{\beta^4(d^2-\delta^2)^2}}{4\beta^2(d^2-\delta^2)};\frac{5d+a\beta\left(\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}-5\right)-(\beta^2+\delta)\sqrt{\frac{(d-a\beta)^2}{a^2\beta^2-2ad\beta+\delta^2}}}{4(d-a\beta)};\frac{1}{2}\left(\right.\right.}}{\beta}$$

$$b(d-\delta)$$

$$\frac{a^2b^2\left(3\sqrt{\frac{(ab+d)^2}{a^2b^2+2abd+\delta^2}}-1\right)+ab\left(6d\sqrt{\frac{(ab+d)^2}{a^2b^2+2abd+\delta^2}}+b^2-d+\delta\right)+\delta\left(3\delta\sqrt{\frac{(ab+d)^2}{a^2b^2+2abd+\delta^2}}+d\right)+b^2d}{(ab+d)\,{}_2F_1\left(-\frac{\sqrt{b^4(d^2-\delta^2)^2}-5b^2(d^2-\delta^2)}{4b^2(d^2-\delta^2)},\frac{5(d^2-\delta^2)b^2+\sqrt{b^4(d^2-\delta^2)^2}}{4b^2(d^2-\delta^2)};\frac{7d-ab\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}-7\right)+(b^2+\delta)\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}}{4(ab+d)};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{(a}{a^2b^2+}}\right.}$$

$$\frac{\dfrac{a^2b^2\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2abd+\delta^2}}-1\right)+ab\left(2d\sqrt{\frac{(ab+d)^2}{a^2b^2+2abd+\delta^2}}+b^2-d-\delta\right)+\delta\left(\delta\sqrt{\frac{(ab+d)^2}{a^2b^2+2abd+\delta^2}}-d\right)+b^2d}{(ab+d)\,{}_2F_1\left(-\frac{\sqrt{b^4(d^2-\delta^2)^2}-3b^2(d^2-\delta^2)}{4b^2(d^2-\delta^2)},\frac{3(d^2-\delta^2)b^2+\sqrt{b^4(d^2-\delta^2)^2}}{4b^2(d^2-\delta^2)};\frac{5d-ab\left(\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}-5\right)+(b^2-\delta)\sqrt{\frac{(ab+d)^2}{a^2b^2+2adb+\delta^2}}}{4(ab+d)};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{(a}{a^2b^2+}}\right.}}{b}$$

$$b^3+2ab^2+\beta b^2-\beta^2 b+4a\beta b+\delta b-\beta^3+2a\beta^2+\beta\delta-(b+\beta)\left(b^2-\beta^2+2a(b+\beta)+2\delta+\epsilon\right)-\frac{(a^2(\delta(3}{}$$

$$\frac{\left(a^2(b-\beta)^2(\delta+2\epsilon)\left(\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}-1\right)+\delta^2(\delta+2\epsilon)\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}+a\delta(b-\beta)\left(b^2-\beta^2-\delta\right)\right){}_2F_1\Bigg(-\frac{\delta(\delta-2\epsilon)b^2-2\beta\delta(\delta-2\epsilon)b+\beta^2\delta(\delta-2\epsilon)+\sqrt{(}}{4(b-\beta)^2\delta^2}}{a\delta\,{}_2F_1\Bigg(\frac{\delta(3\delta+2\epsilon)b^2-2\beta\delta(3\delta+2\epsilon)b+3\beta^2\delta^2+2\beta^2\delta\epsilon+\sqrt{(\beta-b)^4\delta^4}}{4(b-\beta)^2\delta^2},\frac{\delta(3\delta+2\epsilon)b^2-2\beta\delta(3\delta+2\epsilon)b+\beta^2\delta(3\delta+2\epsilon)-\sqrt{(b-\beta)}}{4(b-\beta)^2\delta^2}}$$

$$\frac{\beta^2(}{\frac{\left(a^2\beta^2\left(2\epsilon\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+\delta\left(3\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)\right)+\delta^2(3\delta+2\epsilon)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta\delta(\delta-\beta^2)\right){}_2F_1\Bigg(\frac{\beta^2\delta(\delta+2\epsilon)-\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2},\frac{\delta(\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2};}{a\delta\,{}_2F_1\Bigg(\frac{1}{4}\left(-\frac{\beta^2\delta^2}{\sqrt{\beta^4\delta^4}}+5+\frac{2\epsilon}{\delta}\right),\frac{\delta(5\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2};\frac{\delta\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}(\delta-\beta^2)+a\beta\left(-\delta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-7\right)\right.}{4a\beta\delta}}}$$

$$\frac{\frac{\left(a^2\beta^2(\delta+2\epsilon)\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+\delta^2(\delta+2\epsilon)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta\delta\left(\beta^2+\delta\right)\right){}_2F_1\Bigg(\frac{\sqrt{\beta^4\delta^4}-\beta^2\delta(\delta-2\epsilon)}{4\beta^2\delta^2},\frac{1}{4}\left(-\frac{\beta^2\delta^2}{\sqrt{\beta^4\delta^4}}-1+\frac{2\epsilon}{\delta}\right);\frac{\delta\left(\beta^2+\delta\right)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}}{}}{a\delta\,{}_2F_1\Bigg(\frac{1}{4}\left(-\frac{\beta^2\delta^2}{\sqrt{\beta^4\delta^4}}+3+\frac{2\epsilon}{\delta}\right),\frac{\delta(3\delta+2\epsilon)\beta^2+\sqrt{\beta^4\delta^4}}{4\beta^2\delta^2};\frac{\delta\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}(\beta^2+\delta)+a\beta\left(-\delta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-5\right)-2\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)\right.}{4a\beta\delta}}}{\beta^2}$$

$$\frac{b^2(-\delta-}{\frac{\left(a^2b^2\left(2\epsilon\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+\delta\left(3\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)\right)+\delta^2(3\delta+2\epsilon)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab\delta(b^2+\delta)\right){}_2F_1\Bigg(\frac{b^2\delta(\delta+2\epsilon)-\sqrt{b^4\delta^4}}{4b^2\delta^2},\frac{\delta(\delta+2\epsilon)b^2+\sqrt{b^4\delta^4}}{4b^2\delta^2};\frac{\delta\sqrt{\frac{a^2}{a^2b}}}{}}{a\delta\,{}_2F_1\Bigg(\frac{1}{4}\left(-\frac{b^2\delta^2}{\sqrt{b^4\delta^4}}+5+\frac{2\epsilon}{\delta}\right),\frac{\delta(5\delta+2\epsilon)b^2+\sqrt{b^4\delta^4}}{4b^2\delta^2};\frac{\delta\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}(b^2+\delta)+ab\left(-\delta\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-7\right)-2\left(\sqrt{}\right.\right.}{4ab\delta}}}$$

$$\frac{\frac{\left(a^2b^2(\delta+2\epsilon)\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+\delta^2(\delta+2\epsilon)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab\delta\left(b^2-\delta\right)\right){}_2F_1\Bigg(\frac{\sqrt{b^4\delta^4}-b^2\delta(\delta-2\epsilon)}{4b^2\delta^2},\frac{1}{4}\left(-\frac{b^2\delta^2}{\sqrt{b^4\delta^4}}-1+\frac{2\epsilon}{\delta}\right);\frac{\left(b^2-\delta\right)\delta\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-ab(}{4ab}}{a\delta\,{}_2F_1\Bigg(\frac{1}{4}\left(-\frac{b^2\delta^2}{\sqrt{b^4\delta^4}}+3+\frac{2\epsilon}{\delta}\right),\frac{\delta(3\delta+2\epsilon)b^2+\sqrt{b^4\delta^4}}{4b^2\delta^2};\frac{(b^2-\delta)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}\delta+ab\left(-\delta\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-5\right)-2\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)\epsilon\right)}{4ab\delta};}}{b^2}$$

$$-\frac{}{-\frac{\left(a^2(b+\beta)^2\left(3\sqrt{\frac{a^2(b+\beta)^2}{a^2(b+\beta)^2+\delta^2}}-1\right)+3\delta^2\sqrt{\frac{a^2(b+\beta)^2}{a^2(b+\beta)^2+\delta^2}}+a(b+\beta)(b^2-\beta^2+\delta+2e)\right)\,{}_2F_1\left(\frac{b^2\delta^2+\beta^2\delta^2+2b\beta\delta^2-\sqrt{(b+\beta)^4\delta^2(\delta-2e)^2}}{4(b+\beta)^2\delta^2},\frac{b^2\delta^2 \ldots}{\ldots}\right.}{a\,{}_2F_1\left(\frac{5b^2\delta^2+5\beta^2\delta^2+10b\beta\delta^2-\sqrt{(b+\beta)^4\delta^2(\delta-2e)^2}}{4(b+\beta)^2\delta^2},\frac{5b^2\delta^2+5\beta^2\delta^2+10b\beta\delta^2+\sqrt{(b+\beta)^4\delta^2(\delta-2e)^2}}{4(b+\beta)^2\delta^2};\frac{(b^2-\ldots}{\ldots}\right.}}$$

$$\frac{\left(a^2(b-\beta)^2\left(\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}-1\right)+\delta^2\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}+a(b-\beta)\left(b^2-\beta^2-\delta+2e\right)\right)\,{}_2F_1\left(\frac{-b^2\delta^2-\beta^2\delta^2+2b\beta\delta^2+\sqrt{(b-\beta)^4\delta^2(2e+\delta)^2}}{4(b-\beta)^2\delta^2},-\frac{b^2\delta^2+\beta^2 \ldots}{\ldots}\right.}{a\,{}_2F_1\left(-\frac{-3b^2\delta^2-3\beta^2\delta^2+6b\beta\delta^2+\sqrt{(b-\beta)^4\delta^2(2e+\delta)^2}}{4(b-\beta)^2\delta^2},\frac{3b^2\delta^2+3\beta^2\delta^2-6b\beta\delta^2+\sqrt{(b-\beta)^4\delta^2(2e+\delta)^2}}{4(b-\beta)^2\delta^2};\frac{(b^2-\beta^2 \ldots}{\ldots}\right.}$$

$$\frac{\beta^2(e-\delta)}{\frac{\left(a^2\beta^2\left(3\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+3\delta^2\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta(-\beta^2+\delta+2e)\right)\,{}_2F_1\left(\frac{\beta^2\delta^2-\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2},\frac{\beta^2\delta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{(-\beta^2+2e+\delta)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-\ldots}{4a\beta}\right.}{a\,{}_2F_1\left(-\frac{\sqrt{\beta^4\delta^2(\delta-2e)^2}-5\beta^2\delta^2}{4\beta^2\delta^2},\frac{5\beta^2\delta^2+\sqrt{\beta^4\delta^2(\delta-2e)^2}}{4\beta^2\delta^2};\frac{(-\beta^2+2e+\delta)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-a\beta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-7\right)}{4a\beta};\frac{1}{2}\left(1-\sqrt{\ldots}\right.\right.}}$$

$$\frac{\left(a^2\beta^2\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+\delta^2\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta\left(\beta^2+\delta-2e\right)\right)\,{}_2F_1\left(\frac{\sqrt{\beta^4\delta^2(2e+\delta)^2}-\beta^2\delta^2}{4\beta^2\delta^2},-\frac{\beta^2\delta^2+\sqrt{\beta^4\delta^2(2e+\delta)^2}}{4\beta^2\delta^2};\frac{\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}\left(\beta^2-2e+\delta\right)+a\left(\beta \ldots\right.}{4a\beta}\right.}{a\,{}_2F_1\left(-\frac{\sqrt{\beta^4\delta^2(2e+\delta)^2}-3\beta^2\delta^2}{4\beta^2\delta^2},\frac{3\beta^2\delta^2+\sqrt{\beta^4\delta^2(2e+\delta)^2}}{4\beta^2\delta^2};\frac{(\beta^2-2e+\delta)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-a\beta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-5\right)}{4a\beta};\frac{1}{2}\left(1-\sqrt{\frac{a^2 \ldots}{a^2 \ldots}}\right.\right.}$$

$$\beta^2$$

$$-\frac{b^2(e-\delta)}{-\frac{\left(a^2b^2\left(3\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+3\delta^2\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab(b^2+\delta+2e)\right)\,{}_2F_1\left(\frac{b^2\delta^2-\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2},\frac{b^2\delta^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{(b^2+2e+\delta)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-ab\left(\ldots\right.}{4ab}\right.}{a\,{}_2F_1\left(-\frac{\sqrt{b^4\delta^2(\delta-2e)^2}-5b^2\delta^2}{4b^2\delta^2},\frac{5b^2\delta^2+\sqrt{b^4\delta^2(\delta-2e)^2}}{4b^2\delta^2};\frac{(b^2+2e+\delta)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-ab\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-7\right)}{4ab};\frac{1}{2}\left(1-\sqrt{\frac{a^2 \ldots}{a^2 \ldots}}\right.\right.}}$$

$$\frac{\left(a^2b^2\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+\delta^2\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab\left(b^2-\delta+2e\right)\right)\,{}_2F_1\left(\frac{\sqrt{b^4\delta^2(2e+\delta)^2}-b^2\delta^2}{4b^2\delta^2},-\frac{b^2\delta^2+\sqrt{b^4\delta^2(2e+\delta)^2}}{4b^2\delta^2};\frac{\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}\left(b^2+2e-\delta\right)+a\left(b-b\sqrt{\ldots}\right.}{4ab}\right.}{a\,{}_2F_1\left(-\frac{\sqrt{b^4\delta^2(2e+\delta)^2}-3b^2\delta^2}{4b^2\delta^2},\frac{3b^2\delta^2+\sqrt{b^4\delta^2(2e+\delta)^2}}{4b^2\delta^2};\frac{(b^2+2e-\delta)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-ab\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-5\right)}{4ab};\frac{1}{2}\left(1-\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}\right.\right.}$$

$$b^2$$

$$-\frac{\delta(b \cdots}{\dfrac{a^2(b+\beta)^2\left(3\sqrt{\frac{a^2(b+\beta)^2}{a^2(b+\beta)^2+\delta^2}}-1\right)+3\delta^2\sqrt{\frac{a^2(b+\beta)^2}{a^2(b+\beta)^2+\delta^2}}+a(b+\beta)(b^2-\beta^2+\delta)}{a\,{}_2F_1\left(\frac{5b^2\delta^2+5\beta^2\delta^2+10b\beta\delta^2-\sqrt{(b+\beta)^4\delta^4}}{4(b+\beta)^2\delta^2},\frac{5b^2\delta^2+5\beta^2\delta^2+10b\beta\delta^2+\sqrt{(b+\beta)^4\delta^4}}{4(b+\beta)^2\delta^2};\frac{\left(b^2-\beta^2+\delta\right)\sqrt{\frac{a^2(b+\beta)^2}{a^2(b+\beta)^2+\delta^2}}-a(b+\beta)\left(\sqrt{\frac{a^2(b+\beta)^2}{a^2(b+\beta)^2+\delta^2}}-7\right)}{4a(b+\beta)};\frac{1}{2}\left(1 \cdots\right.\right.} \cdots}$$

$$\frac{\dfrac{a^2(b-\beta)^2\left(\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}-1\right)+\delta^2\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}+a(b-\beta)\left(b^2-\beta^2-\delta\right)}{a\,{}_2F_1\left(-\frac{-3b^2\delta^2-3\beta^2\delta^2+6b\beta\delta^2+\sqrt{(\beta-b)^4\delta^4}}{4(b-\beta)^2\delta^2},\frac{3b^2\delta^2+3\beta^2\delta^2-6b\beta\delta^2+\sqrt{(\beta-b)^4\delta^4}}{4(b-\beta)^2\delta^2};\frac{\left(b^2-\beta^2-\delta\right)\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}-a(b-\beta)\left(\sqrt{\frac{a^2(b-\beta)^2}{a^2(b-\beta)^2+\delta^2}}-5\right)}{4a(b-\beta)};\frac{1}{2}\left(1 \cdots\right.\right.} \cdots}{(b-\beta)^2 \cdots}$$

$$-\frac{\beta^2\delta}{\dfrac{a^2\beta^2\left(3\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+3\delta^2\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a(\beta\delta-\beta^3)}{a\,{}_2F_1\left(\frac{5}{4}-\frac{\beta^2\delta^2}{4\sqrt{\beta^4\delta^4}},\frac{1}{4}\left(\frac{\beta^2\delta^2}{\sqrt{\beta^4\delta^4}}+5\right);\frac{\left(\delta-\beta^2\right)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-a\beta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-7\right)}{4a\beta};\frac{1}{2}\left(1-\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}\right)\right)}+\beta\left(2a\beta-\beta^2+2\delta\right)-2a\beta^2 \cdots}$$

$$\frac{\dfrac{8\beta^2\left(a^2\beta^2\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-1\right)+\delta^2\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}+a\beta\left(\beta^2+\delta\right)\right)}{a\,{}_2F_1\left(\frac{3}{4}-\frac{\beta^2\delta^2}{4\sqrt{\beta^4\delta^4}},\frac{1}{4}\left(\frac{\beta^2\delta^2}{\sqrt{\beta^4\delta^4}}+3\right);\frac{\left(\beta^2+\delta\right)\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-a\beta\left(\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}-5\right)}{4a\beta};\frac{1}{2}\left(1-\sqrt{\frac{a^2\beta^2}{a^2\beta^2+\delta^2}}\right)\right)}-8\beta^5}{8\beta^4}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1)^k k\delta}{ak+(-1)^k(ak+ \cdots}$$

$$-\frac{b^2\delta}{\dfrac{a^2b^2\left(3\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+3\delta^2\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+ab(b^2+\delta)}{a\,{}_2F_1\left(\frac{5}{4}-\frac{b^2\delta^2}{4\sqrt{b^4\delta^4}},\frac{1}{4}\left(\frac{b^2\delta^2}{\sqrt{b^4\delta^4}}+5\right);\frac{\left(b^2+\delta\right)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-ab\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-7\right)}{4ab};\frac{1}{2}\left(1-\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}\right)\right)}+b\left(2ab+b^2+2\delta\right)-2ab^2-b^3 \cdots}$$

$$\frac{\dfrac{8b^2\left(a^2b^2\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-1\right)+\delta^2\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}+a\left(b^3-b\delta\right)\right)}{a\,{}_2F_1\left(\frac{3}{4}-\frac{b^2\delta^2}{4\sqrt{b^4\delta^4}},\frac{1}{4}\left(\frac{b^2\delta^2}{\sqrt{b^4\delta^4}}+3\right);\frac{\left(b^2-\delta\right)\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-ab\left(\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}-5\right)}{4ab};\frac{1}{2}\left(1-\sqrt{\frac{a^2b^2}{a^2b^2+\delta^2}}\right)\right)}-8b^5}{8b^4}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{(-1)^k k\delta}{b+\left(1+(-1)^k\right)ak}$$

$$\frac{b\sqrt{\frac{d}{b^2}}H_{-\frac{e}{d}}\left(\frac{1}{\sqrt{2}\sqrt{\frac{d}{b^2}}}\right)}{\sqrt{2}H_{-\frac{d+e}{d}}\left(\frac{1}{\sqrt{2}\sqrt{\frac{d}{b^2}}}\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e+dk}{b}\ \text{ for }(b,d,e)\in\mathbb{C}^3$$

$$\frac{\sqrt{\frac{2}{\pi}}b\sqrt{\frac{d}{b^2}}e^{-\frac{b^2}{2d}}}{\operatorname{erfc}\left(\frac{1}{\sqrt{2}\sqrt{\frac{d}{b^2}}}\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk}{b}\text{ for }(b,d)\in\mathbb{C}^2$$

$$\frac{(ab+d)\,{}_1F_1\left(\frac{e}{d};\frac{ab+d}{a^2};\frac{d}{a^2}\right)}{a\,{}_1F_1\left(\frac{e}{d}+1;\frac{ab+d}{a^2}+1;\frac{d}{a^2}\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e+dk}{b+ak}\text{ for }(a,b,d,e)\in\mathbb{C}^4$$

$$\frac{d\,{}_1F_1\left(\frac{e}{d};\frac{d}{a^2};\frac{d}{a^2}\right)}{a\,{}_1F_1\left(\frac{d+e}{d};\frac{d}{a^2}+1;\frac{d}{a^2}\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e+dk}{ak}\text{ for }(a,d,e)\in\mathbb{C}^3$$

$$\frac{ae^{-\frac{d}{a^2}}\left(\frac{d}{a^2}\right)^{\frac{ab+d}{a^2}}}{\Gamma\left(\frac{ab+d}{a^2},0,\frac{d}{a^2}\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk}{b+ak}\text{ for }(a,b,d)\in\mathbb{C}^3$$

$$\frac{a\left(\frac{d}{a^2}\right)^{\frac{d}{a^2}}e^{-\frac{d}{a^2}}}{\Gamma\left(\frac{d}{a^2},0,\frac{d}{a^2}\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk}{ak}\text{ for }(a,d)\in\mathbb{C}^2$$

$$-\frac{\qquad}{2b^3+4ab^2+2\beta b^2-2\beta^2b+6db+4eb+8a\beta b-2\beta^3+4a\beta^2+6d\beta+4e\beta-2(b+\beta)\left(b^2-\beta^2+2d+e+2\right.}$$

$$\frac{}{-b^3+\beta b^2+\beta^2b-db-2eb-\beta^3+(d+e)(b-\beta)+d\beta+2e\beta+\frac{(b-\beta)^2\left((d+2e)(b-\beta)\left(\sqrt{\frac{(ab+d-a\beta)^2}{a(b-\beta)(ab+2d-a\beta)}}-1\right)a^2+\right.}{d(d+}}$$

$$-\frac{\qquad}{\frac{\beta\left(a^2(-\beta)\left(d\left(3\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)+2e\left(\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)\right)-ad\left(d\left(6\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)+4e\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-\beta^2-2e\right)+\beta d^2\right)\,{}_2F_1\left(\frac{1}{4}\left(\right.\right.}{d(a\beta+d)\,{}_2F_1\left(\frac{1}{4}\left(-\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+5\right),\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+5\right);\frac{7d^2+\left(2e-\beta\left(\sqrt{\frac{(d}{a\beta(}}\right.\right.}{}\right.}}}$$

$$-\frac{\dfrac{\beta\left(a^2\beta(d+2e)\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-1\right)+ad\left(-2d\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-4e\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}+\beta^2+d+2e\right)-\beta d^2\right){}_2F_1\left(\frac{1}{4}\left(-\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}-1\right),\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}\cdots\right.\right.}{d(d-a\beta)\,{}_2F_1\left(\frac{1}{4}\left(-\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+3\right),\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+\frac{2e}{d}+3\right);\frac{5d^2+\left(2e+\beta\left(a\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-5\right)-\cdots\right.\right.}{\cdots}\right.}}{\beta\cdots}$$

$$-\frac{\cdots}{-\dfrac{2(b+\beta)^2\left(a^2(b+\beta)\left(3\sqrt{\frac{(a(b+\beta)+d)^2}{a(b+\beta)(a(b+\beta)+2d)}}-1\right)+a\left(6d\sqrt{\frac{(a(b+\beta)+d)^2}{a(b+\beta)(a(b+\beta)+2d)}}+b^2-\beta^2-d\right)\cdots\right.}{(a(b+\beta)+d)\,{}_2F_1\left(\frac{5b^2d^2+5\beta^2d^2+10b\beta d^2-\sqrt{d^4(b+\beta)^4}}{4d^2(b+\beta)^2},\frac{5b^2d^2+5\beta^2d^2+10b\beta d^2+\sqrt{d^4(b+\beta)^4}}{4d^2(b+\beta)^2};\frac{7d-a(b+\beta)\left(\sqrt{\frac{(d+a(b+\beta))^2}{a(b+\beta)(2d+a(b+\beta))}}-7\right)+\left(b^2-\beta^2\cdots\right.}{4(d+a(b+\beta))}\cdots\right.}+\frac{(b-\beta)^2\left(a^2(b-\beta)\left(\sqrt{\frac{(ab-a\beta+d)^2}{a(b-\beta)(ab-a\beta+2d)}}-1\right)+a\left(d\left(2\sqrt{\frac{(ab-a\beta+d)^2}{a(b-\beta)(ab-a\beta+2d)}}-1\right)+b^2-\beta^2\right)+d(\cdots\right.}{(a(b-\beta)+d)\,{}_2F_1\left(-\frac{-3b^2d^2-3\beta^2d^2+6b\beta d^2+\sqrt{d^4(b-\beta)^4}}{4d^2(b-\beta)^2},\frac{3b^2d^2+3\beta^2d^2-6b\beta d^2+\sqrt{d^4(b-\beta)^4}}{4d^2(b-\beta)^2};\frac{5d+\sqrt{\frac{(d+a(b-\beta))^2}{a(2d+a(b-\beta))(b-\beta)}}\left(b^2-\beta^2\right)-a(b-\beta)\left(\sqrt{\cdots}\right.}{4(d+a(b-\beta))}\cdots\right.}}{(b-\beta)^2\cdots}$$

$$-\frac{b(d+e\cdots}{-\dfrac{b\left(a^2b\left(2e\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+d\left(3\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)\right)+ad\left(4e\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}+6d\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}+b^2-d-2e\right)+bd^2\right){}_2F_1\left(\frac{b^2d(d+2e)-\cdots}{4b^2d^2}\cdots\right.}{d(ab+d)\,{}_2F_1\left(\frac{1}{4}\left(-\frac{b^2d^2}{\sqrt{b^4d^4}}+5+\frac{2e}{d}\right),\frac{d(5d+2e)b^2+\sqrt{b^4d^4}}{4b^2d^2};\frac{d\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2+7d+2e\right)\cdots}{\cdots}\right.}-\frac{b\left(a^2b(d+2e)\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+ad\left((d+2e)\left(2\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+b^2\right)+bd^2\right){}_2F_1\left(\frac{\sqrt{b^4d^4}-b^2d(d-2e)}{4b^2d^2},\frac{1}{4}\left(-\frac{b^2d^2}{\sqrt{b^4d^4}}-1+\frac{2e}{d}\right);\frac{d\left(\sqrt{\cdots}\right.}{\cdots}\right.}{d(ab+d)\,{}_2F_1\left(\frac{1}{4}\left(-\frac{b^2d^2}{\sqrt{b^4d^4}}+3+\frac{2e}{d}\right),\frac{d(3d+2e)b^2+\sqrt{b^4d^4}}{4b^2d^2};\frac{d\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2+5d+2e\right)-ab\left(d\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-5\right)+\cdots\right.}{4d(ab+d)}\right.}}{b\cdots}$$

$$\frac{\beta d}{-\frac{\beta\left(a^2(-\beta)\left(3\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}-1\right)+a\left(-6d\sqrt{\frac{(a\beta+d)^2}{a\beta(a\beta+2d)}}+\beta^2+d\right)+\beta d\right)}{(a\beta+d)\,{}_2F_1\left(\frac{5}{4}-\frac{d^2\beta^2}{4\sqrt{d^4\beta^4}},\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+5\right);\frac{7d-\beta\left(\sqrt{\frac{(d+a\beta)^2}{a\beta(2d+a\beta)}}\beta+a\left(\sqrt{\frac{(d+a\beta)^2}{a\beta(2d+a\beta)}}-7\right)\right)}{4(d+a\beta)};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{(d+a\beta)^2}{a\beta(2d+a\beta)}}\right)}-2a\beta+\beta(2a-\beta)+}$$

$$-\frac{a^2\beta\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-1\right)+a\left(-2d\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}+\beta^2+d\right)-\beta d}{(d-a\beta)\,{}_2F_1\left(\frac{3}{4}-\frac{d^2\beta^2}{4\sqrt{d^4\beta^4}},\frac{1}{4}\left(\frac{d^2\beta^2}{\sqrt{d^4\beta^4}}+3\right);\frac{5d+\beta\left(a\left(\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}-5\right)-\beta\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}\right)}{4(d-a\beta)};\frac{1}{2}\left(1-\sqrt{\frac{(d-a\beta)^2}{a\beta(a\beta-2d)}}\right)\right)}$$

$$\frac{a^2b\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+a\left(d\left(2\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+b^2\right)+bd}{(ab+d)\,{}_2F_1\left(\frac{3}{4}-\frac{b^2d^2}{4\sqrt{b^4d^4}},\frac{1}{4}\left(\frac{b^2d^2}{\sqrt{b^4d^4}}+3\right);\frac{\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2-a\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-5\right)b+5d}{4(ab+d)};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{}{b}$$

$$\frac{bd}{\frac{b\left(a^2b\left(3\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+a\left(d\left(6\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-1\right)+b^2\right)+bd\right)}{(ab+d)\,{}_2F_1\left(\frac{5}{4}-\frac{b^2d^2}{4\sqrt{b^4d^4}},\frac{1}{4}\left(\frac{b^2d^2}{\sqrt{b^4d^4}}+5\right);\frac{\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}b^2-a\left(\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}-7\right)b+7d}{4(ab+d)};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{(ab+d)^2}{ab(ab+2d)}}\right)}-d}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk}{b-(-1+(-1)^k}$$

$$\log(z+1)=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{kz}{1+k}}{1-\frac{kz}{1+k}}+1}\text{ for }z\in\mathbb{C}\land|z|<1$$

$$\log(z+1)=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{\left(\frac{(1+(-1)^k)k}{8(1+k)}+\frac{(1-(-1)^k)(1+k)}{8k}\right)z}{1}+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=z-\frac{z^2}{2\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{(5-3(-1)^k+(2-6(-1)^k)(1+k)+2(1+k)^2)z}{8(1+k)(2+k)}}{1}+1\right)}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{z\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{z\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}(3+(-1)^k(-1+k)+k)}+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z\left(\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{1}{4}\left(1+(-1)^k\right)kz+\frac{1}{4}\left(1-(-1)^k\right)(3+k)z}{1+(-1)^k+\frac{1}{2}\left(1-(-1)^k\right)(2+k)}+2}+1\right)}{z+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z\left(\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{z\left\lfloor\frac{1+k}{2}\right\rfloor\left\lfloor\frac{3+k}{2}\right\rfloor}{2+k}+2}+1\right)}{z+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{2z}{\operatorname*{K}_{k=1}^{\infty}\frac{-k^2z^2}{(1+2k)(2+z)}+z+2}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{k^2z}{1+k-kz}+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{2\left(\left(-1-(-1)^k\right)\left(-1+i^k\right)+2\left(-1+(-1)^k\right)z\right)+k\left(1-z+(-1)^k(1+z)\right)}{2(4+k)}}{\frac{1}{2}(2+z+(-1)^kz)}+z+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{\left(-\frac{1}{4}(1+(-1)^k)k-\frac{1}{4}(1-(-1)^k)(1+k)\right)z}{1-(-1)^k+\frac{1}{2}(1+(-1)^k)(1+k)(1+z)}+z+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log(z+1)=\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{-k^2z(1+z)}{1+k+(1+2k)z}+z+1}\text{ for }z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\log\left(\frac{2x}{y}+1\right)=\frac{2x}{\operatorname*{K}_{k=1}^{\infty}\frac{x\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(1+k)y}+y}\text{ for }(x,y)\in\mathbb{C}^2\land\left|\arg\left(\frac{2x}{y}+1\right)\right|<\pi$$

$$\log\left(\frac{2x}{y}+1\right)=\frac{2x}{\operatorname*{K}_{k=1}^{\infty}\frac{-k^2x^2}{(1+2k)(x+y)}+x+y}\text{ for }(x,y)\in\mathbb{C}^2\land\left|\arg\left(\frac{2x}{y}+1\right)\right|<\pi$$

$$\log\left(\frac{\sqrt{z}+1}{1-\sqrt{z}}\right)=\frac{2\,\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{z\left(-\frac{(-1+k)^2}{-1+4(-1+k)^2}+\delta_{1-k}\right)}{1}}{\sqrt{z}}\text{ for }z\in\mathbb{C}\wedge|\arg(1-z)|<\pi$$

$$\log\left(\frac{z+1}{1-z}\right)=\frac{2z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k^2z^2}{-1+4k^2}}{1}+1}\text{ for }z\in\mathbb{C}\wedge\left|\arg\left(1-z^2\right)\right|<\pi$$

$$\log\left(\frac{z+1}{1-z}\right)=\frac{2z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k^2z^2}{1+2k}+1}\text{ for }z\in\mathbb{C}\wedge\left|\arg\left(1-z^2\right)\right|<\pi$$

$$\log\left(\frac{z+1}{1-z}\right)=\frac{2z}{1-\frac{z^2}{2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{4}(1+k)^2z^2}{\frac{1}{2}(3+2k)}+\frac{3}{2}\right)}}\text{ for }z\in\mathbb{C}\wedge\left|\arg\left(1-z^2\right)\right|<\pi$$

$$\log\left(\frac{z+1}{1-z}\right)=\frac{2z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(-1+2k)z^2}{1+2k}}{1+\frac{(-1+2k)z^2}{1+2k}}+1}\text{ for }z\in\mathbb{C}\wedge|z|<1$$

$$\log\left(\frac{z+1}{1-z}\right)=\frac{2z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-(-1+2k)^2z^2}{1+2k+(-1+2k)z^2}+1}\text{ for }z\in\mathbb{C}\wedge|z|<1$$

$$\log\left(\frac{z+1}{z-1}\right)=\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k^2}{(1+2k)z}+z}\text{ for }\neg(z\in\mathbb{R}\wedge 0<z<1)\wedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

$$\log\left(\frac{z+1}{z-1}\right)=\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k}{1+k}}{\frac{(1+2k)z}{1+k}}+z}\text{ for }\neg(z\in\mathbb{R}\wedge 0<z<1)\wedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

$$\log\left(\frac{z+1}{z-1}\right)=\frac{2}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{k^2z^{-1-k}}{-1+4k^2}}{1}+1\right)}\text{ for }z\in\mathbb{C}\wedge\neg(z\in\mathbb{R}\wedge-1\leq z\leq 1)$$

$$\log\left(\sqrt{z^2+1}+z\right)=\frac{z\sqrt{z^2+1}}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{2z^2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k}+1}\text{ for }z\in\mathbb{C}\wedge\neg(iz\in\mathbb{R}\wedge(-\infty<iz\leq-1\vee 1\leq iz<\infty))$$

$$\log\Gamma(z)=\frac{\pi^2z^2}{12\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1+k)z\zeta(2+k)}{(2+k)\zeta(1+k)}}{1-\frac{(1+k)z\zeta(2+k)}{(2+k)\zeta(1+k)}}+1\right)}-\gamma z-\log(z)\text{ for }z\in\mathbb{C}\wedge|z|<1$$

$$\operatorname{li}(z)=2\sum_{k=0}^{\infty}\frac{\log^{1+2k}(z)}{(1+2k)(1+2k)!}-\frac{1}{z\log(z)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left\lfloor\frac{1+k}{2}\right\rfloor}{\log(z)}}{1}+1\right)}\text{ for }z\in\mathbb{C}\land|\arg(z)|<\pi$$

$$\operatorname{li}(z)=\frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)\log(z)}+\log(z)}+\frac{1}{2}\left(2\log\left(-z^2\right)+\log\left(-\frac{1}{z}\right)-3\log(-z)\right)\text{ for }z\in\mathbb{C}\land|$$

$$\operatorname{li}\left(e^{-z}\right)=\frac{\pi\sqrt{-z^2}}{z}-\frac{e^{-z}}{\frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{4}\left(3+(-1)^k+2k\right)}{\frac{1}{2}\left(1+(-1)^k\right)+\frac{1}{2}\left(1-(-1)^k\right)z}+1}+z}\text{ for }z\in\mathbb{C}\land|z|<1$$

$$L_\nu=2+\frac{\nu^2\left(\operatorname{csch}^{-1}(2)^2-\frac{\pi^2}{2}\right)}{1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{\nu\left(\left(i\pi-\operatorname{csch}^{-1}(2)\right)^{2+k}+2\operatorname{csch}^{-1}(2)^{2+k}+(-1)^k\left(i\pi+\operatorname{csch}^{-1}(2)\right)^{2+k}\right)}{2(2+k)\left(\operatorname{csch}^{-1}(2)^{1+k}+\frac{1}{2}\left(\left(i\pi-\operatorname{csch}^{-1}(2)\right)^{1+k}-(-1)^k\left(i\pi+\operatorname{csch}^{-1}(2)\right)^{1+k}\right)\right)}}{1+\frac{\nu\left(\left(i\pi-\operatorname{csch}^{-1}(2)\right)^{2+k}+2\operatorname{csch}^{-1}(2)^{2+k}+(-1)^k\left(i\pi+\operatorname{csch}^{-1}(2)\right)^{2+k}\right)}{2(2+k)\left(\operatorname{csch}^{-1}(2)^{1+k}+\frac{1}{2}\left(\left(i\pi-\operatorname{csch}^{-1}(2)\right)^{1+k}-(-1)^k\left(i\pi+\operatorname{csch}^{-1}(2)\right)^{1+k}\right)\right)}}}\text{ for }\nu\in\mathbb{C}$$

$$L_\nu(z)=2-\frac{\nu^2\left(\pi^2-2\log^2\left(\frac{1}{2}\left(\sqrt{z^2+4}+z\right)\right)\right)}{2\left(1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{\nu\left(1+\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{2+k}+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{2+k}\right)\right)\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}{(2+k)\left(1-\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}\right)\right)}}{1+\frac{\nu\left(1+\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{2+k}+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{2+k}\right)\right)\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}{(2+k)\left(1-\frac{1}{2}(-1)^k\left(\left(1-\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}+\left(1+\frac{i\pi}{\log\left(\frac{1}{2}\left(z+\sqrt{4+z^2}\right)\right)}\right)^{1+k}\right)\right)}}\right)}\text{ f}$$

$$L_v(z)=\frac{2\cos^2\left(\frac{\pi\text{CalculateData`Private`nu}}{2}\right)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z\cot\left(\frac{1}{2}(k-\text{CalculateData`Private`nu})\pi\right)\Gamma\left(\frac{k-\text{CalculateData`Private`nu}}{2}\right)\Gamma\left(\frac{k+\text{CalculateData`Private`nu}}{2}\right)}{k\Gamma\left(\frac{1}{2}(-1+k-\text{CalculateData`Private`nu})\right)\Gamma\left(\frac{1}{2}(-1+k+\text{CalculateData`Private`nu})\right)}}{1-\frac{z\cot\left(\frac{1}{2}(k-\text{CalculateData`Private`nu})\pi\right)\Gamma\left(\frac{k-\text{CalculateData`Private`nu}}{2}\right)\Gamma\left(\frac{k+\text{CalculateData`Private`nu}}{2}\right)}{k\Gamma\left(\frac{1}{2}(-1+k-\text{CalculateData`Private`nu})\right)\Gamma\left(\frac{1}{2}(-1+k+\text{CalculateData`Private`nu})\right)}}+1}\text{ for (Calcul}$$

$$D_\nu(z)=\frac{\sqrt{\pi}2^{\nu/2}e^{-\frac{z^2}{4}}}{\Gamma\left(\frac{1-\nu}{2}\right)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\sqrt{2}z\Gamma\left(\frac{k-\nu}{2}\right)}{k\Gamma\left(\frac{1}{2}(-1+k-\nu)\right)}}{1-\frac{\sqrt{2}z\Gamma\left(\frac{k-\nu}{2}\right)}{k\Gamma\left(\frac{1}{2}(-1+k-\nu)\right)}}+1\right)}\text{ for }(z,\nu)\in\mathbb{C}^2\land-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

$$\frac{D_{\nu}(z)}{D_{\nu+1}(z)} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-1+k-\nu}{z} + z} \text{ for } (z,\nu) \in \mathbb{C}^2 \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\frac{D_{-\frac{3}{2}}(z)}{D_{-\frac{1}{2}}(z)} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}+k}{z} + z} \text{ for } z \in \mathbb{C} \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\pi = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2}{6} + 3$$

$$\pi = \frac{16}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{5(1+2k)} + 5} - \frac{4}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{239(1+2k)} + 239}$$

$$\pi = \frac{4}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1-\frac{2}{1+2k}}{\frac{2}{1+2k}} + 1}$$

$$\frac{\pi}{2} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(1+k)}{1} + 1} + 1$$

$$\frac{\pi}{2} = 1 - \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{((-1)^k-k)(1-(-1)^k+k)}{2+(-1)^k} + 3}$$

$$\frac{\pi}{4} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2}{2} + 1}$$

$$\frac{\pi}{4} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{1+2k} + 1}$$

$$\frac{\pi}{4} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^2}{1+2k} + 1}$$

$$\frac{\pi}{16} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2}{10} + 5}$$

$$\frac{\pi}{\sqrt{3}} = 2 - \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k(2+4k)}{6+5k} + 6}$$

$$\frac{1}{\pi} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2}{6} + 3}$$

$$\frac{2}{\pi} = 1 - \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(1+k)}{1} + 2}$$

$$\frac{2}{\pi} = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-1+(-1)^k+(-1+2(-1)^k)k-k^2}{2+(-1)^k} + 2} + 1$$

$$\frac{4}{\pi} = \operatorname*{K}_{k=1}^{\infty} \frac{k^2}{1+2k} + 1$$

$$\frac{4}{\pi} = \operatorname*{K}_{k=1}^{\infty} \frac{(-1+2k)^2}{2} + 1$$

$$\frac{16}{\pi} = \operatorname*{K}_{k=1}^{\infty} \frac{(-1+2k)^2}{10} + 5$$

$$\frac{\pi^2}{6} = \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{\frac{1}{8}(1-(-1)^k+4k+2k^2)}{1} + 1} + 1$$

$$\frac{\pi^2}{12} = \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{k^4}{1+2k} + 1}$$

$$\frac{6}{\pi^2} = 1 - \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{\frac{1}{8}(1-(-1)^k+4k+2k^2)}{1} + 2}$$

$$\frac{12}{\pi^2} = \operatorname*{K}_{k=1}^{\infty} \frac{k^4}{1+2k} + 1$$

$$\frac{6}{\pi^2 - 6} = \operatorname*{K}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor \left\lfloor \frac{2+k}{2} \right\rfloor}{1} + 1$$

$$\frac{2}{4-\pi} = \operatorname*{K}_{k=1}^{\infty} \frac{1}{\frac{(-15+17(-1)^k)(-1+2k)^2(9+16k+8k^2)\left(\left(\frac{1}{4}(5-2k)\right)_{\left\lfloor \frac{1}{2}(-1+k) \right\rfloor}\right)^2}{64(1+k)(1+2k)^2\left(\left(\frac{1}{4}(3-2k)\right)_{\left\lfloor \frac{k}{2} \right\rfloor}\right)^2}} + \frac{9}{2}$$

$$\pi^2 = \frac{6}{\operatorname{K}_{k=1}^{\infty} \frac{-\frac{k^2}{(1+k)^2}}{1+\frac{k^2}{(1+k)^2}} + 1}$$

$$\psi^{(0)}(z) = \frac{\pi^2 z}{6\left(\operatorname{K}_{k=1}^{\infty} \frac{\frac{z\zeta(2+k)}{\zeta(1+k)}}{1-\frac{z\zeta(2+k)}{\zeta(1+k)}} + 1\right)} - \frac{1}{z} - \gamma \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\psi^{(1)}(z) = \frac{1}{z\left(\operatorname{K}_{k=1}^{\infty} \frac{\frac{\left(1+(-1)^k\right)k^2}{16(1+k)} - \frac{\left(1-(-1)^k\right)(1+k)^2}{16k}}{\frac{z}{1}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\psi^{(1)}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{\frac{\left(1+(-1)^{k}\right)k^{2}}{16(1+k)} - \frac{\left(1-(-1)^{k}\right)(1+k)^{2}}{16k}}{z}}{1} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\psi^{(1)}(z) = \frac{1}{z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{-\frac{\left(1+(-1)^{k}\right)k^{2}}{8(1+k)} + \frac{\left(1-(-1)^{k}\right)(1+k)^{2}}{8k}}{2z}}{1} + 1\right)} + \frac{1}{z^{2}} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\psi^{(1)}(z) = \frac{1}{4z^{3}\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{4z^{2}}}{\frac{1}{1+k} + \frac{1}{2+k}} + \frac{3}{2}\right)} + \frac{1}{2z^{2}} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\psi^{(1)}(z) = \frac{1}{2z^{2}\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{4}k(1+k)^{2}(2+k)}{(3+2k)z} + 3z\right)} + \frac{1}{2z^{2}} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\psi^{(1)}(z) = \frac{1}{z^{2}\left(\mathrm{K}_{k=1}^{\infty} \frac{k(1+k)^{2}(2+k)}{2(3+2k)z} + 6z\right)} + \frac{1}{2z^{2}} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\psi^{(1)}(z) = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{k^{4}}{4(-1+2k)(1+2k)}}{-\frac{1}{2}+z} + z - \frac{1}{2}} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\psi^{(1)}(z) = \frac{2}{\mathrm{K}_{k=1}^{\infty} \frac{k^{4}}{(1+2k)(-1+2z)} + 2z - 1} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\psi^{(1)}(z) = \frac{2}{\mathrm{K}_{k=1}^{\infty} \frac{k^{4}}{(1+2k)(1+2z)} + 2z + 1} + \frac{1}{z^{2}} \text{ for } z \in \mathbb{C} \wedge \Re(z) > -\frac{1}{2}$$

$$\psi^{(1)}(z) = \frac{1}{6z\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{k(1+k)^{2}(2+k)}{4\left(3+8k+4k^{2}\right)}}{\frac{1}{2}(1-(-1)^{k}+(1+(-1)^{k})z^{2})} + z^{2}\right)} + \frac{1}{2z^{2}} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\psi^{(1)}(z+1) = \frac{z + \frac{1}{2}}{\mathrm{K}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor^{2}\left(-1+2\left\lfloor \frac{1+k}{2} \right\rfloor\right)^{2}}{\frac{1}{2}(1-(-1)^{k})(1+2k)+\frac{1}{2}(1+(-1)^{k})(1+2k)(z+z^{2})} + z^{2} + z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > -\frac{1}{2}$$

$$\psi^{(1)}\left(z+\frac{1}{2}\right)=\frac{8}{(1-4z^2)\left(\operatorname*{K}_{k=1}^{\infty}\frac{k^2(2+k)^2}{2(3+2k)z}+6z\right)}+\frac{4z}{4z^2-1}\text{ for }z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(1)}\left(z+\frac{1}{2}\right)=\frac{2}{\operatorname*{K}_{k=1}^{\infty}\frac{k^4}{2(1+2k)z}+2z}\text{ for }z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(2)}(z)=\frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{\left\{\begin{array}{ll}\frac{(1+k)^2}{2(17+2k+k^2)} & ((1+k)\bmod 4)=0\\ -\frac{20+4k+k^2}{8k} & ((2+k)\bmod 4)=0\\ \frac{17-2k+k^2}{8+8k} & ((3+k)\bmod 4)=0\\ -\frac{k^2}{32+2k^2} & (k\bmod 4)=0\end{array}\right.}{z}}{1}+1\right)}-\frac{1}{z}\text{ for }z\in\mathbb{C}\wedge\Re(z)>1$$

$$\psi^{(2)}(z)=-\frac{1}{(z-1)z\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{\frac{1}{32}\left(1+(-1)^k\right)k^4+\frac{1}{32}\left(1-(-1)^k\right)(1+k)^4}{(-1+z)z}}{1+k}+1\right)}\text{ for }z\in\mathbb{C}\wedge\neg\left(z\in\mathbb{R}\wedge\frac{1}{2}<z<1\right)\wedge\Re(z)$$

$$\psi^{(2)}(z)=-\frac{1}{2z^3\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{(1+(-1)^k)k(2+k)^2}{32(1+k)}+\frac{(1-(-1)^k)(1+k)^2(3+k)}{32(2+k)}}{z}+z\right)}-\frac{1}{z^3}-\frac{1}{z^2}\text{ for }z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(2)}(z)=-\frac{1}{z^3\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{1}{32}(1+(-1)^k)k(2+k)^3+\frac{1}{32}(1-(-1)^k)(1+k)^3(3+k)}{(2+k)z}+2z\right)}-\frac{1}{z^3}-\frac{1}{z^2}\text{ for }z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(2)}(z)=-\frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty}\frac{\frac{\left(\frac{1}{32}\left(1+(-1)^k\right)k^4+\frac{1}{32}\left(1-(-1)^k\right)(1+k)^4\right)(-1+z)}{z}}{(1+k)(-1+z)}+z-1\right)}\text{ for }z\in\mathbb{C}\wedge\neg\left(z\in\mathbb{R}\wedge\frac{1}{2}<z<1\right)\wedge\Re$$

$$\psi^{(2)}(z)=-\frac{2}{\operatorname*{K}_{k=1}^{\infty}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^3}{\frac{1}{2}(1-(-1)^k)+(1+(-1)^k)(1+k)(-1+z)z}+2(z-1)z}\text{ for }z\in\mathbb{C}\wedge\Re(z)>\frac{1}{2}$$

$$\psi^{(2)}(z) = -\frac{1}{2z^2\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1+(-1)^k)k(2+k)^2}{32(1+k)}+\frac{(1-(-1)^k)(1+k)^2(3+k)}{32(2+k)}}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)z^2}+z^2\right)}-\frac{1}{z^3}-\frac{1}{z^2} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(2)}\left(z+\frac{1}{2}\right) = -\frac{4}{(2z+1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1+(-1)^k)k^4(-1+2z)}{8(1+2z)}+\frac{(1-(-1)^k)(1+k)^4(-1+2z)}{8(1+2z)}}{(1+k)(-1+2z)}+2z-1\right)} \text{ for } z\in\mathbb{C}\wedge\neg\left(z\in\mathbb{R}\right.$$

$$\psi^{(2)}\left(z+\frac{1}{2}\right) = -\frac{4}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor^3}{((1+k)(-1+4z^2))^{\frac{1}{2}(1+(-1)^k)}}+4z^2-1} \text{ for } z\in\mathbb{C}\wedge\neg\left(z\in\mathbb{R}\wedge 0<z<\frac{1}{2}\right)\wedge\Re(z)>0$$

$$\psi^{(\nu)}(z) = \frac{\pi^2 z^{1-\nu}}{6\Gamma(2-\nu)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1+k)z\zeta(2+k)}{(1+k-\nu)\zeta(1+k)}}{1-\frac{(1+k)z\zeta(2+k)}{(1+k-\nu)\zeta(1+k)}}+1\right)}-\frac{\gamma z^{-\nu}}{\Gamma(1-\nu)}+\frac{z^{-\nu-1}(\psi^{(0)}(-\nu)-\log(z)+\gamma)}{\Gamma(-\nu)} \text{ for } (\nu,z)\in\mathbb{C}$$

$$\psi^{(m)}(z) = \frac{(-1)^{m+1}m!\zeta(m+1)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(k+m)z\zeta(1+k+m)}{k\zeta(k+m)}}{1-\frac{(k+m)z\zeta(1+k+m)}{k\zeta(k+m)}}+1}+(-1)^{m-1}m!z^{-m-1} \text{ for } m\in\mathbb{Z}\wedge z\in\mathbb{C}\wedge m>0\wedge|z|<1$$

$$\psi^{(1)}(a)-\psi^{(1)}(b) = \frac{4(b-a)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-4k^4(-(-a+b)^2+k^2)}{(1+2k)(2-2b+a(-2+4b)+2k(1+k))}+a(4b-2)-2b+2} \text{ for } (a,b)\in\mathbb{C}^2\wedge\Re(a+b)>1$$

$$\psi^{(1)}(a)-\psi^{(1)}(b) = \frac{4(b-a)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{8}(1+(-1)^k)k^3+\frac{1}{2}(1-(-1)^k)(1+k)\left(-(-a+b)^2+\frac{1}{4}(1+k)^2\right)}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)((-a+b)^2+(-1+(-1+a+b)^2)(1+k))}+2((a-1)a+(b-1)b)} \text{ for }$$

$$\psi^{(1)}\left(z+\frac{1}{2}\right)-\psi^{(1)}(z) = \frac{2}{z}-\frac{2}{z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\begin{cases}\frac{(1+k)^2}{16\left(-\frac{1}{2}+\frac{1+k}{4}\right)} & ((1+k)\bmod 4)=0\\ \frac{1}{2}+\frac{1}{4}(-2-k) & ((2+k)\bmod 4)=0\\ -\frac{1}{2}+\frac{1}{4}(-1+k) & ((3+k)\bmod 4)=0\\ -\frac{k^2}{16\left(-\frac{1}{2}+\frac{k}{4}\right)} & (k\bmod 4)=0\end{cases}}{2z}}{1}+1\right)} \text{ for } z\in\mathbb{C}\wedge\neg$$

$$\psi^{(1)}\left(z+\frac{1}{2}\right)-\psi^{(1)}(z)=-\frac{1}{z(2z-1)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2}{2z(-1+2z)}}{1}+1\right)} \text{ for } z\in\mathbb{C}\wedge\neg(z\in\mathbb{R}\wedge 0<z<1)\wedge\Re(z)>\frac{1}{2}$$

$$\psi^{(1)}\left(\frac{z+3}{4}\right)-\psi^{(1)}\left(\frac{z+1}{4}\right)=-\frac{8}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{4\left\lfloor\frac{1+k}{2}\right\rfloor^2}{(-1+z^2)^{\frac{1}{2}(1+(-1)^k)}}+z^2-1} \text{ for } z\in\mathbb{C}\wedge\Re(z)>-1$$

$$\psi^{(0)}(a)-\psi^{(0)}(b)=\frac{2(a-b)}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2(-(a-b)^2+k^2)}{(-1+a+b)(1+2k)}+a+b-1} \text{ for } (a,b)\in\mathbb{C}^2\wedge\Re(a+b)>1$$

$$\psi^{(0)}(a)-\psi^{(0)}(b)=\frac{a-b}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{k^2(a-b+k)(-a+b+k)}{4(-1+4k^2)}}{\frac{1}{2}(-1+a+b)}+\frac{1}{2}(a+b-1)} \text{ for } (a,b)\in\mathbb{C}^2\wedge\Re(a)>1\wedge\Re(b)>1$$

$$\psi^{(0)}\left(z+\frac{1}{2}\right)-\psi^{(0)}(z)=\frac{1}{2z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(-1)^k\left\lfloor\frac{1+k}{2}\right\rfloor}{4z}}{1}+1\right)} \text{ for } z\in\mathbb{C}\wedge\Re(z)>\frac{1}{4}$$

$$\psi^{(0)}\left(z+\frac{1}{2}\right)-\psi^{(0)}(z)=\frac{1}{z}-\frac{1}{2z\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(-1)^{-1+k}\left\lfloor\frac{1+k}{2}\right\rfloor}{4z}}{1}+1\right)} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(0)}\left(z+\frac{1}{2}\right)-\psi^{(0)}(z)=\frac{\frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k(1+k)}{4z}+4z}+1}{2z} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(0)}\left(z+\frac{1}{2}\right)-\psi^{(0)}(z)=\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2}{-1+4z}+4z-1} \text{ for } z\in\mathbb{C}\wedge\Re(z)>\frac{1}{4}$$

$$\psi^{(0)}\left(z+\frac{1}{2}\right)-\psi^{(0)}(z)=\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k(1+k)}{4^{1+(-1)^k}z^{1+(-1)^k}}+16z^2}+\frac{1}{2z} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(0)}\left(\frac{z+3}{4}\right)-\psi^{(0)}\left(\frac{z+1}{4}\right)=\frac{2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{k^2}{z}+z} \text{ for } z\in\mathbb{C}\wedge\Re(z)>0$$

$$\psi^{(0)}(z+1)-\psi^{(0)}\left(\frac{z}{3}+1\right)=\frac{2}{3\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{3}{16}(1+(-1)^k)k(2+3k)(4+3k)+\frac{3}{16}(1-(-1)^k)(1+k)(-1+9k^2)}{3(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(1+k)z^2}+z^2\right)}-\frac{1}{z}+\log(3)\ \text{fo}$$

$$-\psi^{(0)}\left(\frac{z}{2(a+b)}\right)+\psi^{(0)}\left(\frac{2a+z}{2(a+b)}\right)+\psi^{(0)}\left(\frac{2b+z}{2(a+b)}\right)-\psi^{(0)}\left(\frac{z}{2(a+b)}+1\right)=\frac{\cdots}{z\left(\mathrm{K}_{k=1}^{\infty}\cdots\right)}$$

$$-\psi^{(0)}\left(\frac{z}{2(a+b)}\right)+\psi^{(0)}\left(\frac{2a+z}{2(a+b)}\right)+\psi^{(0)}\left(\frac{2b+z}{2(a+b)}\right)-\psi^{(0)}\left(\frac{z}{2(a+b)}+1\right)=\frac{\cdots}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-1)^k)k^2\left(a+\cdots\right.}{\cdots}\right.}$$

$$-\psi^{(0)}\left(\frac{z}{2(a+b)}\right)+\psi^{(0)}\left(\frac{2a+z}{2(a+b)}\right)+\psi^{(0)}\left(\frac{2b+z}{2(a+b)}\right)-\psi^{(0)}\left(\frac{z}{2(a+b)}+1\right)=\frac{4a\cdots}{\mathrm{K}_{k=1}^{\infty}\frac{-\frac{2k^2(-a^2+(a+b)^2k^2)\cdots}{-1+4\cdots}}{-a^2-b^2+(a+b)^2(1-2(1\cdots}}$$

$$-\psi^{(0)}\left(\frac{-a+b+z}{4b}\right)-\psi^{(0)}\left(\frac{a+b+z}{4b}\right)+\psi^{(0)}\left(\frac{-a+b+z}{4b}+\frac{1}{2}\right)+\psi^{(0)}\left(\frac{a+b+z}{4b}+\frac{1}{2}\right)=\frac{\cdots}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}(1+(-\cdots}{\cdots}}$$

$$-\psi^{(0)}\left(\frac{1}{2}(-a-b+z+1)\right)+\psi^{(0)}\left(\frac{1}{2}(a-b+z+1)\right)+\psi^{(0)}\left(\frac{1}{2}(-a+b+z+1)\right)-\psi^{(0)}\left(\frac{1}{2}(a+b+z+1)\right)\cdots$$

$$-\psi^{(0)}\left(\frac{1}{4}(-a+z+1)\right)+\psi^{(0)}\left(\frac{1}{4}(-a+z+3)\right)+\psi^{(0)}\left(\frac{1}{4}(a+z+1)\right)-\psi^{(0)}\left(\frac{1}{4}(a+z+3)\right)=\frac{\cdots}{\mathrm{K}_{k=1}^{\infty}\frac{-\frac{1}{2}\cdots}{\cdots}}$$

$$\mathrm{Li}_{\nu}(z)=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{-\left(\frac{k}{1+k}\right)^{\nu}z}{1+\left(\frac{k}{1+k}\right)^{\nu}z}+1}\ \text{for}\ (\nu,z)\in\mathbb{C}^2\wedge|z|<1$$

$$\mathrm{Li}_m(z)=\frac{(-1)^{m-1}}{z\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{k^m(1+k)^{-m}}{z}}{1+\frac{k^m(1+k)^{-m}}{z}}+1\right)}-\frac{(2i\pi)^m B_m\left(\frac{1}{2}-\frac{i\log(-z)}{2\pi}\right)}{m!}\ \text{for}\ m\in\mathbb{Z}\wedge z\in\mathbb{C}\wedge m>0\wedge|z|>1$$

$$S_{\nu,p}(z) = \frac{p^{-\nu} z^p}{p! \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left(1-\frac{1}{k+p}\right)^{\nu} z S_{k+p}^{(p)}}{(k+p) S_{-1+k+p}^{(p)}}}{1-\frac{\left(1-\frac{1}{k+p}\right)^{\nu} z S_{k+p}^{(p)}}{(k+p) S_{-1+k+p}^{(p)}}} + 1 \right)} \text{ for } p \in \mathbb{Z} \wedge (\nu, z) \in \mathbb{C}^2 \wedge p > 0 \wedge |z| < 1$$

$$z^a = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{a(-1+k)! \log(z)}{k!}}{1+\frac{a(-1+k)! \log(z)}{k!}} + 1} \text{ for } (a, z) \in \mathbb{C}^2 \wedge |\arg(z)| < \pi$$

$$(z+1)^a = \frac{az}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)}{4(1+k)} + \frac{\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}\right)}{4k} \right) z}{1} + 1} + 1 \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{(-1-a+k)z}{k}}{1-\frac{(-1-a+k)z}{k}} + 1} \text{ for } (a, z) \in \mathbb{C}^2 \wedge |z| < 1$$

$$(z+1)^a = \frac{z^a}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{-1-a+k}{kz}}{1-\frac{-1-a+k}{kz}} + 1} \text{ for } (a, z) \in \mathbb{C}^2 \wedge |z| > 1$$

$$(z+1)^a = \frac{1}{1 - \frac{az}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1+(-1)^k\right)\left(-a+\frac{k}{2}\right)}{4(1+k)} + \frac{\left(1-(-1)^k\right)\left(a+\frac{1+k}{2}\right)}{4k} \right) z}{1} + 1}} \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \frac{1}{1 - \frac{az}{(z+1) \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left(\frac{\left(1-(-1)^k\right)\left(a+\frac{1}{2}(-1-k)\right)}{4k} + \frac{\left(1+(-1)^k\right)\left(-a-\frac{k}{2}\right)}{4(1+k)} \right) z}{1+z}}{1} + 1 \right)}} \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \frac{az}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k(1+k) \left(\frac{\left(1+(-1)^k\right)\left(a+\frac{k}{2}\right)}{4(1+k)} + \frac{\left(1-(-1)^k\right)\left(-a+\frac{1+k}{2}\right)}{4k} \right) z}{1+k} + 1} + 1 \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z \left(-(-1)^k a + \left\lfloor \frac{k}{2} \right\rfloor \right)}{1 + (-1)^k + \frac{1}{2}\left(1-(-1)^k\right) k} + 1 \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \cfrac{1}{1 - \cfrac{az}{\operatorname*{K}_{k=1}^{\infty} \cfrac{k(1+k)\left(\frac{(1+(-1)^k)\left(-a+\frac{k}{2}\right)}{4(1+k)} + \frac{(1-(-1)^k)\left(a+\frac{1+k}{2}\right)}{4k}\right)z}{1+k} + 1}} \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \cfrac{1}{\operatorname*{K}_{k=1}^{\infty} \frac{z\left((-1)^k a + \left\lfloor \frac{k}{2} \right\rfloor\right)}{1+(-1)^k+\frac{1}{2}(1-(-1)^k)k} + 1} \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \cfrac{az}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{(-a+k)z}{1+k}}{1-\frac{(-a+k)z}{1+k}} + 1} + 1 \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$(z+1)^a = \cfrac{1}{\operatorname*{K}_{k=1}^{\infty} \frac{z\left((-1)^k a - \left\lfloor \frac{k}{2} \right\rfloor\right)}{1+(-1)^k+\frac{1}{2}(1-(-1)^k)k(1+z)} + 1} \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$(z+1)^a = \cfrac{1}{1 - \cfrac{az}{\operatorname*{K}_{k=1}^{\infty} \frac{-k(a+k)z(1+z)}{1+k+(1+a+2k)z} + (a+1)z + 1}} \text{ for } z \in \mathbb{C} \wedge \Re(z) > -\frac{1}{2}$$

$$(z+1)^a = \cfrac{1}{1 - \cfrac{az}{\operatorname*{K}_{k=1}^{\infty} \frac{k(-a+k)z}{1+k-(-a+k)z} + az + 1}} \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$(z+1)^a = \cfrac{2az}{\operatorname*{K}_{k=1}^{\infty} \frac{(a^2-k^2)z^2}{(1+2k)(2+z)} + (1-a)z + 2} + 1 \text{ for } (a,z) \in \mathbb{C}^2 \wedge |\arg(z+1)| < \pi$$

$$\frac{1}{z+1} = 1 - \cfrac{z}{\operatorname*{K}_{k=1}^{\infty} \frac{z}{1-z} + 1} \text{ for } z \in \mathbb{C} \wedge |z| < 1$$

$$\left(\frac{z+1}{z-1}\right)^a = \cfrac{2a}{\operatorname*{K}_{k=1}^{\infty} \frac{a^2-k^2}{(1+2k)z} - a + z} + 1 \text{ for } (a,z) \in \mathbb{C}^2 \wedge \neg(z \in \mathbb{R} \wedge 0 < z < 1) \wedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

$$\left(\frac{az+1}{bz+1}\right)^\nu = \cfrac{2\nu z(a-b)}{\operatorname*{K}_{k=1}^{\infty} \frac{-(a-b)^2 z^2 (k^2-\nu^2)}{(1+2k)(2+(a+b)z)} + z(-\nu(a-b)+a+b) + 2} + 1 \text{ for } (a,b,z,\nu) \in \mathbb{C}^4 \wedge |az| < 1 \wedge |bz| < 1$$

$$(x^p+y)^{m/p} = \cfrac{my}{\operatorname*{K}_{k=1}^{\infty} \frac{y\left((-1)^k m + p\left\lfloor \frac{1+k}{2} \right\rfloor\right)}{(1-(-1)^k)x^m + \frac{1}{2}(1+(-1)^k)(1+k)px^{-m+p}} + px^{p-m}} + x^m \text{ for } (m,p) \in \mathbb{Z}^2 \wedge (x,y) \in \mathbb{C}^2 \wedge m > 0$$

$$\left(az^2+bz+c\right)^r=\frac{c^r}{\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{2az(-1+k)!\,{}_2F_1\left(-k,-r;1-k+r;\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)(-r)_k}{\left(-b+\sqrt{b^2-4ac}\right)k!\,{}_2F_1\left(1-k,-r;2-k+r;\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)(-r)_{-1+k}}}{1+\frac{2az(-1+k)!\,{}_2F_1\left(-k,-r;1-k+r;\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)(-r)_k}{\left(-b+\sqrt{b^2-4ac}\right)k!\,{}_2F_1\left(1-k,-r;2-k+r;\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)(-r)_{-1+k}}}+1}\text{ for }(a,b,c,r,z)\in\mathbb{C}^5$$

$$\frac{(z+1)^a-(1-z)^a}{(1-z)^a+(z+1)^a}=\frac{az}{\operatorname*{K}_{k=1}^{\infty}\frac{(a^2-k^2)z^2}{1+2k}+1}\text{ for }a\in\mathbb{C}\land z\in\mathbb{C}\land\neg(z\in\mathbb{R}\land(-\infty<z\leq-1\lor 1\leq z<\infty))$$

$$\frac{(z-1)^a+(z+1)^a}{(z+1)^a-(z-1)^a}=\frac{\operatorname*{K}_{k=1}^{\infty}\frac{a^2-k^2}{(1+2k)z}+z}{a}\text{ for }a\in\mathbb{C}\land z\in\mathbb{C}\land-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

$$z^{\frac{1}{z}}=\frac{z-1}{\operatorname*{K}_{k=1}^{\infty}\frac{(-1+z)\left((-1)^k+z\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1-(-1)^k+\frac{1}{2}(1+(-1)^k)(1+k)z}+z}+1\text{ for }z\in\mathbb{C}\land|\arg(z)|<\pi$$

$$z^{\frac{1}{z}}=\frac{2(z-1)}{\operatorname*{K}_{k=1}^{\infty}\frac{(-1+z)^2(1-k^2z^2)}{(1+2k)z(1+z)}+z^2+1}+1\text{ for }z\in\mathbb{C}\land|\arg(z)|<\pi$$

$$\frac{\prod_{k=0}^{\infty}\left(1+\frac{4a^2}{(1+2k+z)^2}\right)-\frac{\Gamma\left(\frac{z+1}{2}\right)^2}{\Gamma\left(\frac{1}{2}(-2a+z+1)\right)\Gamma\left(\frac{1}{2}(2a+z+1)\right)}}{\prod_{k=0}^{\infty}\left(1+\frac{4a^2}{(1+2k+z)^2}\right)+\frac{\Gamma\left(\frac{z+1}{2}\right)^2}{\Gamma\left(\frac{1}{2}(-2a+z+1)\right)\Gamma\left(\frac{1}{2}(2a+z+1)\right)}}=\frac{\operatorname*{K}_{k=1}^{\infty}\frac{4a^4+(-1+k)^4}{(-1+2k)z}}{2a^2}\text{ for }(a,z)\in\mathbb{C}^2\land\Re(z)>2\left|\Re(a)\right|$$

$$\frac{\prod_{k=1}^{\infty}\left(1+\frac{a^3}{(k+z)^3}\right)-\prod_{k=1}^{\infty}\left(1-\frac{a^3}{(k+z)^3}\right)}{\prod_{k=1}^{\infty}\left(1-\frac{a^3}{(k+z)^3}\right)+\prod_{k=1}^{\infty}\left(1+\frac{a^3}{(k+z)^3}\right)}=\frac{\operatorname*{K}_{k=1}^{\infty}\frac{a^6-(-1+k)^6}{(-1+2k)((-1+k)^2+k+2z+2z^2)}}{a^3}\text{ for }(a,z)\in\mathbb{C}^2$$

$$W(z)=\frac{z}{\operatorname*{K}_{k=1}^{\infty}\frac{\frac{\left(1+\frac{1}{k}\right)^k kz}{1+k}}{1-\frac{\left(1+\frac{1}{k}\right)^k kz}{1+k}}+1}\text{ for }z\in\mathbb{C}\land|z|<\frac{1}{e}$$

$$\frac{1}{(-q;q)_\infty\left(q^2;q^2\right)_\infty}-1=\operatorname*{K}_{k=1}^{\infty}\frac{-\frac{1}{2}\left(1-(-1)^k\right)q^k+\frac{1}{2}\left(1+(-1)^k\right)q^{k/2}\left(1-q^{k/2}\right)}{1}\text{ for }q\in\mathbb{C}\land 0<|q|<1$$

$$\frac{\left(-q;q^2\right)_\infty}{\left(-q^2;q^2\right)_\infty}-1=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{2}\left(1-(-1)^k\right)q^k+\frac{1}{2}\left(1+(-1)^k\right)q^{k/2}\left(1+q^{k/2}\right)}{1}\text{ for }q\in\mathbb{C}\wedge 0<|q|<1$$

$$\frac{\left(-q;q^4\right)_\infty}{\left(-q^3;q^4\right)_\infty}-1=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{2}\left(1-(-1)^k\right)q^{-1+2k}+\frac{1}{2}\left(1+(-1)^k\right)q^k\left(1+q^{-1+k}\right)}{1}\text{ for }q\in\mathbb{C}\wedge 0<|q|<1$$

$$\frac{\left(q^3;q^8\right)_\infty\left(q^5;q^8\right)_\infty}{\left(q;q^8\right)_\infty\left(q^7;q^8\right)_\infty}-1=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{2}\left(1+(-1)^k\right)q^{2k}+\frac{1}{2}\left(1-(-1)^k\right)\left(q^k+q^{2k}\right)}{1}\text{ for }q\in\mathbb{C}\wedge 0<|q|<1$$

$$\frac{\left(b^2v^2+cq\right)\left((-v;q)_\infty\left(-\frac{cq}{b^2v};q\right)_\infty+(v;q)_\infty\left(\frac{cq}{b^2v};q\right)_\infty\right)}{bv\left((-v;q)_\infty\left(-\frac{cq}{b^2v};q\right)_\infty-(v;q)_\infty\left(\frac{cq}{b^2v};q\right)_\infty\right)}+bq-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{cq^k+cq^{3k}+b^2q^{2k}\left(\frac{c^2q}{b^4v^2}+\frac{v^2}{q}\right)}{b-bq^{1+2k}}\text{ for }(b,$$

$$\frac{2bq\left(\left((-q;q)_\infty\right)^2+\left((q;q)_\infty\right)^2\right)}{\left((-q;q)_\infty\right)^2-\left((q;q)_\infty\right)^2}+bq-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{b^2q^{1+k}+2b^2q^{1+2k}+b^2q^{1+3k}}{b-bq^{1+2k}}\text{ for }(b,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\left(1-q^2\sigma5z^2\right)\left(1-q^3\sigma5z^2\right)\left(q^{10}\sigma5^2\left(-z^5\right)+q^6\sigma1\sigma5z^3-q^4\sigma4z^2+1\right)\left(\frac{\left(q^2z;q\right)_\infty\left(\frac{qz}{\mathrm{a1}};q\right)_\infty\left(\frac{qz}{\mathrm{a2}};q\right)_\infty}{(qz;q)_\infty\left(\frac{q^2z}{\mathrm{a1}};q\right)_\infty\left(\frac{q^2z}{\mathrm{a2}};q\right)_\infty\left(\frac{q^2z}{\mathrm{a3}};\right.}\right.$$

$$-cq^6+cq^3+\frac{2c(q-1)^2q\left(q^3;q^2\right)_\infty}{\left((-q;-q)_\infty-(q;-q)_\infty\right)\left(q^2;q^4\right)_\infty}+cq-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{c^2q^{4k}\left(1-q^{4k}\right)\left(1-q^{-1+4k}\right)\left(1-q^{1+4k}\right)}{c-cq^{1+4k}\left(1+q+q^2\right)+cq^{2+8k}\left(1+q^4\right)}\text{ for }(c$$

$$\frac{ab^2c^2(d;q)_\infty(e;q)_\infty\left(bc-deq^2\right)\left(\frac{de}{abc};q\right)_\infty{}_3\phi_2\left(a,b,c;d,e;q,\frac{de}{abc}\right)}{(dq;q)_\infty(eq;q)_\infty\left(\frac{deq}{abc};q\right)_\infty{}_3\phi_2\left(aq,b,c;dq,eq;q,\frac{deq}{abc}\right)}+(bc-deq)\left(a\left(b^2c(c(d+e-1)-de(q+1)\right.\right.$$

$$\frac{c(q+1)^2}{q\,{}_2\phi_2\left(q,q^2;-q,-q^3;q^2,q\right)}-2cq-\frac{c}{q}-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{c^2q^{-2+2k}\left(1-q^{4k}\right)\left(1+q^{-2+2k}\right)}{c+cq^{1+2k}+cq^{-1+4k}\left(1+q^2\right)}\text{ for }(c,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$${}_2\phi_1(a,b;bq;q,z)=\frac{ab(b;q)_\infty\left(\frac{az}{q};q\right)_\infty}{(bq;q)_\infty(z;q)_\infty\left(ab\,\mathrm{K}_{k=1}^{\infty}\frac{abq^{-2+k}z-b^2q^{-3+4k}z^2-bq^{-3+2k}z(bq+a(q+z))+bq^{-3+3k}z(az+b(q+z))}{1+bq^{-1+2k}(1+q)z-q^{-1+k}(az+b(q+z))}+\right.}$$

$$ {}_2\phi_1(a,q;cq;q,z) = \cfrac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\left(\frac{(1-(-1)^k) q^{\frac{1}{2}(-1+k)}\left(1-aq^{\frac{1}{2}(-1+k)}\right)\left(-1+cq^{\frac{1}{2}(-1+k)}\right)}{2\left(1-cq^{-1+k}\right)\left(1-cq^k\right)} + \frac{(1+(-1)^k) q^{-1+\frac{k}{2}}\left(1-q^{k/2}\right)\left(-a+cq^{k/2}\right)}{2\left(1-cq^{-1+k}\right)\left(1-cq^k\right)}\right) z}{1} \ldots} $$

$$ {}_2\phi_1(a,q;cq;q,z) = \cfrac{1-c}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1-(-1)^k) q^{\frac{1}{2}(-1+k)}\left(1-aq^{\frac{1}{2}(-1+k)}\right)\left(-1+cq^{\frac{1}{2}(-1+k)}\right)z+\frac{1}{2}(1+(-1)^k) q^{\frac{1}{2}(-2+k)}\left(1-q^{k/2}\right)\left(-a+cq \ldots\right.}{1-cq^k} \ldots} $$

$$ {}_2\phi_1(a,q;cq;q,z) = \frac{(1-c)q}{\mathrm{K}_{k=1}^{\infty} \frac{q(1-q^k)(-a+cq^k)z}{q(1-cq^k)+(a-q^{1+k})z} + z(a-q) + (1-c)q} \text{ for } (a,c,q,z) \in \mathbb{C}^4 \wedge 0<|q|<1 \wedge |z| < \ldots $$

$$ {}_2\phi_1(a,q;cq;q,z) = \frac{1-c}{\mathrm{K}_{k=1}^{\infty} \frac{q^{-1+k}(1-aq^{-1+k})(1-q^k)z(c-aq^{-1+k}z)}{1-cq^k+q^k\left(-1+\frac{a\left(-1+q^k+q^{1+k}\right)}{q}\right)z} + az - c - z + 1} \text{ for } (a,c,q,z) \in \mathbb{C}^4 \wedge 0<|q| < \ldots $$

$$ {}_2\phi_1\left(q,q;q^2;q,z\right) = \cfrac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\left(-\frac{(1-(-1)^k) q^{\frac{1}{2}(-1+k)}\left(1-q^{\frac{1+k}{2}}\right)}{2(1-q^k)\left(1+q^{\frac{1+k}{2}}\right)} - \frac{(1+(-1)^k) q^{k/2}\left(1-q^{k/2}\right)}{2\left(1+q^{k/2}\right)\left(1-q^{1+k}\right)}\right) z}{1} + 1} \text{ for } (q,z) \in \mathbb{C}^2 \wedge 0<|q| \ldots $$

$$ {}_2\phi_1\left(0,aq;aq^2;q,z\right) = \frac{1-aq}{(z;q)_\infty \left(\mathrm{K}_{k=1}^{100} \frac{-\frac{1}{2}(1+(-1)^k) aq^{k/2}\left(1-q^{k/2}\right)z - \frac{1}{2}(1-(-1)^k) aq^{\frac{1+k}{2}}\left(1-q^{\frac{1}{2}(-1+k)}z\right)}{1} + 1\right)} \text{ for } (a \ldots $$

$$ {}_1\phi_1(a;aq;q,z) = \frac{q(a;q)_\infty \left(\frac{z}{q};q\right)_\infty}{(aq;q)_\infty \left(q\left(\mathrm{K}_{k=1}^{\infty} \frac{-aq^{-2+k}(-1+q^k)z}{1-aq^k-q^{-1+k}z}\right) - aq + q - z\right)} \text{ for } (a,q,z) \in \mathbb{C}^3 \wedge 0<|q|<1 $$

$$ {}_2\phi_2\left(a,q;c,\frac{aqz}{c};q,z\right) = -\frac{cq\left(\frac{c}{q};q\right)_\infty \left(\frac{az}{c};q\right)_\infty}{(c;q)_\infty \left(\frac{aqz}{c};q\right)_\infty \left(-cq\left(\mathrm{K}_{k=1}^{100} \frac{-\frac{q^{-3+k}(-1+q^k)(aq-cq^k)z(c-q^kz)}{c}}{1+q^{-1+2k}(1+q)z-\frac{q^{-1+k}(c^2+cz+aqz)}{c}}\right) + aqz + c^2 - c \ldots\right.} $$

$$ {}_2\phi_2\left(q,q^2;-q,-q^3;q^2,q\right) = \frac{(q+1)^2}{q\left(\mathrm{K}_{k=1}^{\infty} \frac{-q^{-4+2k}(-1+q^{2k})(1+q^{2k})(q^2+q^{2k})}{1+q^{1+2k}+q^{-1+4k}+q^{1+4k}}\right) + 2q^2 + q + 1} \text{ for } q \in \mathbb{C} \wedge 0<|q|<1 $$

$$\frac{\text{QHypergeometricPFQ}(\{\},\{b\},q,z)}{\text{QHypergeometricPFQ}(\{\},\{b\},q,qz)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{q^{-1+k}z}{1-bq^{-1+k}} + 1 \text{ for } (b,q,z) \in \mathbb{C}^3 \wedge 0 < |q| < 1$$

$$\frac{\text{QHypergeometricPFQ}(\{\},\{b\},q,z)}{\text{QHypergeometricPFQ}(\{\},\{b\},q,qz)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}\left(1-(-1)^k\right)q^{-1+k}z + \frac{1}{2}\left(1+(-1)^k\right)\left(-bq^{-1+\frac{k}{2}} + q^{-1+k}z\right)}{1} +$$

$$\frac{{}_1\phi_1(a;b;q,z)}{{}_1\phi_1(a;b;q,qz)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}\left(1+(-1)^k\right)\left(-bq^{-1+\frac{k}{2}} + aq^{-1+k}z\right) + \frac{1}{2}\left(1-(-1)^k\right)\left(aq^{-1+k}z - q^{-1+\frac{1+k}{2}}z\right)}{1} + 1 \text{ for}$$

$$\frac{{}_1\phi_1(a;b;q,z)}{{}_1\phi_1(a;bq;q,qz)} = \frac{(bq;q)_\infty \left(\frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-q^{-1+k}\left(-a+bq^k\right)r^2 z}{r-q^k r(b+z)}}{r} - b - z + 1\right)}{(b;q)_\infty} \text{ for } (a,b,z,q) \in \mathbb{C}^4 \wedge 0 < |q| < 1$$

$$\frac{{}_1\phi_1(0;-q;q,z)}{{}_1\phi_1(0;-q;q,qz)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}\left(1+(-1)^k\right)q^{-1+\frac{3k}{2}}z - \frac{1}{2}\left(1-(-1)^k\right)q^{-1+\frac{1+k}{2}}z}{1+q^k} + 1 \text{ for } (q,z) \in \mathbb{C}^2 \wedge 0 < |q| < 1$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,b;c;q,qz)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{2}\left(1-(-1)^k\right)\left(1-aq^{\frac{1}{2}(-1+k)}\right)\left(1-bq^{\frac{1}{2}(-1+k)}\right)z + \frac{1}{2}\left(1+(-1)^k\right)\left(-cq^{-1+\frac{k}{2}} + abq\right.}{\frac{1}{2}\left(1+(-1)^k\right) + \frac{1}{2}\left(1-(-1)^k\right)(1-z)}$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,b;cq;q,qz)} = \frac{(cq;q)_\infty (qz;q)_\infty \left(ab \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{q^{-1+k}\left(-a+cq^k\right)\left(-b+cq^k\right)z\left(-ab+abq^k z\right)}{ab}}{\frac{abq^k\left(-b+cq^k(1+q)\right)z + a\left(b-bcq^k-abq^k z\right)}{ab}} + abz(-b+cq+c) + a\right.}{ab(c;q)_\infty (z;q)_\infty}$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,bq;cq;q,z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(\frac{\left(1-(-1)^k\right)q^{\frac{1}{2}(-1+k)}\left(1-aq^{\frac{1}{2}(-1+k)}\right)\left(-b+cq^{\frac{1}{2}(-1+k)}\right)}{2\left(1-cq^{-1+k}\right)\left(1-cq^k\right)} + \frac{\left(1+(-1)^k\right)q^{-1+\frac{k}{2}}\left(1-bq^{k/2}\right)\left(-a+cq^{k/2}\right.}{2\left(1-cq^{-1+k}\right)\left(1-cq^k\right)}\right.}{1}$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,bq;cq;q,z)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\left(-\frac{\left(-1+(-1)^k\right)q^{\frac{1}{2}(-1+k)}\left(\sqrt{q}-aq^{k/2}\right)\left(b\sqrt{q}-cq^{k/2}\right)}{2\left(q-cq^k\right)\left(-1+cq^k\right)} - \frac{\left(1+(-1)^k\right)q^{k/2}\left(-1+bq^{k/2}\right)\left(-a+cq^{k/2}\right)}{2\left(-1+cq^k\right)\left(-q+cq^k\right)}\right) z}{1}$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,bq;cq;q,z)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{2}\left(1-(-1)^k\right)q^{\frac{1}{2}(-1+k)}\left(1-aq^{\frac{1}{2}(-1+k)}\right)\left(-b+cq^{\frac{1}{2}(-1+k)}\right)z+\frac{1}{2}\left(1+(-1)^k\right)q^{\frac{1}{2}(-2+k)}\left(1-bq^{k/2}\right)\left(-a \cdots\right)}{1-cq^k}}{1-c}$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,bq;cq;q,z)} = \operatorname*{K}_{k=1}^{\infty} \frac{\left(-\frac{\left(-1+(-1)^k\right)q^{\frac{1}{2}(-1+k)}\left(\sqrt{q}-aq^{k/2}\right)\left(b\sqrt{q}-cq^{k/2}\right)}{2(q-cq^k)(-1+cq^k)} - \frac{\left(1+(-1)^k\right)q^{k/2}\left(-1+bq^{k/2}\right)\left(-a+cq^{k/2}\right)}{2(-1+cq^k)(-q+cq^k)}\right) z \cdots}{1}$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(a,bq;cq;q,z)} = \frac{az\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{q^3\left(-1+bq^k\right)\left(a-cq^k\right)}{a^2 z}}{\frac{q\left(q+az-q^{1+k}(c+bz)\right)}{az}}\right)}{(1-c)q^2} + \frac{z(a-bq)}{(1-c)q} + 1 \text{ for } (a,b,c,q,z) \in \mathbb{C}^5 \wedge 0<|q|<1 \wedge 0< \Big| \cdots$$

$$\frac{{}_2\phi_1(a,b;c;q,z)}{{}_2\phi_1(aq,bq;cq;q,z)} = \frac{\operatorname*{K}_{k=1}^{\infty} \frac{q^{-1+k}\left(1-aq^k\right)\left(1-bq^k\right)\left(cz-abq^k z^2\right)}{1-cq^k-q^k(a+b-abq^k-abq^{1+k})z}}{1-c} - \frac{z(-abq+a(-b)+a+b)}{1-c} + 1 \text{ for } (a,b,c,q,z) \cdots$$

$$\frac{{}_2\phi_1(aq,b;c;q,z)}{{}_2\phi_1(a,b;c;q,qz)} = \operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{2}\left(1+(-1)^k\right)\left(a-cq^{-1+\frac{k}{2}}\right)+\frac{1}{2}\left(1-(-1)^k\right)\left(1-bq^{\frac{1}{2}(-1+k)}\right)z}{\frac{1}{2}\left(1+(-1)^k\right)+\frac{1}{2}\left(1-(-1)^k\right)(1-a)(1-z)} + 1 \text{ for } (a,b,c,q,z) \cdots$$

$$\frac{{}_2\phi_1\left(a,aq;q^3;q^2,z\right)}{{}_2\phi_1\left(a,\frac{a}{q};q;q^2,z\right)} = \frac{1-q}{\operatorname*{K}_{k=1}^{\infty} \frac{q^{-1+k}\left(\sqrt{z}-aq^{-1+k}\sqrt{z}\right)\left(-\frac{a\sqrt{z}}{q}+q^k\sqrt{z}\right)}{1-q^{1+2k}} - q + 1} \text{ for } (a,q,z) \in \mathbb{C}^3 \wedge 0<|q|<1 \wedge \Big| \frac{\cdots}{(1 \cdots}$$

Undefined for $(a,b,c,e,\text{Undefined},q) \in \mathbb{C}^6 \wedge 0<|q|<1$

$$\frac{{}_3\phi_2\left(a,b,c;d,e;q,\frac{de}{abc}\right)}{{}_3\phi_2\left(aq,b,c;dq,eq;q,\frac{deq}{abc}\right)} = \frac{(dq;q)_\infty(eq;q)_\infty\left(\frac{deq}{abc};q\right)_\infty\left(ab^2c^2\left(bc-deq^2\right)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\cdots}{-\frac{\left(bc-deq^{1+2k}\right)\left(deq^k\left(b^2c+\cdots\right.\right.}{\cdots}}\right.\right.}{\cdots}$$

Undefined for $(a,b,c,e,\text{Undefined},q) \in \mathbb{C}^6 \wedge 0<|q|<1$

$$\frac{{}_5\phi_7\left(a,b,c,-\sqrt{c}q,\sqrt{c}q;0,0,0,-\sqrt{c},\sqrt{c},\frac{cq}{a},\frac{cq}{b};q,\frac{c^2q^2}{ab}\right)}{{}_5\phi_7\left(a,b,cq,-\sqrt{c}q^{3/2},\sqrt{c}q^{3/2};0,0,0,-\sqrt{c}\sqrt{q},\sqrt{c}\sqrt{q},\frac{cq^2}{a},\frac{cq^2}{b};q,\frac{c^2q^4}{ab}\right)} = \frac{(cq;q)_\infty\left(\frac{cq^2}{a};q\right)_\infty\left(\frac{cq^2}{b};q\right)_\infty\left(\operatorname{K}\cdots\right.}{(cq^2;q)_\infty \cdots}$$

$$\frac{{}_1\phi_2\left(a;d,e;q,\frac{de}{a}\right)}{{}_1\phi_2\left(aq;dq,eq;q,\frac{deq}{a}\right)}=\frac{(dq;q)_\infty(eq;q)_\infty\left(\frac{ade(aq-1)}{-a\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{deq^k\left(-1+aq^{1+k}\right)}{a}}{1-dq^{1+k}-eq^{1+k}}\right)+adq+aeq-a}+a(-d)-ae+a\right)}{a(d;q)_\infty(e;q)_\infty}\text{ fo}$$

$$\frac{{}_2\phi_2\left(a,b;d,e;q,\frac{de}{ab}\right)}{{}_2\phi_2\left(aq,b;dq,eq;q,\frac{deq}{ab}\right)}=\frac{(dq;q)_\infty(eq;q)_\infty\left(\frac{abcde(aq-1)(b-dq)(b-eq)}{(bc-deq^2)\left(-a\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{deq^k\left(-1+aq^{1+k}\right)\left(-b+dq^{1+k}\right)\left(-b+eq^{1+k}\right)}{a}}{b-\frac{(de+ab(d+e))q^{1+k}}{a}+deq^{2+2k}(1+q)}+a(b(dq\cdots\right.}\right.}{ab(d;q)_\infty(e;q)_\infty}$$

$$\frac{{}_2\phi_2\left(a,b;c,\frac{abz}{c};q,z\right)}{{}_2\phi_2\left(a,bq;cq,\frac{abqz}{c};q,qz\right)}=-\frac{(cq;q)_\infty\left(\frac{abqz}{c};q\right)_\infty\left(-c\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{q^{-1+k}\left(-1+bq^k\right)\left(-a+cq^k\right)z\left(c-bq^kz\right)}{c}}{1+bq^{2k}(1+q)z-\frac{q^k\left(c^2+abz+cz\right)}{c}}\right)+abz-c(\cdots\right.}{c(c;q)_\infty\left(\frac{abz}{c};q\right)_\infty}$$

$$\frac{{}_8\phi_7\left(z,q\sqrt{z},-q\sqrt{z},\mathrm{Symbol}(\mathrm{a}_1),\mathrm{Symbol}(\mathrm{a}_2),\mathrm{Symbol}(\mathrm{a}_3),\mathrm{Symbol}(\mathrm{a}_4),\mathrm{Symbol}(\mathrm{a}_5);\sqrt{z},-\sqrt{z},\frac{qz}{\mathrm{Symbol}(\mathrm{a}\cdots}\right.}{{}_8\phi_7\left(qz,q\sqrt{qz},-q\sqrt{qz},\mathrm{Symbol}(\mathrm{a}_1),\mathrm{Symbol}(\mathrm{a}_2),\mathrm{Symbol}(\mathrm{a}_3),\mathrm{Symbol}(\mathrm{a}_4),\mathrm{Symbol}(\mathrm{a}_5);\sqrt{qz},-\sqrt{qz},\frac{q\cdots}{\mathrm{Symb}\cdots}\right.}$$

$$\frac{\mathrm{QHypergeometricPFQ}(\{\},\{0\},q,z)}{\mathrm{QHypergeometricPFQ}(\{\},\{0\},q,qz)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{q^{-1+k}z}{1}+1\text{ for }(q,z)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{\mathrm{QHypergeometricPFQ}(\{\},\{0\},q,qz)}{\mathrm{QHypergeometricPFQ}(\{\},\{0\},q,z)}=\frac{q\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{q^{-2+k}z}{1}}{z}\text{ for }(q,z)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{\left(\frac{2e}{\sqrt{b^2+4e}+b}+b\right){}_1\phi_1\left(-\frac{d}{e};0;q,-\frac{2eq}{b^2+\sqrt{b^2+4e}b+2e}\right)}{{}_1\phi_1\left(-\frac{dq}{e};0;q,-\frac{2eq}{b^2+\sqrt{b^2+4e}b+2e}\right)}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e+dq^k}{b}\text{ for }(b,d,e,q)\in\mathbb{C}^4\wedge 0<|q|<1$$

$$\frac{dq\text{QHypergeometricPFQ}\left(\{\},\{0\},q,\frac{dq^3}{b^2}\right)}{b\text{QHypergeometricPFQ}\left(\{\},\{0\},q,\frac{dq^2}{b^2}\right)} = \operatorname*{K}_{k=1}^{\infty} \frac{dq^k}{b} \text{ for } (b,d,q)\in\mathbb{C}^3\wedge 0<|q|<1$$

$$\frac{b\left(q;q^5\right)_\infty\left(q^4;q^5\right)_\infty}{\left(q^2;q^5\right)_\infty\left(q^3;q^5\right)_\infty} = \operatorname*{K}_{k=1}^{\infty} \frac{b^2q^{-1+k}}{b} \text{ for } (b,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{b\left(-q^{3/2};q^4\right)_\infty\left(-q^{5/2};q^4\right)_\infty}{\left(\sqrt{q}+1\right)\left(-\sqrt{q};q^4\right)_\infty\left(-q^{7/2};q^4\right)_\infty} - \frac{b}{\sqrt{q}+1} = \operatorname*{K}_{k=1}^{\infty} \frac{-\frac{b^2\sqrt{q}}{\left(1+\sqrt{q}\right)^2}+\frac{b^2q^k}{\left(1+\sqrt{q}\right)^2}}{b} \text{ for } (b,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{\left(a+\frac{2e}{\sqrt{b^2+4e}+b}+b\right){}_1\phi_1\left(-\frac{d}{e};-\frac{a\left(b+\sqrt{b^2+4e}\right)}{b^2+\sqrt{b^2+4e}b+2e};q,-\frac{2eq}{b^2+\sqrt{b^2+4e}b+2e}\right)}{{}_1\phi_1\left(-\frac{dq}{e};-\frac{a\left(b+\sqrt{b^2+4e}\right)q}{b^2+\sqrt{b^2+4e}b+2e};q,-\frac{2eq}{b^2+\sqrt{b^2+4e}b+2e}\right)} - a - b = \operatorname*{K}_{k=1}^{\infty} \frac{e+dq^k}{b+aq^k} \text{ for } (a,b,d,e,q)\in$$

$$\frac{(a+b)\text{QHypergeometricPFQ}\left(\{\},\left\{-\frac{a}{b}\right\},q,\frac{2dq}{b^2+\sqrt{b^2}b}\right)}{\text{QHypergeometricPFQ}\left(\{\},\left\{-\frac{aq}{b}\right\},q,\frac{2dq^2}{b^2+\sqrt{b^2}b}\right)} - a - b = \operatorname*{K}_{k=1}^{\infty} \frac{dq^k}{b+aq^k} \text{ for } (a,b,d,q)\in\mathbb{C}^4\wedge 0<|q|<1$$

$$\frac{dq\text{QHypergeometricPFQ}\left(\{\},\{0\},\frac{q}{p^2},\frac{dq^3}{a^2p^5}\right)}{ap\text{QHypergeometricPFQ}\left(\{\},\{0\},\frac{q}{p^2},\frac{dq^2}{a^2p^3}\right)} = \operatorname*{K}_{k=1}^{\infty} \frac{dq^k}{ap^k} \text{ for } (a,d,p,q)\in\mathbb{C}^4\wedge 0<|q|<1$$

$$\frac{aq\left(a^2+d\right)\left(-\frac{d}{a^2q};q^2\right)_\infty}{\left(a^2q+d\right)\left(-\frac{d}{a^2};q^2\right)_\infty} - a = \operatorname*{K}_{k=1}^{\infty} \frac{dq^k}{a+aq^k} \text{ for } (a,d,q)\in\mathbb{C}^3\wedge 0<|q|<1$$

$$\frac{a\left(q;q^4\right)_\infty\left(q^{3/2};q^4\right)_\infty\left(q^{7/2};q^4\right)_\infty}{\left(\sqrt{q};q^4\right)_\infty\left(q^{5/2};q^4\right)_\infty\left(q^3;q^4\right)_\infty} - a = \operatorname*{K}_{k=1}^{\infty} \frac{a^2q^{-\frac{1}{2}+k}}{a+aq^k} \text{ for } (a,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{a\left(\sqrt{q};q^2\right)_\infty}{\left(q^{3/2};q^2\right)_\infty} - a = \operatorname*{K}_{k=1}^{\infty} \frac{-a^2q^{-\frac{1}{2}+k}}{a+aq^k} \text{ for } (a,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{b\sqrt[4]{q}\left(\left(-\sqrt[4]{q};\sqrt{q}\right)_\infty+\left(\sqrt[4]{q};\sqrt{q}\right)_\infty\right)}{\left(-\sqrt[4]{q};\sqrt{q}\right)_\infty-\left(\sqrt[4]{q};\sqrt{q}\right)_\infty} + b\sqrt{q} - b = \operatorname*{K}_{k=1}^{\infty} \frac{b^2q^k}{b-bq^{\frac{1}{2}+k}} \text{ for } (b,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{b\left(q^{3/2};q^4\right)_\infty \left(q^{5/2};q^4\right)_\infty}{\left(\sqrt{q};q^4\right)_\infty \left(q^{7/2};q^4\right)_\infty}+b\left(-\sqrt{q}\right)-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{b^2q^k}{b+bq^{\frac{1}{2}+k}} \text{ for } (b,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{b\left(q^{3/2};q^4\right)_\infty \left(q^{5/2};q^4\right)_\infty}{\left(\sqrt{q};q^4\right)_\infty \left(q^{7/2};q^4\right)_\infty}+b\left(-\sqrt{q}\right)-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{b^2q^k}{b+bq^{\frac{1}{2}+k}} \text{ for } (b,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{\left(-q^2;q^2\right)_\infty}{\left(-q;q^2\right)_\infty}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}\left(1-(-1)^k\right)q^k+\frac{1}{2}\left(1+(-1)^k\right)\left(q^{k/2}+q^k\right)}{1}+1} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(q^2;q^3\right)_\infty}{\left(q;q^3\right)_\infty}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-q^{-1+2k}}{1+q^k}+1} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(q^3;q^4\right)_\infty}{\left(q;q^4\right)_\infty}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{-q^{-1+2k}}{1+q^{2k}}+1} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(-\frac{b}{q};q^2\right)_\infty}{\left(-b;q^2\right)_\infty}=\frac{(b+q)\left(\mathrm{K}_{k=1}^{\infty}\frac{bq^k}{1+q^k}+1\right)}{(b+1)q} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(-\frac{b}{q^3};q^4\right)_\infty}{\left(-\frac{b}{q};q^4\right)_\infty}=\frac{\left(b+q^3\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{bq^{-1+2k}}{1+q^{2k}}+1\right)}{q^2(b+q)} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(q;q^2\right)_\infty}{\left(\left(q^3;q^6\right)_\infty\right)^3}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{q^k+q^{2k}}{1}+1} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\sqrt[5]{q}\left(q;q^5\right)_\infty\left(q^4;q^5\right)_\infty}{\left(q^2;q^5\right)_\infty\left(q^3;q^5\right)_\infty}=\sqrt[5]{q}\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{q^{-1+k}}{1} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(q;q^8\right)_\infty\left(q^7;q^8\right)_\infty}{\left(q^3;q^8\right)_\infty\left(q^5;q^8\right)_\infty}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{1}{2}\left(1+(-1)^k\right)q^{2k}+\frac{1}{2}\left(1-(-1)^k\right)\left(q^k+q^{2k}\right)}{1}+1} \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(a^2q^3;q^4\right)_\infty\left(b^2q^3;q^4\right)_\infty}{\left(a^2q;q^4\right)_\infty\left(b^2q;q^4\right)_\infty}=\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{\left(b-aq^{-1+2k}\right)\left(a-bq^{-1+2k}\right)}{(1-ab)\left(1+q^{2k}\right)}-ab+1} \text{ for } (a,b,q)\in\mathbb{C}^3\wedge 0<|q|<1$$

$$\frac{\left(q^2;q^8\right)_\infty\left(q^3;q^8\right)_\infty\left(q^7;q^8\right)_\infty}{\left(q;q^8\right)_\infty\left(q^5;q^8\right)_\infty\left(q^6;q^8\right)_\infty}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{q^{-1+2k}}{1+q^{2k}}+1 \text{ for } q\in\mathbb{C}\wedge|q|<1$$

$$\frac{\left(q^3;q^8\right)_\infty \left(q^5;q^8\right)_\infty}{\left(q;q^8\right)_\infty \left(q^7;q^8\right)_\infty} = \operatorname*{K}_{k=1}^{\infty} \frac{q^{2k}}{1+q^{1+2k}} + q + 1 \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{(-a;q)_\infty(b;q)_\infty - (a;q)_\infty(-b;q)_\infty}{(a;q)_\infty(-b;q)_\infty + (-a;q)_\infty(b;q)_\infty} = \frac{a-b}{\operatorname{K}_{k=1}^{\infty} \frac{q^{-1+k}(-b+aq^k)(a-bq^k)}{1-q^{1+2k}} - q + 1} \text{ for } (a,b,q) \in \mathbb{C}^3 \wedge |q| < 1$$

$$\frac{(a;q)_\infty(b;q)_\infty}{(aq;q)_\infty(bq;q)_\infty} = \operatorname*{K}_{k=1}^{\infty} \frac{-q^{-1+k}\left(-1+aq^k\right)\left(-1+bq^k\right)\left(-c+abq^k\right)}{1-bq^k-cq^k+aq^k\left(-1+bq^k(1+q)\right)} + a(bq+b-1) - b - c + 1 \text{ for } (a,b,q) \in \mathbb{C}^3 \wedge$$

$$\frac{\left((-q;-q)_\infty - (q;-q)_\infty\right)\left(q^2;q^4\right)_\infty}{2(1-q)q\left(q^3;q^2\right)_\infty} = \frac{1-q}{\operatorname{K}_{k=1}^{\infty} \frac{q^{4k}(1-q^{4k})(1-q^{-1+4k})(1-q^{1+4k})}{1-q^{1+4k}(1+q+q^2)+q^{2+8k}(1+q^4)} + q^6 - q^3 - q + 1} \text{ for } q \in \mathbb{C} \wedge |q| <$$

$$(-q;q)_\infty \left(q^2;q^2\right)_\infty = \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{-\frac{1}{2}(1-(-1)^k)q^k + \frac{1}{2}(1+(-1)^k)q^{k/2}\left(1-q^{k/2}\right)}{1} + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\left(q^2;q^2\right)_\infty}{\left(q;q^2\right)_\infty} = \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{-\frac{1}{2}(1-(-1)^k)q^k + \frac{1}{2}(1+(-1)^k)q^{k/2}\left(1-q^{k/2}\right)}{1} + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\left(-q^3;q^4\right)_\infty}{\left(-q;q^4\right)_\infty} = \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1-(-1)^k)q^{-1+2k} + \frac{1}{2}(1+(-1)^k)q^k(1+q^{-1+k})}{1} + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\sqrt{q}\left(\left(q^4;q^4\right)_\infty\right)^2}{\left(\left(q^2;q^4\right)_\infty\right)^2} = \frac{\sqrt{q}}{\operatorname{K}_{k=1}^{\infty} \frac{q(1-q^{-1+2k})^2}{(1-q)(1+q^{2k})} - q + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\left((-q;q)_\infty\right)^2 - \left((q;q)_\infty\right)^2}{\left((-q;q)_\infty\right)^2 + \left((q;q)_\infty\right)^2} = \frac{2q}{\operatorname{K}_{k=1}^{\infty} \frac{q^{1+k}(1+q^k)^2}{1-q^{1+2k}} - q + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\left(-q;q^2\right)_\infty - \left(q;q^2\right)_\infty}{\left(-q;q^2\right)_\infty + \left(q;q^2\right)_\infty} = \frac{q}{\operatorname{K}_{k=1}^{\infty} \frac{q^{4k}}{1-q^{2+4k}} - q^2 + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\sqrt[3]{q}\left(q;q^6\right)_\infty \left(q^5;q^6\right)_\infty}{\left(\left(q^3;q^6\right)_\infty\right)^2} = \frac{\sqrt[3]{q}}{\operatorname{K}_{k=1}^{\infty} \frac{q^k+q^{2k}}{1} + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\left(q^8;q^{20}\right)_\infty \left(q^{12};q^{20}\right)_\infty}{\left(q^4;q^{20}\right)_\infty \left(q^{16};q^{20}\right)_\infty} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-q}{1+q+q^{1+2k}} + q + 1 \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\left(-q^3;q^8\right)_\infty \left(-q^5;q^8\right)_\infty}{\left(-q;q^8\right)_\infty \left(-q^7;q^8\right)_\infty} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-q+q^{2k}}{1+q} + 1 \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\sqrt{q}\left(q;q^8\right)_\infty \left(q^7;q^8\right)_\infty}{\left(q^3;q^8\right)_\infty \left(q^5;q^8\right)_\infty} = \frac{\sqrt{q}}{\mathrm{K}_{k=1}^{\infty} \frac{q^{2k}}{1+q^{1+2k}} + q + 1} \text{ for } q \in \mathbb{C} \wedge |q| < 1$$

$$\frac{\sqrt{q}\left(q;q^8\right)_\infty \left(q^7;q^8\right)_\infty}{\left(q^3;q^8\right)_\infty \left(q^5;q^8\right)_\infty} = \frac{\sqrt{q}}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)q^{2k}+\frac{1}{2}(1-(-1)^k)(q^k+q^{2k})}{1} + 1} \text{ for } q \in \mathbb{C}\wedge|q| < 1$$

$$\frac{\left(\sqrt{b^2+4e}+b+2\sqrt{cq}\right)\, {}_2\phi_1\left(\frac{d\left(b+\sqrt{b^2+4e}\right)\left(\sqrt{1-\frac{4ce}{d^2}}+1\right)q}{2\left(2e+b\left(b+\sqrt{b^2+4e}\right)\right)\sqrt{cq}}, \frac{2c}{d\left(\sqrt{1-\frac{4ce}{d^2}}-1\right)}; \frac{\left(b-\sqrt{b^2+4e}\right)\sqrt{cq}}{2e}; q, -\frac{4e\sqrt{cq}}{d\left(b+\sqrt{b^2+4e}\right)\left(\sqrt{1-\frac{4}{d}\cdots}\right.}\right.}{2\, {}_2\phi_1\left(\frac{d\left(b+\sqrt{b^2+4e}\right)\left(\sqrt{1-\frac{4ce}{d^2}}+1\right)q}{2\left(2e+b\left(b+\sqrt{b^2+4e}\right)\right)\sqrt{cq}}, \frac{2cq}{d\left(\sqrt{1-\frac{4ce}{d^2}}-1\right)}; \frac{\left(b-\sqrt{b^2+4e}\right)q\sqrt{cq}}{2e}; q, -\frac{4e\sqrt{cq}}{d\left(b+\sqrt{b^2+4e}\right)\left(\sqrt{1-\frac{4ce}{d^2}}+1\right)}\right)}$$

$$\frac{\left(b+\sqrt{cq}\right)\, {}_1\phi_1\left(\frac{d\sqrt{cq}}{bc}; -\frac{\sqrt{cq}}{b}; q, \frac{\sqrt{cq}}{b}\right)}{{}_1\phi_1\left(\frac{d\sqrt{cq}}{bc}; -\frac{q\sqrt{cq}}{b}; q, \frac{q\sqrt{cq}}{b}\right)} - b = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{dq^k+cq^{2k}}{b} \text{ for } (b,c,d,q) \in \mathbb{C}^4\wedge 0 < |q| < 1$$

$$\frac{\left(\frac{2e}{\sqrt{b^2+4e}+b}+b\right)\, {}_1\phi_1\left(-\frac{c}{e}; 0; q^2, -\frac{2eq^2}{b^2+\sqrt{b^2+4e}b+2e}\right)}{{}_1\phi_1\left(-\frac{cq^2}{e}; 0; q^2, -\frac{2eq^2}{b^2+\sqrt{b^2+4e}b+2e}\right)} - b = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{e+cq^{2k}}{b} \text{ for } (b,c,e,q) \in \mathbb{C}^4\wedge 0 < |q| < 1$$

$$\frac{cq^2\text{QHypergeometricPFQ}\left(\{\},\{0\},q^2,\frac{cq^6}{b^2}\right)}{b\text{QHypergeometricPFQ}\left(\{\},\{0\},q^2,\frac{cq^4}{b^2}\right)} = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{cq^{2k}}{b} \text{ for } (b,c,q) \in \mathbb{C}^3\wedge 0 < |q| < 1$$

$$\frac{b\left(\left(q^3;q^6\right)_\infty\right)^3}{\left(q;q^2\right)_\infty} - b = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{b^2q^k+b^2q^{2k}}{b} \text{ for } (b,q) \in \mathbb{C}^2 \wedge 0 < |q| < 1$$

$$\frac{b\left(\left(q^3;q^6\right)_\infty\right)^2}{\left(q;q^6\right)_\infty \left(q^5;q^6\right)_\infty} - b = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{b^2q^k+b^2q^{2k}}{b} \text{ for } (b,q) \in \mathbb{C}^2 \wedge 0 < |q| < 1$$

$$\frac{\left(a\left(b\left(\sqrt{b^2+4e}+b\right)+2e\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)+2cq\left(\sqrt{b^2+4e}+b\right)\right)\,{}_2\phi_1\left(\frac{d\left(b+\sqrt{b^2+4e}\right)\left(\sqrt{1-\frac{4ce}{d^2}}+1\right)q}{a\left(2e+b\left(b+\sqrt{b^2+4e}\right)\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}, \frac{\cdots}{d\left(\cdots\right.}\cdots\right.}{a\left(\sqrt{b^2+4e}+b\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)\,{}_2\phi_1\left(\frac{d\left(b+\sqrt{b^2+4e}\right)\left(\sqrt{1-\frac{4ce}{d^2}}+1\right)q}{a\left(2e+b\left(b+\sqrt{b^2+4e}\right)\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}, \frac{2cq}{d\left(\sqrt{1-\frac{4ce}{d^2}}-1\right)}; -\frac{\cdots}{a(2\cdots}\cdots\right.}$$

$$\frac{\left(ab\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)+2cq\right)\,{}_1\phi_1\left(\frac{2dq}{ab\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}; \frac{2cq}{a\left(b-b\sqrt{\frac{4cq}{a^2}+1}\right)}; q, \frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}\right)}{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)\,{}_1\phi_1\left(\frac{2dq}{ab\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}; \frac{2cq^2}{a\left(b-b\sqrt{\frac{4cq}{a^2}+1}\right)}; q, \frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}\right)}-a-b=\operatorname*{K}_{k=1}^{\infty}\frac{dq^k+cq^{2k}}{b+aq^k}\cdots$$

$$\frac{b\left(-\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b};q\right)_\infty\,{}_1\phi_1\left(\frac{2dq}{ab\left(\sqrt{\frac{a^2+4cq}{a^2}}-1\right)}; -\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b}; q, \frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}\right)}{\left(-\frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b};q\right)_\infty\,{}_1\phi_1\left(\frac{2dq}{ab\left(\sqrt{\frac{a^2+4cq}{a^2}}-1\right)}; -\frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b}; q, \frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}\right)}-a-b=\operatorname*{K}_{k=1}^{\infty}\frac{dq^k+cq^{2k}}{b+aq^k}\ \text{f}\cdots$$

$$-\frac{c+d}{-\frac{b\left(\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2bq};q\right)_\infty\left(-\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2bq};q\right)_\infty\,{}_1\phi_2\left(-\frac{c}{dq}; \frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2bq}, -\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2bq}; q, \frac{d}{b^2}\right)}{\left(\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b};q\right)_\infty\left(-\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b};q\right)_\infty\,{}_1\phi_2\left(-\frac{c}{d}; \frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}, -\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b}; q, \frac{dq}{b^2}\right)}+\frac{a}{q}+b}-a-b=\operatorname*{K}_{k=1}^{\infty}\frac{dq^{k}\cdots}{b\cdots}$$

$$\frac{b\left(\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b};q\right)_\infty\left(-\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b};q\right)_\infty}{{}_1\phi_1\left(\frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}; \frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}; q, -\frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}+1\right)}{2b}\right)\left(\frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b};q\right)_\infty}-a-b=\operatorname*{K}_{k=1}^{\infty}\frac{-cq^k+cq^{2k}}{b+aq^k}\ \text{for}\cdots$$

$$\frac{\left(a\left(b\sqrt{b^2+4e}+b^2+2e\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)+2cq\left(\sqrt{b^2+4e}+b\right)\right)\,{}_2\phi_1\left(\frac{2\sqrt{-ce}\left(b+\sqrt{b^2+4e}\right)q}{a\left(2e+b\left(b+\sqrt{b^2+4e}\right)\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}, \frac{c}{\sqrt{-c\cdots}}\cdots\right.}{a\left(\sqrt{b^2+4e}+b\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)\,{}_2\phi_1\left(\frac{2\sqrt{-ce}\left(b+\sqrt{b^2+4e}\right)q}{a\left(2e+b\left(b+\sqrt{b^2+4e}\right)\right)\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}, \frac{cq}{\sqrt{-ce}}; -\frac{2c\left(b+\cdots\right.}{a\left(2e+b\left(b+\sqrt{b^2\cdots}\right.\right.}\cdots\right.}$$

$$\frac{\left(ab\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)+2cq\right)\,{}_1\phi_1\left(0; \frac{2cq}{ab\left(1-\sqrt{\frac{4cq}{a^2}+1}\right)}; q, \frac{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}\right)}{a\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)\,{}_1\phi_1\left(0; \frac{2cq^2}{ab\left(1-\sqrt{\frac{4cq}{a^2}+1}\right)}; q, \frac{aq\left(\sqrt{\frac{4cq}{a^2}+1}-1\right)}{2b}\right)}-a-b=\operatorname*{K}_{k=1}^{\infty}\frac{cq^{2k}}{b+aq^k}\ \text{for } (a,b,c,q)\in\cdots$$

$$-\frac{a\left(\sqrt{q}u^2;q^2\right)_\infty\left(\sqrt{q}v^2;q^2\right)_\infty}{(uv-1)\left(q^{3/2}u^2;q^2\right)_\infty\left(q^{3/2}v^2;q^2\right)_\infty}-a=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c0}+\mathrm{c1}q^k+\mathrm{c0}q^{-1+2k}}{a+aq^k}\text{ for }\mathrm{c0}=\frac{a^2uv}{(uv-1)^2}\wedge\mathrm{c1}=-\frac{a^2\left(u^2\right.}{\sqrt{q}(uv}$$

$$\frac{a\left(\left(q;q^2\right)_\infty\right)^2}{\left(1-\sqrt{q}\right)\left(\left(q^2;q^2\right)_\infty\right)^2}-a=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c0}-2\mathrm{c0}q^{-\frac{1}{2}+k}+\mathrm{c0}q^{-1+2k}}{a+aq^k}\text{ for }\mathrm{c0}=\frac{a^2\sqrt{q}}{\left(\sqrt{q}-1\right)^2}\wedge(a,\mathrm{c0},q)\in\mathbb{C}^3\wedge 0<|q|<$$

$$\frac{a\left(q;q^3\right)_\infty}{\left(q^2;q^3\right)_\infty}-a=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-a^2q^{-1+2k}}{a+aq^k}\text{ for }(a,q)\in\mathbb{C}^2\wedge 0<|q|<1$$

$$\frac{c\left(\frac{v}{q+1};q\right)_\infty\left(\frac{a}{cv};q\right)_\infty{}_2\phi_1\left((q+1)uv,-\frac{v\left(au(q+1)^2+b(q+1)+cv\right)}{a(q+1)};\frac{v}{q+1};q,\frac{a}{cv}\right)}{\left(\frac{qv}{q+1};q\right)_\infty\left(\frac{aq}{cv};q\right)_\infty{}_2\phi_1\left((q+1)uv,-\frac{v(au(q+1)^2+b(q+1)+cv)}{a(q+1)};\frac{qv}{q+1};q,\frac{aq}{cv}\right)}-a-b-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c1}q^k+\mathrm{c2}q^{2k}+\mathrm{c3}}{c+bq^k+}$$

$$\frac{c^2(q(1-u)+1)(a-cq(q+1)v)(a(qu+q+1)+q(q+1)u(b+cqv))}{q(q+1)u^2\left(a-c^2(q+1)\right)\left(-\frac{cq^2(v;q)_\infty\left(\frac{qu(-b-cqv)-a}{cq^2u};q\right)_\infty{}_2\phi_2\left(\frac{u}{q+1},-\frac{(q+1)v(a+qu(b+cqv))}{au};\frac{qu(-b-cqv)-a}{cq^2u},v;q,\frac{a}{cq^2u}\right)}{(qv;q)_\infty\left(\frac{qu(-b-cqv)-a}{cqu};q\right)_\infty{}_2\phi_2\left(\frac{qu}{q+1},-\frac{(q+1)v(a+qu(b+cqv))}{au};\frac{qu(-b-cqv)-a}{cqu},qv;q,\frac{a}{\frac{cuq^2}{q+1}+\frac{cuq}{q+1}}\right)}\right.}$$

$$\frac{c(v;q)_\infty\left(-\frac{a+(q+1)u(b+cv)}{c(q+1)u};q\right)_\infty{}_2\phi_2\left(-\frac{v(a+(q+1)u(b+cv))}{au},u;v,-\frac{a+(q+1)u(b+cv)}{c(q+1)u};q,\frac{a}{cu+cqu}\right)}{(qv;q)_\infty\left(-\frac{q(a+(q+1)u(b+cv))}{c(q+1)u};q\right)_\infty{}_2\phi_2\left(-\frac{v(a+(q+1)u(b+cv))}{au},qu;qv,-\frac{q(a+(q+1)u(b+cv))}{c(q+1)u};q,\frac{aq}{cu+cqu}\right)}-a-b-c=$$

$$\frac{c\left(\frac{v}{q};q\right)_\infty\left(-\frac{a+b+bq}{qc+c}-\frac{v}{q};q\right)_\infty}{(v;q)_\infty\left(-\frac{q(a+b+bq)}{c(q+1)}-v;q\right)_\infty{}_2\phi_2\left(-\frac{v(aq+(q+1)(bq+cv))}{aq^2},q;v,-\frac{q(a+b+bq)}{c(q+1)}-v;q,\frac{aq}{qc+c}\right)}-a-b-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c1}q^k+}{}$$

$$\frac{c(u;q)_\infty\left(-\frac{a+b+bq+cu+cqu}{c(q+1)};q\right)_\infty}{(qu;q)_\infty\left(\frac{aq}{c(q+1)u};q\right)_\infty{}_2\phi_1\left(-\frac{u(a+(q+1)(b+cu))}{a},u;qu;q,\frac{aq}{c(q+1)u}\right)}-a-b-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c1}q^k+\mathrm{c2}q^{2k}+\mathrm{c3}q^{3k}+\mathrm{c4}q}{c+bq^k+aq^{2k}}$$

$$\frac{c(u;q)_\infty\left(\frac{a}{c(q+1)u};q\right)_\infty}{(qu;q)_\infty\left(\frac{aq}{c(q+1)u};q\right)_\infty}-a-b-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c1}q^k+\mathrm{c2}q^{2k}+\mathrm{c3}q^{3k}+\mathrm{c4}q^{4k}}{c+bq^k+aq^{2k}}\text{ for }\mathrm{c1}=-\frac{c(a+(q+1)u(b+cu))}{q(q+1)u}\wedge\mathrm{c2}$$

$$\frac{c(b;q)_\infty\left(\frac{a}{bc(q+1)};q\right)_\infty}{(bq;q)_\infty\left(\frac{aq}{bc(q+1)};q\right)_\infty}-a-b-c=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\mathrm{c1}q^k+\mathrm{c2}q^{2k}+\mathrm{c3}q^{3k}+\mathrm{c4}q^{4k}}{c+bq^k+aq^{2k}}\text{ for }\mathrm{c1}=-\frac{c(a+b(q+1)(bc+b))}{bq(q+1)}\wedge\mathrm{c2}=$$

$$\frac{(b\sqrt{c}+c+d)\;{}_2F_1\left(\frac{d-\sqrt{d^2-4ce}}{2c},\frac{d+\sqrt{d^2-4ce}}{2c};\frac{\sqrt{c}b+c+d}{2c};\frac{1}{2}\right)}{\sqrt{c}\,{}_2F_1\left(\frac{d-\sqrt{d^2-4ce}}{2c}+1,\frac{d+\sqrt{d^2-4ce}}{2c}+1;\frac{\sqrt{c}b+3c+d}{2c};\frac{1}{2}\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{e+dk+ck^2}{b}\text{ for }(b,c,d,e)\in\mathbb{C}^4$$

$$\frac{b+\frac{c+d}{\sqrt{c}}}{{}_2F_1\left(1,\frac{c+d}{c};\frac{\sqrt{c}b+3c+d}{2c};\frac{1}{2}\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{dk+ck^2}{b}\text{ for }(b,c,d)\in\mathbb{C}^3$$

$$\frac{(b+\sqrt{c})\;{}_2F_1\left(\frac{e}{\sqrt{-ce}},\frac{\sqrt{-ce}}{c};\frac{1}{2}\left(\frac{b}{\sqrt{c}}+1\right);\frac{1}{2}\right)}{{}_2F_1\left(\frac{e}{\sqrt{-ce}}+1,\frac{c+\sqrt{-ce}}{c};\frac{1}{2}\left(\frac{b}{\sqrt{c}}+3\right);\frac{1}{2}\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{e+ck^2}{b}\text{ for }(b,c,e)\in\mathbb{C}^3$$

$$-\frac{2\sqrt{c}}{\psi^{(0)}\left(\frac{1}{4}\left(\frac{b}{\sqrt{c}}+1\right)\right)-\psi^{(0)}\left(\frac{1}{4}\left(\frac{b}{\sqrt{c}}+3\right)\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{ck^2}{b}\text{ for }(b,c)\in\mathbb{C}^2$$

$$\frac{\left(\frac{\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)(c+d)}{a}-a(c+d)+2bc\right)\;{}_2F_1\left(\frac{d-\sqrt{d^2-4ce}}{2c},\frac{d+\sqrt{d^2-4ce}}{2c};\frac{1}{2}\left(\frac{d}{c}+\frac{\sqrt{\frac{a^2}{a^2+4c}}(2bc-a(c+d))}{ac}+1\right);\frac{1}{2}\left(1\right.\right.}{2c\,{}_2F_1\left(\frac{d-\sqrt{d^2-4ce}}{2c}+1,\frac{d+\sqrt{d^2-4ce}}{2c}+1;\frac{1}{2}\left(\frac{d}{c}+\frac{\sqrt{\frac{a^2}{a^2+4c}}(2bc-a(c+d))}{ac}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}$$

$$\frac{aU\left(\frac{2\left(\sqrt{ea^2+d^2}-d\right)}{a^2},\frac{4\sqrt{ea^2+d^2}}{a^2}+1,\frac{2ab+4d}{a^2}-1\right)}{2U\left(\frac{2\left(\sqrt{ea^2+d^2}-d\right)}{a^2}+1,\frac{4\sqrt{ea^2+d^2}}{a^2}+1,\frac{2ab+4d}{a^2}-1\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{e+dk-\frac{a^2k^2}{4}}{b+ak}\text{ for }(a,b,d,e)\in\mathbb{C}^4$$

$$\frac{\frac{\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)(c+d)}{a}-a(c+d)+2bc}{2c\,{}_2F_1\left(\frac{d-\sqrt{d^2}}{2c}+1,\frac{d+\sqrt{d^2}}{2c}+1;\frac{1}{2}\left(\frac{d}{c}+\frac{\sqrt{\frac{a^2}{a^2+4c}}(2bc-a(c+d))}{ac}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{dk+ck^2}{b+ak}\text{ for}$$

$$\frac{ae^{1-\frac{2(ab+2d)}{a^2}}}{2E_{1-\frac{4d}{a^2}}\left(\frac{2ab+4d}{a^2}-1\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{dk-\frac{a^2k^2}{4}}{b+ak}\text{ for }(a,b,d)\in\mathbb{C}^3$$

$$\frac{\left(\frac{c\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)}{a}-ac+2bc\right)\;{}_2F_1\left(-\frac{\sqrt{-ce}}{c},\frac{\sqrt{-ce}}{c};\frac{1}{2}\left(\frac{\sqrt{\frac{a^2}{a^2+4c}}(2bc-ac)}{ac}+1\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}{2c\,{}_2F_1\left(1-\frac{\sqrt{-ce}}{c},\frac{\sqrt{-ce}}{c}+1;\frac{1}{2}\left(\frac{\sqrt{\frac{a^2}{a^2+4c}}(2bc-ac)}{ac}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{e}{b}$$

$$-\frac{ae\left(K_{\frac{2e}{\sqrt{a^2e}}-\frac{1}{2}}\left(\frac{b}{a}-\frac{1}{2}\right)+K_{\frac{2e}{\sqrt{a^2e}}+\frac{1}{2}}\left(\frac{b}{a}-\frac{1}{2}\right)\right)}{\sqrt{a^2e}\left(K_{\frac{2e}{\sqrt{a^2e}}-\frac{1}{2}}\left(\frac{b}{a}-\frac{1}{2}\right)-K_{\frac{2e}{\sqrt{a^2e}}+\frac{1}{2}}\left(\frac{b}{a}-\frac{1}{2}\right)\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e-\frac{a^2k^2}{4}}{b+ak} \text{ for } (a,b,e)\in\mathbb{C}^3$$

$$\frac{\frac{c\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)}{a}-ac+2bc}{2c\,{}_2F_1\left(1,1;\frac{1}{2}\left(\frac{\sqrt{\frac{a^2}{a^2+4c}}(2bc-ac)}{ac}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{ck^2}{b+ak} \text{ for } (a,b,c)\in\mathbb{C}^3$$

$$-\frac{ae^{1-\frac{2b}{a}}}{2\left(\mathrm{Chi}\left(\frac{2b}{a}-1\right)+\mathrm{Shi}\left(1-\frac{2b}{a}\right)\right)}-b=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\frac{1}{4}a^2k^2}{b+ak} \text{ for } (a,b)\in\mathbb{C}^2$$

$$b\left(\frac{\sqrt{\frac{c}{b^2}}}{\tan^{-1}\left(\sqrt{\frac{c}{b^2}}\right)}-1\right)=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{ck^2}{b+2bk} \text{ for } (b,c)\in\mathbb{C}^2$$

$$\frac{\left(\frac{\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)(c+d)}{a}-a(c+d)\right){}_2F_1\left(\frac{d-\sqrt{d^2-4ce}}{2c},\frac{d+\sqrt{d^2-4ce}}{2c};\frac{1}{2}\left(\frac{d}{c}-\frac{\sqrt{\frac{a^2}{a^2+4c}}(c+d)}{c}+1\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right.}{2c\,{}_2F_1\left(\frac{d-\sqrt{d^2-4ce}}{2c}+1,\frac{d+\sqrt{d^2-4ce}}{2c}+1;\frac{1}{2}\left(\frac{d}{c}-\frac{\sqrt{\frac{a^2}{a^2+4c}}(c+d)}{c}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}$$

$$\frac{aU\left(\frac{2\left(\sqrt{ea^2+d^2}-d\right)}{a^2},\frac{4\sqrt{ea^2+d^2}}{a^2}+1,\frac{4d}{a^2}-1\right)}{2U\left(\frac{2\left(\sqrt{ea^2+d^2}-d\right)}{a^2}+1,\frac{4\sqrt{ea^2+d^2}}{a^2}+1,\frac{4d}{a^2}-1\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e+dk-\frac{a^2k^2}{4}}{ak} \text{ for } (a,d,e)\in\mathbb{C}^3$$

$$\frac{\frac{\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)(c+d)}{a}-a(c+d)}{2c\,{}_2F_1\left(\frac{d-\sqrt{d^2}}{2c}+1,\frac{d+\sqrt{d^2}}{2c}+1;\frac{1}{2}\left(\frac{d}{c}-\frac{\sqrt{\frac{a^2}{a^2+4c}}(c+d)}{c}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk+ck^2}{ak} \text{ for } (a,c,d)\in$$

$$\frac{ae^{1-\frac{4d}{a^2}}}{2E_{1-\frac{4d}{a^2}}\left(\frac{4d}{a^2}-1\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{dk-\frac{a^2k^2}{4}}{ak} \text{ for } (a,d)\in\mathbb{C}^2$$

$$\frac{\left(\frac{c\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)}{a}-ac\right){}_2F_1\left(-\frac{\sqrt{-ce}}{c},\frac{\sqrt{-ce}}{c};\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}{2c\,{}_2F_1\left(1-\frac{\sqrt{-ce}}{c},\frac{\sqrt{-ce}}{c}+1;\frac{1}{2}\left(3-\sqrt{\frac{a^2}{a^2+4c}}\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}=\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{e+ck^2}{ak} \text{ for } (a,c,e)$$

$$-\frac{ae\left(K_{\frac{2e}{\sqrt{a^2e}}-\frac{1}{2}}\left(-\frac{1}{2}\right)+K_{\frac{2e}{\sqrt{a^2e}}+\frac{1}{2}}\left(-\frac{1}{2}\right)\right)}{\sqrt{a^2e}\left(K_{\frac{2e}{\sqrt{a^2e}}-\frac{1}{2}}\left(-\frac{1}{2}\right)-K_{\frac{2e}{\sqrt{a^2e}}+\frac{1}{2}}\left(-\frac{1}{2}\right)\right)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{e-\frac{a^2k^2}{4}}{ak} \text{ for } (a,e)\in\mathbb{C}^2$$

$$\frac{\frac{c\sqrt{\frac{a^2}{a^2+4c}}\left(a^2+4c\right)}{a}-ac}{2c\,{}_2F_1\left(1,1;\frac{1}{2}\left(3-\sqrt{\frac{a^2}{a^2+4c}}\right);\frac{1}{2}\left(1-\sqrt{\frac{a^2}{a^2+4c}}\right)\right)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{ck^2}{ak} \text{ for } (a,c)\in\mathbb{C}^2$$

$$-\frac{ea}{2(\mathrm{Chi}(-1)+\mathrm{Shi}(1))}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{1}{4}a^2k^2}{ak} \text{ for } a\in\mathbb{C}$$

$$\mathrm{RamanujanTauTheta}(z)=\frac{z\left(\frac{137}{60}-\gamma-\log(2\pi)\right)}{\mathrm{K}_{k=1}^{\infty}\frac{\frac{(-1)^k z^2\psi^{(2k)}(6)}{(1+2k)!\left(\delta_{1-k}\log(2\pi)+\frac{(-1)^k\psi^{(2(-1+k))}(6)}{(-1+2k)!}\right)}}{1-\frac{(-1)^k z^2\psi^{(2k)}(6)}{(1+2k)!\left(\delta_{1-k}\log(2\pi)+\frac{(-1)^k\psi^{(2(-1+k))}(6)}{(-1+2k)!}\right)}}+1} \text{ for } z\in\mathbb{C}\wedge|z|<1$$

$$\frac{a+z+1}{z+1}=\frac{a+z}{\mathrm{K}_{k=1}^{\infty}\frac{a(1+k)+z}{-1+ak+z}+z-1} \text{ for } (a,z)\in\mathbb{C}^2$$

$$\frac{z^2+z+1}{z^2-z+1}=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{k+z}{-3+k+z}+z-3} \text{ for } z\in\mathbb{C}$$

$$\frac{z^3+2z+1}{(z-1)^3+2(z-1)+1}=\frac{z}{\mathrm{K}_{k=1}^{\infty}\frac{k+z}{-4+k+z}+z-4} \text{ for } z\in\mathbb{C}$$

$$\frac{\left(b^2uv+d\right)\,{}_1F_1\left(\frac{e}{d};\frac{d}{b^2u}+v;\frac{d}{b^2u}\right)}{buv\,{}_1F_1\left(\frac{d+e}{d};\frac{d}{b^2u}+v+1;\frac{d}{b^2u}\right)}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{e+dk}{u(-1+k+v)(k+v)}}{b} \text{ for } (b,d,e,u,v)\in\mathbb{C}^5$$

$$\frac{be^{-\frac{d}{b^2u}}\left(\frac{d}{b^2u}\right)^{\frac{d}{b^2u}+v}}{v\Gamma\left(\frac{d}{b^2u}+v,0,\frac{d}{b^2u}\right)}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{dk}{u(-1+k+v)(k+v)}}{b} \text{ for } (b,d,u,v)\in\mathbb{C}^4$$

$$\frac{\sqrt{e}I_{v-1}\left(\frac{2\sqrt{e}}{b\sqrt{u}}\right)}{\sqrt{u}vI_v\left(\frac{2\sqrt{e}}{b\sqrt{u}}\right)}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{e}{u(-1+k+v)(k+v)}}{b} \text{ for } (b,e,u,v)\in\mathbb{C}^4$$

$$\frac{be\,{}_1F_1\left(\frac{e}{d};\frac{d}{b^2}+1;\frac{d}{b^2}\right)}{d\,{}_1F_1\left(\frac{e}{d}-1;\frac{d}{b^2};\frac{d}{b^2}\right)}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{e+dk}{k(1+k)}}{b} \text{ for } (b,d,e)\in\mathbb{C}^3$$

$$\frac{\sqrt{e}I_0\left(\frac{2\sqrt{e}}{b}\right)}{I_1\left(\frac{2\sqrt{e}}{b}\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{e}{k(1+k)}}{b}\text{ for }(b,e)\in\mathbb{C}^2$$

$$b\left(\frac{d}{b^2}\right)^{1-\frac{d}{b^2}}e^{\frac{d}{b^2}}\Gamma\left(\frac{d}{b^2},0,\frac{d}{b^2}\right)-b=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{d}{k}}{b}\text{ for }(b,d)\in\mathbb{C}^2$$

$$\frac{b\left(-d\sqrt{\frac{b^2u}{b^2u+4c}}+2cv\sqrt{\frac{b^2u}{b^2u+4c}}-c\sqrt{\frac{b^2u}{b^2u+4c}}+c+d\right)\,{}_2F_1\left(\frac{d-b^2\sqrt{\frac{d^2-4ce}{b^4u^2}}u}{2c},\frac{\sqrt{\frac{d^2-4ce}{b^4u^2}}ub^2+d}{2c};\frac{1}{2}\left(\frac{d}{c}+\frac{\sqrt{\frac{b^2u}{ub^2+4c}}(c(2v}{c}\right.\right.}{2cv\sqrt{\frac{b^2u}{b^2u+4c}}\,{}_2F_1\left(\frac{-\sqrt{\frac{d^2-4ce}{b^4u^2}}ub^2+2c+d}{2c},\frac{\sqrt{\frac{d^2-4ce}{b^4u^2}}ub^2+2c+d}{2c};\frac{1}{2}\left(\frac{d}{c}+\frac{\sqrt{\frac{b^2u}{ub^2+4c}}(c(2v-1)-d)}{c}+3\right);\frac{1}{2}\right.}$$

$$\frac{bU\left(2\left(\sqrt{\frac{d^2}{b^4u^2}+\frac{e}{b^2u}}-\frac{d}{b^2u}\right),4\sqrt{\frac{d^2}{b^4u^2}+\frac{e}{b^2u}}+1,\frac{4d}{b^2u}+2v-1\right)}{2vU\left(2\left(\sqrt{\frac{d^2}{b^4u^2}+\frac{e}{b^2u}}-\frac{d}{b^2u}\right)+1,4\sqrt{\frac{d^2}{b^4u^2}+\frac{e}{b^2u}}+1,\frac{4d}{b^2u}+2v-1\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{e+dk-\frac{1}{4}b^2k^2u}{u(-1+k+v)(k+v)}}{b}\text{ for }(b,d,e,u$$

$$\frac{b\left(\frac{-d\sqrt{\frac{b^2u}{b^2u+4c}}+2cv\sqrt{\frac{b^2u}{b^2u+4c}}-c\sqrt{\frac{b^2u}{b^2u+4c}}+c+d}{2c\sqrt{\frac{b^2u}{b^2u+4c}}\,{}_2F_1\left(\frac{-\sqrt{\frac{d^2}{b^4u^2}}ub^2+2c+d}{2c},\frac{\sqrt{\frac{d^2}{b^4u^2}}ub^2+2c+d}{2c};\frac{1}{2}\left(\frac{d}{c}+\frac{\sqrt{\frac{b^2u}{ub^2+4c}}(c(2v-1)-d)}{c}+3\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2u}{ub^2+4c}}\right)\right)}-v\right)}{v}=\operatorname*{K}_{k=1}^{\infty}\frac{}{u(-}$$

$$\frac{be^{-\frac{4d}{b^2u}-2v+1}}{2vE_{1-\frac{4d}{b^2u}}\left(\frac{4d}{b^2u}+2v-1\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{dk-\frac{1}{4}b^2k^2u}{u(-1+k+v)(k+v)}}{b}\text{ for }(b,d,u,v)\in\mathbb{C}^4$$

$$\frac{b\left(2v\sqrt{\frac{b^2u}{b^2u+4c}}-\sqrt{\frac{b^2u}{b^2u+4c}}+1\right)\,{}_2F_1\left(-\frac{b^2\sqrt{-\frac{ce}{b^4u^2}}u}{c},\frac{b^2\sqrt{-\frac{ce}{b^4u^2}}u}{c};\frac{1}{2}\left(\sqrt{\frac{b^2u}{ub^2+4c}}(2v-1)+1\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2u}{ub^2+4c}}\right)\right.}{2v\sqrt{\frac{b^2u}{b^2u+4c}}\,{}_2F_1\left(1-\frac{b^2\sqrt{-\frac{ce}{b^4u^2}}u}{c},\frac{\sqrt{-\frac{ce}{b^4u^2}}ub^2}{c}+1;\frac{1}{2}\left(\sqrt{\frac{b^2u}{ub^2+4c}}(2v-1)+3\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2u}{ub^2+4c}}\right)\right)}$$

$$-\frac{b\sqrt{\frac{e}{b^2u}}\left(K_{-2\sqrt{\frac{e}{b^2u}}-\frac{1}{2}}\left(v-\frac{1}{2}\right)+K_{\frac{1}{2}-2\sqrt{\frac{e}{b^2u}}}\left(v-\frac{1}{2}\right)\right)}{v\left(K_{2\sqrt{\frac{e}{b^2u}}-\frac{1}{2}}\left(v-\frac{1}{2}\right)-K_{2\sqrt{\frac{e}{b^2u}}+\frac{1}{2}}\left(v-\frac{1}{2}\right)\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{e-\frac{1}{4}b^2k^2u}{u(-1+k+v)(k+v)}}{b}\text{ for }(b,e,u,v)\in\mathbb{C}^4$$

$$\frac{b\left(2v\sqrt{\frac{b^2u}{b^2u+4c}}-\sqrt{\frac{b^2u}{b^2u+4c}}+1\right)}{2v\sqrt{\frac{b^2u}{b^2u+4c}}\,{}_2F_1\left(1,1;\frac{1}{2}\left(\sqrt{\frac{b^2u}{ub^2+4c}}(2v-1)+3\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2u}{ub^2+4c}}\right)\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{\frac{ck^2}{u(-1+k+v)(k+v)}}{b}\text{ for }(b,c,u,v)$$

$$-\frac{be^{1-2v}}{2v(\operatorname{Chi}(2v-1)+\operatorname{Shi}(1-2v))}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{b^2k^2}{4(-1+k+v)(k+v)}}{b}\text{ for }(b,v)\in\mathbb{C}^2$$

$$-\frac{2bce\,{}_2F_1\left(\frac{d-\sqrt{\frac{d^2-4ce}{u^2}}u}{2c},\frac{d+\sqrt{\frac{d^2-4ce}{u^2}}u}{2c};\frac{\sqrt{\frac{b^2u}{ub^2+4c}}c+c+d-d\sqrt{\frac{b^2u}{ub^2+4c}}}{2c};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{b^2u}{ub^2+4c}}\right)}{(c-d)\left(b^2u\left(\sqrt{\frac{b^2u}{b^2u+4c}}-1\right)+4c\sqrt{\frac{b^2u}{b^2u+4c}}\right)\,{}_2F_1\left(\frac{-2c+d-\sqrt{\frac{d^2-4ce}{u^2}}u}{2c},\frac{-2c+d+\sqrt{\frac{d^2-4ce}{u^2}}u}{2c};\frac{(c-d)\left(\sqrt{\frac{b^2u}{ub^2+4c}}-1\right)}{2c};\frac{1}{2}\right.}$$

$$\frac{2eU\left(\frac{2\left(u\sqrt{\frac{eub^2+d^2}{u^2}}-d\right)}{b^2u},\frac{b^2+4\sqrt{\frac{eub^2+d^2}{u^2}}}{b^2},\frac{4d}{b^2u}+1\right)}{buU\left(-\frac{ub^2+2d-2u\sqrt{\frac{eub^2+d^2}{u^2}}}{b^2u},\frac{b^2+4\sqrt{\frac{eub^2+d^2}{u^2}}}{b^2},\frac{4d}{b^2u}+1\right)}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{e+dk-\frac{1}{4}b^2k^2u}{k(1+k)u}}{b}\text{ for }(b,d,e,u)\in\mathbb{C}^4$$

$$-\frac{2be\,{}_2F_1\left(\frac{e}{\sqrt{-\frac{ce}{u^2}}u},\frac{\sqrt{-\frac{ce}{u^2}}u}{c};\frac{1}{2}\left(\sqrt{\frac{b^2u}{ub^2+4c}}+1\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2u}{ub^2+4c}}\right)\right)}{\left(b^2u\left(\sqrt{\frac{b^2u}{b^2u+4c}}-1\right)+4c\sqrt{\frac{b^2u}{b^2u+4c}}\right)\,{}_2F_1\left(-\frac{c+\sqrt{-\frac{ce}{u^2}}u}{c},\frac{\sqrt{-\frac{ce}{u^2}}u}{c}-1;\frac{1}{2}\left(\sqrt{\frac{b^2u}{ub^2+4c}}-1\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2u}{ub^2+4c}}\right)\right)}$$

$$-\frac{b\left(\left(u\sqrt{\frac{b^2e}{u}}+e\right)K_{\frac{2\sqrt{\frac{b^2e}{u}}}{b^2}-\frac{1}{2}}\left(\frac{1}{2}\right)+\left(e-u\sqrt{\frac{b^2e}{u}}\right)K_{\frac{2\sqrt{\frac{b^2e}{u}}}{b^2}+\frac{1}{2}}\left(\frac{1}{2}\right)\right)}{u\sqrt{\frac{b^2e}{u}}\left(K_{\frac{2\sqrt{\frac{b^2e}{u}}}{b^2}-\frac{1}{2}}\left(\frac{1}{2}\right)-K_{\frac{2\sqrt{\frac{b^2e}{u}}}{b^2}+\frac{1}{2}}\left(\frac{1}{2}\right)\right)}=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{e-\frac{1}{4}b^2k^2u}{k(1+k)u}}{b}\text{ for }(b,e,u)\in\mathbb{C}^3$$

$$b\left(\frac{\sqrt{\frac{c}{b^2}}}{\tan^{-1}\left(\sqrt{\frac{c}{b^2}}\right)}-1\right)=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{ck^2}{-1+4k^2}}{b}\text{ for }(b,c)\in\mathbb{C}^2$$

$$\frac{2bc\,{}_2F_1\left(\frac{c+d-\sqrt{(c-d)^2}}{2c},\frac{c+d+\sqrt{(c-d)^2}}{2c};\frac{2c-\sqrt{\frac{b^2}{b^2+4c}}d+d}{2c};\frac{1}{2}-\frac{1}{2}\sqrt{\frac{b^2}{b^2+4c}}\right)}{b^2\left(\sqrt{\frac{b^2}{b^2+4c}}-1\right)+4c\sqrt{\frac{b^2}{b^2+4c}}}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{c+\frac{d}{k}}{b}\text{ for }(b,d,c)\in\mathbb{C}^3$$

$$\frac{2de^{\frac{4d}{b^2}}E_{-\frac{4d}{b^2}}\left(\frac{4d}{b^2}\right)}{b}-b=\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{b^2}{4}+\frac{d}{k}}{b}\text{ for }(b,d)\in\mathbb{C}^2$$

$$\frac{2c\sqrt{\frac{b^2}{b^2+4c}}\,{}_2F_1\left(1,1;\frac{1}{2}\left(3-\sqrt{\frac{b^2}{b^2+4c}}\right);\frac{1}{2}\left(1-\sqrt{\frac{b^2}{b^2+4c}}\right)\right)}{b\left(1-\sqrt{\frac{b^2}{b^2+4c}}\right)}-b=\operatorname*{K}_{k=1}^{\infty}\frac{c+\frac{c}{k}}{b}\text{ for }(b,c)\in\mathbb{C}^2$$

$$\vartheta(z)=-\frac{z^3\psi^{(2)}\left(\frac{1}{4}\right)}{48\left(\operatorname{K}_{k=1}^{\infty}\frac{\frac{z^2\psi^{(2(1+k))}\left(\frac{1}{4}\right)}{8\left(3+5k+2k^2\right)\psi^{(2k)}\left(\frac{1}{4}\right)}}{1-\frac{z^2\psi^{(2(1+k))}\left(\frac{1}{4}\right)}{8\left(3+5k+2k^2\right)\psi^{(2k)}\left(\frac{1}{4}\right)}}+1\right)}-\frac{1}{2}z\left(\log(\pi)-\psi^{(0)}\left(\frac{1}{4}\right)\right)\text{ for }z\in\mathbb{C}\wedge|z|<\frac{1}{2}$$

$$\sec(z)=\frac{z^2}{2\left(\operatorname{K}_{k=1}^{\infty}\frac{\frac{z^2}{2(1+k)(1+2k)}}{1-\frac{z^2}{2(1+k)(1+2k)}}-\frac{z^2}{2}+1\right)}+1\text{ for }z\in\mathbb{C}\wedge\frac{z}{\pi}-\frac{1}{2}\notin\mathbb{Z}$$

$$\sec(z)=\frac{1}{1+\operatorname{K}_{k=1}^{100}\frac{\frac{z^2(\operatorname{Li}_{-2k}(-i)-\operatorname{Li}_{-2k}(i))}{2k(-1+2k)(\operatorname{Li}_{2-2k}(-i)-\operatorname{Li}_{2-2k}(i))}}{1-\frac{z^2(\operatorname{Li}_{-2k}(-i)-\operatorname{Li}_{-2k}(i))}{2k(-1+2k)(\operatorname{Li}_{2-2k}(-i)-\operatorname{Li}_{2-2k}(i))}}}\text{ for }z\in\mathbb{C}\wedge\frac{z}{\pi}-\frac{1}{2}\notin\mathbb{Z}$$

$$\operatorname{sech}(z)=1-\frac{z^2}{2\left(\operatorname{K}_{k=1}^{\infty}\frac{-\frac{z^2}{2(1+k)(1+2k)}}{1+\frac{z^2}{2(1+k)(1+2k)}}+\frac{z^2}{2}+1\right)}\text{ for }z\in\mathbb{C}\wedge-\frac{1}{2}+\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\operatorname{sech}(z)=\frac{1}{1+\operatorname{K}_{k=1}^{\infty}\frac{-\frac{z^2(\operatorname{Li}_{-2k}(-i)-\operatorname{Li}_{-2k}(i))}{2k(-1+2k)(\operatorname{Li}_{2-2k}(-i)-\operatorname{Li}_{2-2k}(i))}}{1+\frac{z^2(\operatorname{Li}_{-2k}(-i)-\operatorname{Li}_{-2k}(i))}{2k(-1+2k)(\operatorname{Li}_{2-2k}(-i)-\operatorname{Li}_{2-2k}(i))}}}\text{ for }z\in\mathbb{C}\wedge-\frac{1}{2}+\frac{iz}{\pi}\notin\mathbb{Z}$$

$$\sin(z)=z-\frac{z^3}{6\left(\operatorname{K}_{k=1}^{\infty}\frac{\frac{z^2}{2(1+k)(3+2k)}}{1-\frac{z^2}{2(1+k)(3+2k)}}+1\right)}\text{ for }z\in\mathbb{C}$$

$$\sin(z)=\frac{z}{\operatorname{K}_{k=1}^{\infty}\frac{\frac{z^2}{2k(1+2k)}}{1-\frac{z^2}{2k(1+2k)}}+1}\text{ for }z\in\mathbb{C}$$

$$\sin(z)=z\left(1-\frac{z}{\pi\left(\operatorname{K}_{k=1}^{\infty}\frac{\frac{1-(-1)^k+k}{2+k}+\frac{\left(-1+3(-1)^k+2(-1)^k k\right)z}{(1+k)(2+k)\pi}}{\frac{1+(-1)^k}{2+k}+\frac{\left(1-3(-1)^k-2(-1)^k k\right)z}{(1+k)(2+k)\pi}}+1\right)}\right)\text{ for }z\in\mathbb{C}$$

$$\sin(z) = z\left(1 - \frac{z}{\pi\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{1}{8}(1+2k+2k^2-(-1)^k(1+2k))+\frac{(-1+(-1)^k+2(-1)^k k)z}{4\pi}}{\frac{1}{2}\left(1+(-1)^k-\frac{2(-1)^k z}{\pi}\right)} + 1\right)}\right) \text{ for } z \in \mathbb{C}$$

$$\sin(z) = \frac{z}{\frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{2k(1+2k)z^2}{2(1+k)(3+2k)-z^2} - z^2+6} + 1} \text{ for } z \in \mathbb{C}$$

$$\frac{\sin(\pi z)}{\pi z} = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{1}{2}(1+(-1)^k)\left\lfloor\frac{1+k}{2}\right\rfloor\left(-z+\left\lfloor\frac{1+k}{2}\right\rfloor\right)-\frac{1}{2}(1-(-1)^k)\left\lfloor\frac{1+k}{2}\right\rfloor\left(z+\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{\frac{1}{2}(1+(-1)^k)+z} + 1} + 1 \text{ for } z \in \mathbb{C}$$

$$\frac{\sin(\pi z)}{\pi z} = 1 - \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-k^2(k^2-z^2)}{1+2k(1+k)-z^2} + 1} \text{ for } z \in \mathbb{C}$$

$$\frac{\sin\left(\frac{\pi z}{2}\right)}{z} = \frac{1-z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-2k(1+2k)((1+2k)^2-z^2)}{(1+2k)^2+(2+2k)(3+2k)-z^2} + 6} + 1 \text{ for } z \in \mathbb{C}$$

$$\operatorname{sinc}(z) = \frac{1}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{2k(1+2k)}}{1-\frac{z^2}{2k(1+2k)}} + 1} \text{ for } z \in \mathbb{C}$$

$$\sin^m(z) = \frac{2^{1-m} z^m \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-1)^i (-2i+m)^m \binom{m}{i}}{m!\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2(2(-1+k)+m)! \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-1)^i(-2i+m)^{2k+m}\binom{m}{i}}{(2k+m)! \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-1)^i(-2i+m)^{2(-1+k)+m}\binom{m}{i}}}{1-\frac{z^2(2(-1+k)+m)! \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-1)^i(-2i+m)^{2k+m}\binom{m}{i}}{(2k+m)! \sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor} (-1)^i(-2i+m)^{2(-1+k)+m}\binom{m}{i}}} + 1\right)} \text{ for } m \in \mathbb{Z} \land z \in \mathbb{C} \land m > 0$$

$$\frac{\sinh(\pi z) - \sin(\pi z)}{\sin(\pi z) + \sinh(\pi z)} = \frac{2z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^4+4z^4}{1+2k} + 1} \text{ for } z \in \mathbb{C}$$

$$\sinh(z) = \frac{z^3}{6\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z^2}{2(1+k)(3+2k)}}{1+\frac{z^2}{2(1+k)(3+2k)}} + 1\right)} + z \text{ for } z \in \mathbb{C}$$

$$\sinh(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-\frac{z^2}{2k(1+2k)}}{1+\frac{z^2}{2k(1+2k)}} + 1} \text{ for } z \in \mathbb{C}$$

$$\sinh(z) = z\left(1-\frac{iz}{\pi\left(1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1-(-1)^k+k}{2+k}+\frac{i\left(-1+3(-1)^k+2(-1)^k k\right)z}{(1+k)(2+k)\pi}}{\frac{1+(-1)^k}{2+k}+\frac{i\left(1-3(-1)^k-2(-1)^k k\right)z}{(1+k)(2+k)\pi}}\right)}\right) \text{ for } z\in\mathbb{C}$$

$$\sinh(z) = z\left(1-\frac{iz}{\pi\left(1+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{1}{8}(1+2k+2k^2-(-1)^k(1+2k))+\frac{i\left(-1+(-1)^k+2(-1)^k k\right)z}{4\pi}}{\frac{1}{2}\left(1+(-1)^k-\frac{2i(-1)^k z}{\pi}\right)}\right)}\right) \text{ for } z\in\mathbb{C}$$

$$\sinh(z) = \frac{e^{-z}z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{\left(-1-(-1)^k+2(-1)^k(1+k)\right)z}{2k(1+k)}}{1}+1} \text{ for } z\in\mathbb{C}$$

$$\sinh(z) = \frac{z}{1-\frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-2k(1+2k)z^2}{2(1+k)(3+2k)+z^2}+z^2+6}} \text{ for } z\in\mathbb{C}$$

$$\frac{\sinh(\pi z)}{\pi z} = \frac{z^2}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-k^2(k^2+z^2)}{1+2k(1+k)+z^2}+1}+1 \text{ for } z\in\mathbb{C}$$

$$\frac{\sinh\left(\frac{\pi z}{2}\right)}{z} = \frac{z^2+1}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-2k(1+2k)((1+2k)^2+z^2)}{(1+2k)^2+(2+2k)(3+2k)+z^2}+6}+1 \text{ for } z\in\mathbb{C}$$

$$\sinh^m(z) = \frac{2^{1-m}z^m\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor}(-1)^i(-2i+m)^m\binom{m}{i}}{m!\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{z^2(2(-1+k)+m)!\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor}(-1)^i(-2i+m)^{2k+m}\binom{m}{i}}{(2k+m)!\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor}(-1)^i(-2i+m)^{2(-1+k)+m}\binom{m}{i}}}{1+\frac{z^2(2(-1+k)+m)!\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor}(-1)^i(-2i+m)^{2k+m}\binom{m}{i}}{(2k+m)!\sum_{i=0}^{\left\lfloor\frac{1}{2}(-1+m)\right\rfloor}(-1)^i(-2i+m)^{2(-1+k)+m}\binom{m}{i}}}+1\right)} \text{ for } m\in\mathbb{Z}\land z\in\mathbb{C}\land m>0$$

$$\mathrm{Shi}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{(1-2k)z^2}{2k(1+2k)^2}}{1-\frac{(1-2k)z^2}{2k(1+2k)^2}}+1} \text{ for } z\in\mathbb{C}$$

$$\mathrm{Si}(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-\frac{(1-2k)z^2}{2k(1+2k)^2}}{1+\frac{(1-2k)z^2}{2k(1+2k)^2}}+1} \text{ for } z\in\mathbb{C}$$

$$j_\nu(z) = \frac{\sqrt{\pi}2^{-\nu-1}z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}}+1\right)} \text{ for } (\nu,z)\in\mathbb{C}^2\land\neg\left(\nu+\frac{1}{2}\in\mathbb{Z}\land\nu+\frac{1}{2}\leq 0\right)$$

$$j_{\nu}(z) = \frac{i\sqrt{\pi}(-2)^{\nu} z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2k(-1+2k-2\nu)}}{1-\frac{z^2}{2k(-1+2k-2\nu)}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \nu + \frac{1}{2} \in \mathbb{Z} \wedge \nu \leq -\frac{1}{2}$$

$$\frac{j_{\nu}(z)}{j_{\nu-1}(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{-z^2}{1+2k+2\nu} + 2\nu + 1} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg \left(\nu + \frac{1}{2} \in \mathbb{Z} \wedge \nu + \frac{1}{2} \leq 0\right)$$

$$\frac{j_{\nu+1}(z)}{j_{\nu}(z)} = \frac{z}{(2\nu+3)\left(\mathrm{K}_{k=1}^{\infty} \frac{-\frac{z^2}{4\left(\frac{1}{2}+k+\nu\right)\left(\frac{3}{2}+k+\nu\right)}}{1} + 1\right)} \text{ for } (\nu, z) \in \mathbb{C}^2$$

$$\frac{j_{\nu+1}(z)}{j_{\nu}(z)} = \frac{z}{\mathrm{K}_{k=1}^{\infty} \frac{iz(2+2k+2\nu)}{3+k-2iz+2\nu} + 2\nu - iz + 3} \text{ for } (\nu, z) \in \mathbb{C}^2$$

$$\frac{j_{\nu+1}(z)}{j_{\nu}(z)} = -\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{-1}{\frac{1+2k+2\nu}{z}} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg \left(\nu + \frac{1}{2} \in \mathbb{Z} \wedge \nu + \frac{1}{2} \leq 0\right)$$

$$\frac{j_{\nu}\left(2i\sqrt{z}\right)}{j_{\nu-1}\left(2i\sqrt{z}\right)} = \frac{i\sqrt{z}}{\mathrm{K}_{k=1}^{\infty} \frac{z}{\frac{1}{2}+k+\nu} + \nu + \frac{1}{2}} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge \neg \left(\nu + \frac{1}{2} \in \mathbb{Z} \wedge \nu + \frac{1}{2} \leq 0\right)$$

$$y_{\nu}(z) = \sec(\pi\nu)\left(-\frac{\sqrt{\pi}2^{\nu} z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2k(-1+2k-2\nu)}}{1-\frac{z^2}{2k(-1+2k-2\nu)}} + 1\right)} - \frac{\sqrt{\pi}2^{-\nu-1}\sin(\pi\nu) z^{\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}} + 1\right)}\right) \text{ for}$$

$$y_{\nu}(z) = -\frac{2^{\nu}\left(\nu-\frac{1}{2}\right)! z^{-\nu-1}}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{-\frac{1}{2}+\nu} \frac{-\frac{z^2}{2k(1-2k+2\nu)}}{1+\frac{z^2}{2k(1-2k+2\nu)}} + 1\right)} + \frac{2^{-\nu} z^{\nu} \log\left(\frac{z}{2}\right)}{\sqrt{\pi}\left(\nu+\frac{1}{2}\right)!\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}} + 1\right)} - \frac{2^{-}}{\sqrt{\pi}\left(\nu+\frac{1}{2}\right)!\left(\mathrm{K}\right.}$$

$$h_{\nu}^{(1)}(z) = \frac{\sqrt{\pi}2^{-\nu-1}(1-i\tan(\pi\nu)) z^{\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}} + 1\right)} - \frac{i\sqrt{\pi}2^{\nu}\sec(\pi\nu) z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{z^2}{2k(-1+2k-2\nu)}}{1-\frac{z^2}{2k(-1+2k-2\nu)}} + 1\right)} \text{ for } (\nu, z) \in \mathbb{C}^2 \wedge -$$

$$h_\nu^{(1)}(z) = -\frac{i2^\nu\left(\nu-\frac{1}{2}\right)!z^{-\nu-1}}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{-\frac{1}{2}+\nu}\frac{-\frac{z^2}{2k(1-2k+2\nu)}}{1+\frac{z^2}{2k(1-2k+2\nu)}}+1\right)}+\frac{2^{-\nu-1}z^\nu\left(\pi+2i\log\left(\frac{z}{2}\right)\right)}{\sqrt{\pi}\left(\nu+\frac{1}{2}\right)!\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}}+1\right)}-\frac{i2}{\sqrt{\pi}\left(\nu+\frac{1}{2}\right)!\left(\mathrm{K}\right.}$$

$$h_\nu^{(2)}(z) = \frac{i\sqrt{\pi}2^\nu\sec(\pi\nu)z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{2k(-1+2k-2\nu)}}{1-\frac{z^2}{2k(-1+2k-2\nu)}}+1\right)}+\frac{\sqrt{\pi}2^{-\nu-1}(1+i\tan(\pi\nu))z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}}+1\right)} \text{ for } (\nu,z)\in\mathbb{C}^2\land-$$

$$h_\nu^{(2)}(z) = \frac{i2^\nu\left(\nu-\frac{1}{2}\right)!z^{-\nu-1}}{\sqrt{\pi}\left(\mathrm{K}_{k=1}^{-\frac{1}{2}+\nu}\frac{-\frac{z^2}{2k(1-2k+2\nu)}}{1+\frac{z^2}{2k(1-2k+2\nu)}}+1\right)}+\frac{2^{-\nu-1}z^\nu\left(\pi-2i\log\left(\frac{z}{2}\right)\right)}{\sqrt{\pi}\left(\nu+\frac{1}{2}\right)!\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{2k(1+2k+2\nu)}}{1-\frac{z^2}{2k(1+2k+2\nu)}}+1\right)}+\frac{i2^{-}}{\sqrt{\pi}\left(\nu+\frac{1}{2}\right)!\left(\mathrm{K}_{k}^{\infty}\right.}$$

$$\sqrt{z} = \mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-1+z}{2}+1 \text{ for } z\in\mathbb{C}\land|\arg(z)|<\pi$$

$$\sqrt{z} = \mathop{\mathrm{K}}_{k=1}^{\infty}\frac{z-z^2}{2z}+z \text{ for } z\in\mathbb{C}\land-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

$$\sqrt{z+1} = 2\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z}{4}}{1}+1 \text{ for } z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\sqrt{z+1} = 4\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{\frac{z}{16}}{\frac{1}{2}}+1 \text{ for } z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\sqrt{z+1} = \mathop{\mathrm{K}}_{k=1}^{\infty}\frac{z\left(-\frac{1}{2}(-1)^k+\left\lfloor\frac{k}{2}\right\rfloor\right)}{1+(-1)^k+\frac{1}{2}\left(1-(-1)^k\right)k}+1 \text{ for } z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\sqrt{x^2+y} = \frac{y}{\mathrm{K}_{k=1}^{\infty}\frac{y\left((-1)^k+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{\left(1-(-1)^k\right)x+\left(1+(-1)^k\right)(1+k)x}+2x}+x \text{ for } (x,y)\in\mathbb{C}^2\land\left|\arg\left(x^2+y\right)\right|<\pi$$

$$\sqrt{x^2+y} = \frac{y}{\mathrm{K}_{k=1}^{\infty}\frac{y}{2x}+2x}+x \text{ for } (x,y)\in\mathbb{C}^2\land\left|\arg\left(x^2+y\right)\right|<\pi$$

$$\frac{1}{\sqrt{z+1}+1} = \frac{2\,\mathrm{K}_{k=1}^{\infty}\frac{\frac{z}{4}}{1}}{z} \text{ for } z\in\mathbb{C}\land|\arg(z+1)|<\pi$$

$$\frac{2z(z+1)}{\sqrt{(2z+1)^2}+1} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{z(1+z)}{1} \text{ for } z \in \mathbb{C} \wedge |\arg(z+1)| < \pi$$

$$\frac{2z}{b\left(\sqrt{\frac{4z}{b^2}+1}+1\right)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{z}{b} \text{ for } (b,z) \in \mathbb{C}^2$$

$$\frac{2e}{z\left(\sqrt{\frac{4e}{z^2}+1}+1\right)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{e}{z} \text{ for } (e,z) \in \mathbb{C}^2$$

$$\frac{(\alpha\beta+a+b)\sqrt{1-\frac{4ab}{(\alpha\beta+a+b)^2}}-\alpha\beta+a-b}{2\alpha} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\left\{ \begin{array}{ll} a & (k \bmod 2) = 1 \\ b & (k \bmod 2) = 0 \end{array}\right.}{\left\{ \begin{array}{ll} \alpha & (k \bmod 2) = 1 \\ \beta & (k \bmod 2) = 0 \end{array}\right.} \text{ for } (a,b,\alpha,\beta) \in \mathbb{C}^4 \wedge \left|\arg\left(1-\right.\right.$$

$$\frac{(\alpha\beta+a+b)\sqrt{1-\frac{4ab}{(\alpha\beta+a+b)^2}}-\alpha\beta+a-b}{2\alpha} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{a(k \bmod 2)+b((1+k) \bmod 2)}{\alpha(k \bmod 2)+\beta((1+k) \bmod 2)} \text{ for } (a,b,\alpha,\beta) \in \mathbb{C}^4 \wedge \left|\arg\right.$$

$$\frac{(\alpha\beta+2a)\sqrt{1-\frac{4a^2}{(\alpha\beta+2a)^2}}-\alpha\beta}{2\alpha} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{a}{\alpha(k \bmod 2)+\beta((1+k) \bmod 2)} \text{ for } (a,\alpha,\beta) \in \mathbb{C}^3 \wedge \left|\arg\left(1-\frac{4a^2}{(2a+\alpha\right.\right.$$

$$\frac{-\alpha^2+\left(\alpha^2+a+b\right)\sqrt{1-\frac{4ab}{(\alpha^2+a+b)^2}}+a-b}{2\alpha} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{a(k \bmod 2)+b((1+k) \bmod 2)}{\alpha} \text{ for } (a,b,\alpha) \in \mathbb{C}^3 \wedge \left|\arg\right.$$

$$\frac{(\alpha\beta\gamma+a\beta+b\gamma+\alpha c)\sqrt{\frac{4abc}{(\alpha\beta\gamma+a\beta+b\gamma+\alpha c)^2}+1}-\alpha\beta\gamma+a\beta-b\gamma+\alpha(-c)}{2(\alpha\beta+b)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\left\{ \begin{array}{ll} a & (k \bmod 3) = 1 \\ b & (k \bmod 3) = 2 \\ c & (k \bmod 3) = 0 \end{array}\right.}{\left\{ \begin{array}{ll} \alpha & (k \bmod 3) = 1 \\ \beta & (k \bmod 3) = 2 \\ \gamma & (k \bmod 3) = 0 \end{array}\right.} \text{ f}$$

$$\frac{(\alpha\beta\gamma+a\beta+b\gamma+\alpha c)\sqrt{\frac{4abc}{(\alpha\beta\gamma+a\beta+b\gamma+\alpha c)^2}+1}-\alpha\beta\gamma+a\beta-b\gamma+\alpha(-c)}{2(\alpha\beta+b)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{1}{9}(a+4b-2c)(k \bmod 3)+}{\frac{1}{9}(\alpha+4\beta-2\gamma)(k \bmod 3)+}$$

$$-\frac{-(a(\alpha+\beta+\gamma)+\alpha\beta\gamma)\sqrt{\frac{4a^3}{(a(\alpha+\beta+\gamma)+\alpha\beta\gamma)^2}+1}+a\alpha+\alpha\beta\gamma-a\beta+a\gamma}{2(\alpha\beta+a)} = \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{a}{\left\{ \begin{array}{ll} \alpha & (k \bmod 3) = 1 \\ \beta & (k \bmod 3) = 2 \\ \gamma & (k \bmod 3) = 0 \end{array}\right.} \text{ f}$$

$$\frac{\alpha\left(-\alpha^2+\left(\alpha^2+a+b+c\right)\sqrt{\frac{4abc}{\alpha^2(\alpha^2+a+b+c)^2}+1}+a-b-c\right)}{2\left(\alpha^2+b\right)} = \operatorname*{K}_{k=1}^{\infty} \frac{\{ \begin{matrix} a & (k \bmod 3) = 1 \\ b & (k \bmod 3) = 2 \\ c & (k \bmod 3) = 0 \end{matrix}}{\alpha} \text{ for } (a, b, c, \alpha$$

$$\frac{(a(\beta\gamma+c)+b(\gamma\delta+d)+\alpha(\beta\gamma\delta+c\delta+\beta d))\sqrt{1-\frac{4abcd}{(\alpha\beta\gamma\delta+a\beta\gamma+ac+b\gamma\delta+bd+\alpha c\delta+\alpha\beta d)^2}}+a(\beta\gamma+c)-b(\gamma\delta+d)}{2(\alpha\beta\gamma+b\gamma+\alpha c)}$$

$$\frac{(a(\beta\gamma+c)+b(\gamma\delta+d)+\alpha(\beta\gamma\delta+c\delta+\beta d))\sqrt{1-\frac{4abcd}{(\alpha\beta\gamma\delta+a\beta\gamma+ac+b\gamma\delta+bd+\alpha c\delta+\alpha\beta d)^2}}+a(\beta\gamma+c)-b(\gamma\delta+d)}{2(\alpha\beta\gamma+b\gamma+\alpha c)}$$

$$\frac{\left(2a^2+a(\alpha+\gamma)(\beta+\delta)+\alpha\beta\gamma\delta\right)\sqrt{1-\frac{4a^4}{(2a^2+a(\alpha+\gamma)(\beta+\delta)+\alpha\beta\gamma\delta)^2}}-\alpha(a(\beta+\delta)+\beta\gamma\delta)+a(a+\beta\gamma)-a(a+\gamma}{2(a(\alpha+\gamma)+\alpha\beta\gamma)}$$

$$\frac{\left(a\left(\alpha^2+c\right)+b\left(\alpha^2+d\right)+\alpha^2\left(\alpha^2+c+d\right)\right)\sqrt{1-\frac{4abcd}{(a(\alpha^2+c)+b(\alpha^2+d)+\alpha^2(\alpha^2+c+d))^2}}+a\left(\alpha^2+c\right)-b\left(\alpha^2+d\right)+}{2\alpha\left(\alpha^2+b+c\right)}$$

$$\frac{(c(\delta(\alpha\epsilon+a)+\alpha e)+a\beta(\gamma\delta+d)+(\alpha\beta+b)(\gamma\delta\epsilon+d\epsilon+\gamma e))\sqrt{\frac{4abcde}{(c(\delta(\alpha\epsilon+a)+\alpha e)+a\beta(\gamma\delta+d)+(\alpha\beta+b)(\epsilon(\gamma\delta+d)+\gamma e))^2}}}{2(\alpha\beta\gamma\delta+b\gamma\delta+bd+\alpha c\delta+\alpha\beta d)}$$

$$\frac{(c(\delta(\alpha\epsilon+a)+\alpha e)+a\beta(\gamma\delta+d)+(\alpha\beta+b)(\gamma\delta\epsilon+d\epsilon+\gamma e))\sqrt{\frac{4abcde}{(c(\delta(\alpha\epsilon+a)+\alpha e)+a\beta(\gamma\delta+d)+(\alpha\beta+b)(\epsilon(\gamma\delta+d)+\gamma e))^2} \cdots}}{2(\alpha\beta\gamma\delta+b\gamma\delta+bd+\alpha c\delta+\alpha\beta d)}$$

$$\frac{\left(a^2(\alpha+\beta+\gamma+\delta+\epsilon)+a(\alpha(\beta(\gamma+\epsilon)+\delta\epsilon)+\gamma\delta(\beta+\epsilon))+\alpha\beta\gamma\delta\epsilon\right)\sqrt{\frac{4a^5}{(a^2(\alpha+\beta+\gamma+\delta+\epsilon)+a(\alpha(\beta(\gamma+\epsilon)+\delta\epsilon)+\gamma\delta(\beta+ \cdots}}}{2\left(a^2+\alpha a\beta+\alpha a\delta+\alpha\beta\gamma\delta+a\gamma \cdots\right.}$$

$$\frac{\alpha\left(\left(c\left(\alpha^2+a+e\right)+a\left(\alpha^2+d\right)+\left(\alpha^2+b\right)\left(\alpha^2+d+e\right)\right)\sqrt{\frac{4abcde}{\alpha^2(c(\alpha^2+a+e)+a(\alpha^2+d)+(\alpha^2+b)(\alpha^2+d+e))^2}+1}-c\left(\cdots\right.\right.}{2\left(\alpha^4+\alpha^2 b+bd+\alpha^2 c+\alpha^2 d\right)}$$

$$\boldsymbol{H}_\nu(z)=\frac{2^{-\nu}z^{\nu+1}}{\sqrt{\pi}\Gamma\left(\nu+\frac{3}{2}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{(1+2k)(1+2k+2\nu)}}{1-\frac{z^2}{(1+2k)(1+2k+2\nu)}}+1\right)} \text{ for } (\nu,z)\in\mathbb{C}^2\wedge\neg(\nu\in\mathbb{Z}\wedge\nu\leq 0)$$

$$\boldsymbol{H}_{-m-\frac{3}{2}}(z)=\frac{(-1)^{m-1}2^{-m-\frac{3}{2}}z^{m+\frac{3}{2}}}{\Gamma\left(m+\frac{5}{2}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{\frac{z^2}{2k(3+2k+2m)}}{1-\frac{z^2}{2k(3+2k+2m)}}+1\right)} \text{ for } m\in\mathbb{Z}\wedge z\in\mathbb{C}\wedge m\geq 0$$

$$\boldsymbol{L}_\nu(z)=\frac{2^{-\nu}z^{\nu+1}}{\sqrt{\pi}\Gamma\left(\nu+\frac{3}{2}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^2}{(1+2k)(1+2k+2\nu)}}{1+\frac{z^2}{(1+2k)(1+2k+2\nu)}}+1\right)} \text{ for } (\nu,z)\in\mathbb{C}^2\wedge\neg(\nu\in\mathbb{Z}\wedge\nu\leq 0)$$

$$\boldsymbol{L}_{-m-\frac{3}{2}}(z)=\frac{2^{-m-\frac{3}{2}}z^{m+\frac{3}{2}}}{\Gamma\left(m+\frac{5}{2}\right)\left(\mathrm{K}_{k=1}^{\infty}\frac{-\frac{z^2}{2k(3+2k+2m)}}{1+\frac{z^2}{2k(3+2k+2m)}}+1\right)} \text{ for } m\in\mathbb{Z}\wedge z\in\mathbb{C}\wedge m\geq 0$$

$$\sum_{k=1}^{\infty}\frac{(-1)^k}{2k+z}=\frac{\frac{1}{\mathrm{K}_{k=1}^{\infty}\frac{k(1+k)}{z}+z}-1}{2z} \text{ for } z\in\mathbb{C}\wedge\Re(z)>1$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{(k+z)^2} = \frac{\frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{8}\left(1-(-1)^k\right)(1+k)^2+\frac{1}{8}\left(1+(-1)^k\right)k(2+k)}{z}+z} - 1}{2z^2} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k+z)^2} = \frac{1}{2\left(\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)(1+k)^2}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2)} + z^2 - 1\right)} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 1$$

$$\sum_{k=1}^{\infty} \frac{(-1)^{-1+k}}{(a+k)(b+k)} = \frac{1}{\mathrm{K}_{k=1}^{\infty} \frac{(a+k)^2(b+k)^2}{1+a+b+2k} + (a+1)(b+1)} \text{ for } (a,b) \in \mathbb{C}^2 \wedge \Re(a) > 0 \wedge \Re(b) > 0$$

$$\sum_{k=0}^{\infty} \left(\frac{(-1)^k}{1-b+2k+z} - \frac{(-1)^k}{1+b+2k+z} \right) = \frac{b}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(-1)^k)k^2+\frac{1}{2}(1-(-1)^k)(-b^2+(1+k)^2)}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(-1+z^2)} + z^2 - 1} \text{ for } (b,z) \in \ldots$$

$$\sum_{k=0}^{\infty} \left(\frac{1}{1-a-b+2k+z} - \frac{1}{1+a-b+2k+z} - \frac{1}{1-a+b+2k+z} + \frac{1}{1+a+b+2k+z} \right) = \frac{\ldots}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{2}(1+(\ldots}{\frac{1}{2}\ldots}}$$

$$\sum_{k=0}^{\infty} \left(\frac{1}{(1-b+2k+z)^2} - \frac{1}{(1+b+2k+z)^2} \right) = \frac{b}{\mathrm{K}_{k=1}^{\infty} \frac{\frac{1}{8}(1+(-1)^k)k^3+\frac{1}{8}(1-(-1)^k)(1+k)(-4b^2+(1+k)^2)}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)(b^2+(1+k)(-1+z^2))} + b^2 + z^2 \ldots}$$

$$\sum_{k=0}^{\infty} \left(\frac{1}{(1-b+2k+z)^2} - \frac{1}{(1+b+2k+z)^2} \right) = \frac{b}{\mathrm{K}_{k=1}^{\infty} \frac{4k^4(b^2-k^2)}{(1+2k)(1-b^2+2k+2k^2+z^2)} - b^2 + z^2 + 1} \text{ for } (b,z) \in \mathbb{C}^2 \ldots$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{-1+\sqrt{1+z^2}}{z}\right)^{1+2k}}{1+2k+\frac{a}{\sqrt{1+z^2}}} = \frac{z}{2\left(\mathrm{K}_{k=1}^{\infty} \frac{k^2 z^2}{1+a+2k} + a + 1\right)} \text{ for } (a,z) \in \mathbb{C}^2$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{-1+\sqrt{1+z^2}}{z}\right)^{2k}}{2k+\frac{a}{\sqrt{1+z^2}}} = \frac{z^2}{2a\left(\mathrm{K}_{k=1}^{\infty} \frac{k(1+k)z^2}{2+a+2k} + a + 2\right)} - \frac{\sqrt{z^2+1}-1}{2a} \text{ for } (a,z) \in \mathbb{C}^2$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{-1+\sqrt{1+z^2}}{z}\right)^{2k} (b)_k}{\left(b+2k+\frac{a}{\sqrt{1+z^2}}\right) k!} = \frac{2^{-b} z \left(\frac{1}{z^2}+1\right)^{\frac{1-b}{2}} \left(\frac{\sqrt{z^2+1}-1}{z}\right)^{-b}}{\mathrm{K}_{k=1}^{\infty} \frac{k(-1+b+k)z^2}{a+b+2k} + a + b} \text{ for } (a,b,z) \in \mathbb{C}^3$$

$$\sum_{k=1}^{\infty} 2^{-\lfloor \phi k \rfloor} = \operatorname*{K}_{k=1}^{\infty} \frac{1}{2^{F_{-1+k}}}$$

$$\tan(z) = \frac{z}{\operatorname{K}_{k=1}^{\infty} \frac{-\frac{z^2}{(-1+2k)(1+2k)}}{1} + 1} \text{ for } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = \frac{z}{\operatorname{K}_{k=1}^{\infty} \frac{-\frac{(-1+4^{1+k})z^2\zeta(2+2k)}{(-1+4^k)\pi^2\zeta(2k)}}{1+\frac{(-1+4^{1+k})z^2\zeta(2+2k)}{(-1+4^k)\pi^2\zeta(2k)}} + 1} \text{ for } z \in \mathbb{C} \wedge \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = -\frac{\operatorname{K}_{k=1}^{\infty} \frac{-z^2}{-1+2k}}{z} \text{ for } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = \frac{z}{1 - \frac{4z^2}{\pi^2\left(\operatorname{K}_{k=1}^{\infty} \frac{k^4 - \frac{4k^2z^2}{\pi^2}}{1+2k} + 1\right)}} \text{ for } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan\left(\frac{\pi z}{4}\right) = \frac{z}{\operatorname{K}_{k=1}^{\infty} \frac{(-1+2k)^2 - z^2}{2} + 1} \text{ for } \frac{z}{4} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = -\operatorname*{K}_{k=1}^{\infty} \frac{-1}{\frac{-1+2k}{z}} \text{ for } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = \frac{z^3}{3\left(\operatorname{K}_{k=1}^{\infty} \frac{\frac{2(-1+4^{2+k})z^2 B_{2(2+k)}}{(-1+4^{1+k})(2+k)(3+2k)B_{2(1+k)}}}{1-\frac{2(-1+4^{2+k})z^2 B_{2(2+k)}}{(-1+4^{1+k})(2+k)(3+2k)B_{2(1+k)}}} + 1\right)} + z \text{ for } \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = \frac{\operatorname{K}_{k=1}^{\infty} \frac{(7-4k)(1+4k)z^4}{(-3+4k)(-1+4k)(1+4k)-(-2+8k)z^2}}{3z} + z \text{ for } z \in \mathbb{C} \wedge \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(z) = \frac{1}{\operatorname{K}_{k=1}^{\infty} \frac{1}{\frac{1}{2}(1-(-1)^k)+\frac{1}{2}(1+(-1)^k)\left(-2+\frac{1+k}{z}\right)} + \frac{1}{z} - 1} \text{ for } z \in \mathbb{C} \wedge \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

$$\tan(mz) = \frac{m\tan(z)}{\operatorname{K}_{k=1}^{-1+m} \frac{(k^2-m^2)\tan^2(z)}{1+2k} + 1} \text{ for } m \in \mathbb{Z} \wedge z \in \mathbb{C} \wedge m > 0$$

$$\frac{a\tan\left(\frac{\pi b}{2}\right) - b\tan\left(\frac{\pi a}{2}\right)}{a\tan\left(\frac{\pi a}{2}\right) - b\tan\left(\frac{\pi b}{2}\right)} = -\frac{ab}{\operatorname{K}_{k=1}^{\infty} \frac{(-a^2+k^2)(-b^2+k^2)}{1+2k} + 1} \text{ for } (a,b,z) \in \mathbb{C}^3 \wedge \Re\left(a^2 - b^2\right) > 0$$

$$\tanh(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{z^2}{(-1+2k)(1+2k)}}{1} + 1} \text{ for } -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(z) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{\left(-1+4^{1+k}\right) z^2 \zeta(2+2k)}{\left(-1+4^k\right) \pi^2 \zeta(2k)}}{1-\frac{\left(-1+4^{1+k}\right) z^2 \zeta(2+2k)}{\left(-1+4^k\right) \pi^2 \zeta(2k)}} + 1} \text{ for } z \in \mathbb{C} \land -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(z) = \frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{z^2}{-1+2k}}{z} \text{ for } -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(z) = \frac{z}{\frac{4z^2}{\pi^2 \left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{k^4 + \frac{4k^2 z^2}{\pi^2}}{1+2k} + 1\right)} + 1} \text{ for } -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh\left(\frac{\pi z}{4}\right) = \frac{z}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(-1+2k)^2 + z^2}{2} + 1} \text{ for } -\frac{1}{2} + \frac{iz}{4} \notin \mathbb{Z}$$

$$\tanh(z) = \mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{\frac{-1+2k}{z}} \text{ for } -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(z) = z - \frac{z^3}{3\left(\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{\frac{2\left(1-4^{2+k}\right) z^2 B_{2(2+k)}}{\left(-1+4^{1+k}\right)(2+k)(3+2k) B_{2(1+k)}}}{1-\frac{2\left(1-4^{2+k}\right) z^2 B_{2(2+k)}}{\left(-1+4^{1+k}\right)(2+k)(3+2k) B_{2(1+k)}}} + 1\right)} \text{ for } -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(z) = z - \frac{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(7-4k)(1+4k) z^4}{(-3+4k)(-1+4k)(1+4k) + (-2+8k) z^2}}{3z} \text{ for } z \in \mathbb{C} \land -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(z) = \frac{i}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{1}{\frac{1}{2}(1-(-1)^k) + \frac{1}{2}(1+(-1)^k)\left(-2+\frac{i(1+k)}{z}\right)} + \frac{i}{z} - 1} \text{ for } z \in \mathbb{C} \land -\frac{1}{2} + \frac{iz}{\pi} \notin \mathbb{Z}$$

$$\tanh(mz) = \frac{m \tanh(z)}{\mathop{\mathrm{K}}_{k=1}^{-1+m} \frac{\left(-k^2+m^2\right) \tanh^2(z)}{1+2k} + 1} \text{ for } m \in \mathbb{Z} \land z \in \mathbb{C} \land m > 0$$

$$\frac{a \tanh\left(\frac{\pi b}{2}\right) - b \tanh\left(\frac{\pi a}{2}\right)}{a \tanh\left(\frac{\pi a}{2}\right) - b \tanh\left(\frac{\pi b}{2}\right)} = \frac{ab}{\mathop{\mathrm{K}}_{k=1}^{\infty} \frac{(a^2+k^2)(b^2+k^2)}{1+2k} + 1} \text{ for } (a, b, z) \in \mathbb{C}^3 \land \Re\left(b^2 - a^2\right) > 0$$

$$M_{\nu,\mu}(z) = e^{-z/2} z^{\mu+\frac{1}{2}} \left(\frac{z\left(\mu-\nu+\frac{1}{2}\right)}{(2\mu+1)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z\left(\frac{1}{2}+k+\mu-\nu\right)}{(1+k)(1+k+2\mu)}}{1+\frac{z\left(\frac{1}{2}+k+\mu-\nu\right)}{(1+k)(1+k+2\mu)}} + 1\right)} + 1 \right) \text{ for } (\nu,\mu,z) \in \mathbb{C}^3$$

$$M_{\nu,\mu}(z) = \frac{e^{-z/2} z^{\mu+\frac{1}{2}}}{\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z(-1+2k+2\mu-2\nu)}{2k(k+2\mu)}}{1+\frac{z(-1+2k+2\mu-2\nu)}{2k(k+2\mu)}} + 1} \text{ for } (\nu,\mu,z) \in \mathbb{C}^3$$

$$W_{\nu,\mu}(z) = \frac{e^{-z/2}\Gamma(2\mu) z^{\frac{1}{2}-\mu}}{\Gamma\left(\mu-\nu+\frac{1}{2}\right)\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z(1-2k+2\mu+2\nu)}{2k(k-2\mu)}}{1-\frac{z(1-2k+2\mu+2\nu)}{2k(k-2\mu)}} + 1\right)} + \frac{e^{-z/2}\Gamma(-2\mu) z^{\mu+\frac{1}{2}}}{\Gamma\left(-\mu-\nu+\frac{1}{2}\right)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z(-1+2k+2\mu-2\nu)}{2k(k+2\mu)}}{1+\frac{z(-1+2k+2\mu-2\nu)}{2k(k+2\mu)}} + 1\right)} \text{ fo} \ldots$$

$$W_{\nu,0}(z) = -\frac{e^{-z/2}\sqrt{z}\left(\psi^{(0)}\left(\frac{1}{2}-\nu\right) + \log(z) + 2\gamma\right)}{\Gamma\left(\frac{1}{2}-\nu\right)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z(-1+2k-2\nu)\left(\log(z)-2\psi^{(0)}(1+k)+\psi^{(0)}\left(\frac{1}{2}+k-\nu\right)\right)}{2k^2\left(\log(z)-2\psi^{(0)}(k)+\psi^{(0)}\left(-\frac{1}{2}+k-\nu\right)\right)}}{1+\frac{z(-1+2k-2\nu)\left(\log(z)-2\psi^{(0)}(1+k)+\psi^{(0)}\left(\frac{1}{2}+k-\nu\right)\right)}{2k^2\left(\log(z)-2\psi^{(0)}(k)+\psi^{(0)}\left(-\frac{1}{2}+k-\nu\right)\right)}} + 1\right)} \text{ for } (\nu,z) \in \mathbb{C}^2$$

$$W_{\nu,\frac{m}{2}}(z) = -\frac{(-1)^m e^{-z/2} z^{\frac{m+1}{2}} \log(z)}{m!\,\Gamma\left(\frac{1-m}{2}-\nu\right)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z(-1+2k+m-2\nu)}{2k(k+m)}}{1+\frac{z(-1+2k+m-2\nu)}{2k(k+m)}} + 1\right)} - \frac{(-1)^m e^{-z/2} z^{\frac{m+1}{2}} \left(\psi^{(0)}\left(\frac{m+1}{2} - \ldots\right.\right.}{m!\,\Gamma\left(\frac{1-m}{2}-\nu\right)\left(\operatorname*{K}_{k=1}^{\infty} \frac{-\frac{z(-1+2k+m-2\nu)\left(\psi^{(0)} \ldots\right.}{2k(k+m)\left(\psi^{(0)}(k) \ldots\right.}}{1+\frac{z(-1+2k+m-2\nu)\left(\psi^{(0)} \ldots\right.}{2k(k+m)\left(\psi^{(0)}(k) \ldots\right.}} \ldots\right.}$$

$$\zeta(z+1) = -\frac{\gamma_1 z}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{z\gamma_{1+k}}{\gamma_k+k\gamma_k}}{1-\frac{z\gamma_{1+k}}{\gamma_k+k\gamma_k}} + 1} + \frac{1}{z} + \gamma \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\zeta(2,z) = \frac{1}{z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\left\lfloor \frac{1+k}{2} \right\rfloor^2}{\frac{\left(1+(-1)^k(1+2k)\right)z}{1}} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\zeta(2,z) = \frac{1}{2z^2\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{4}k(1+k)^2(2+k)}{(3+2k)z} + 3z\right)} + \frac{1}{2z^2} + \frac{1}{z} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\zeta(2,z) = \frac{2}{\operatorname*{K}_{k=1}^{\infty} \frac{k^4}{(1+2k)(-1+2z)} + 2z - 1} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\zeta(2,z) = \frac{1}{\operatorname*{K}_{k=1}^{\infty} \frac{\frac{k^4}{4\left(-1+4k^2\right)}}{-\frac{1}{2}+z} + z - \frac{1}{2}} \text{ for } z \in \mathbb{C} \wedge \Re(z) > \frac{1}{2}$$

$$\zeta(3,z) = \frac{1}{2(z-1)z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\frac{1}{32}\left(1+(-1)^k\right)k^4+\frac{1}{32}\left(1-(-1)^k\right)(1+k)^4}{(-1+z)z}}{1+k} + 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg \left(z \in \mathbb{R} \wedge \frac{1}{2} < z < 1\right) \wedge \Re(z) >$$

$$\zeta(3,z) = \frac{1}{4z^3\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\left(1+(-1)^k\right)k(2+k)^2}{32(1+k)}+\frac{\left(1-(-1)^k\right)(1+k)^2(3+k)}{32(2+k)}}{z} + z\right)} + \frac{1}{2z^3} + \frac{1}{2z^2} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\zeta(3,z) = \frac{1}{2z^3\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{1}{4}(1+(-1)^k)\left(1+\frac{k}{2}\right)^3 k+\frac{1}{16}(1-(-1)^k)(1+k)^3\left(1+\frac{1+k}{2}\right)}{(2+k)z} + 2z\right)} + \frac{1}{2z^3} + \frac{1}{2z^2} \text{ for } z \in \mathbb{C} \wedge \Re(z) > 0$$

$$\zeta(3,z) = \frac{1}{2z\left(\operatorname*{K}_{k=1}^{\infty} \frac{\frac{\left(\frac{1}{32}\left(1+(-1)^k\right)k^4+\frac{1}{32}\left(1-(-1)^k\right)(1+k)^4\right)(-1+z)}{z}}{(1+k)(-1+z)} + z - 1\right)} \text{ for } z \in \mathbb{C} \wedge \neg \left(z \in \mathbb{R} \wedge \frac{1}{2} < z < 1\right) \wedge \Re($$

$$\zeta\left(3, \frac{z+1}{2}\right) = \frac{2}{\operatorname*{K}_{k=1}^{\infty} \frac{2\left\lfloor \frac{1+k}{2} \right\rfloor^3}{((1+k)(-1+z^2))^{\frac{1}{2}\left(1+(-1)^k\right)}} + z^2 - 1} \text{ for } z \in \mathbb{C} \wedge \neg (z \in \mathbb{R} \wedge 0 < z < 1) \wedge \Re(z) > 0$$