Theorem examples

Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg

Structure of $\mathrm{Gal}\left(\mathbb{k}_2^{(2)}/\mathbb{k}\right)$ for some fields $\mathbb{k} = \mathbb{Q}\left(\sqrt{2p_1p_2}, i\right)$ with $\mathbf{Cl}_2(\mathbb{k}) \simeq (2, 2, 2)$

Author(s): Abdelmalek Azizi · Abdelkader Zekhnini · Mohammed Taous
Source: Abh. Math. Semin. Univ. Hambg. (2014) 84:203–231

Theorem 3 *Let $2^n = h(p_1p_2)$, $2^{m+1} = h(-p_1p_2)$, where $n \geq 1$ and $m \geq 2$.*

(1) $\#\kappa_{\mathbb{K}_j} = 4$, *for all* $j \neq 3$. *If* $j = 3$, *then* $\#\kappa_{\mathbb{K}_3} = \begin{cases} 4 \text{ if } q = 1, \\ 2 \text{ if } q = 2. \end{cases}$

(2) *All the extensions $\mathbb{K}_j$ satisfy Taussky's condition (A) i.e. $\#\kappa_{\mathbb{K}_j} \cap N_j > 1$, for details see* [17].

(3) *The order of $\kappa_{\mathbb{L}_j}$ is 8 (total 2-capitulation), for all j, and $\mathbb{L}_j$ are of type (A).*

(4) *The abelian type invariants of the 2-class groups $\mathbf{Cl}_2(\mathbb{K}_j)$ are given by:*

(i) $\mathbf{Cl}_2(\mathbb{K}_1) \simeq \mathbf{Cl}_2(\mathbb{K}_2) \simeq \begin{cases} (2, 2, 2) \text{ if } \left(\frac{p_1}{p_2}\right) = 1, \\ (2, 4) \text{ otherwise.} \end{cases}$

(ii) *If* $\left(\frac{p_1}{p_2}\right) = 1$, *then* $\mathbf{Cl}_2(\mathbb{K}_4)$, $\mathbf{Cl}_2(\mathbb{K}_5)$, $\mathbf{Cl}_2(\mathbb{K}_6)$ *and* $\mathbf{Cl}_2(\mathbb{K}_7)$ *are of type* $(2, 2, 2)$ *if* $\left(\frac{\pi_1}{\pi_3}\right) = -1$, *and of type* $(2, 4)$ *otherwise.*

(iii) *Assume* $\left(\frac{p_1}{p_2}\right) = -1$.

If $\left(\frac{\pi_1}{\pi_3}\right) = -1$, *then* $\begin{cases} \mathbf{Cl}_2(\mathbb{K}_4) \simeq \mathbf{Cl}_2(\mathbb{K}_7) \simeq (2, 4), \\ \mathbf{Cl}_2(\mathbb{K}_5) \simeq \mathbf{Cl}_2(\mathbb{K}_6) \simeq (2, 2, 2). \end{cases}$

If $\left(\frac{\pi_1}{\pi_3}\right) = 1$, *then* $\begin{cases} \mathbf{Cl}_2(\mathbb{K}_4) \simeq \mathbf{Cl}_2(\mathbb{K}_7) \simeq (2, 2, 2), \\ \mathbf{Cl}_2(\mathbb{K}_5) \simeq \mathbf{Cl}_2(\mathbb{K}_6) \simeq (2, 4). \end{cases}$

(5) *The abelian type invariants of the 2-class groups $\mathbf{Cl}_2(\mathbb{L}_j)$ are given by:*

(i) $\mathbf{Cl}_2(\mathbb{L}_1) = \mathbf{Cl}_2(\mathbb{k}^{(*)}) \simeq \begin{cases} (2^m, 2^n) \text{ if } q = 1, \\ (2^{\min(m,n)}, 2^{\max(m+1,n+1)}) \text{ if } q = 2. \end{cases}$

(ii) *If* $\left(\frac{p_1}{p_2}\right) = -1$ *or* $\left(\frac{p_1}{p_2}\right) = \left(\frac{\pi_1}{\pi_3}\right) = 1$, *then* $\mathbf{C}l_2(\mathbb{L}_2)$, $\mathbf{C}l_2(\mathbb{L}_3)$, $\mathbf{C}l_2(\mathbb{L}_4)$ *and* $\mathbf{C}l_2(\mathbb{L}_5)$ *are of type* $(2, 4)$.
If $\left(\frac{p_1}{p_2}\right) = -\left(\frac{\pi_1}{\pi_3}\right) = 1$, *then* $\mathbf{C}l_2(\mathbb{L}_2)$, $\mathbf{C}l_2(\mathbb{L}_3)$, $\mathbf{C}l_2(\mathbb{L}_4)$ *and* $\mathbf{C}l_2(\mathbb{L}_5)$ *are of type* $(2, 2, 2)$.

(iii) (*a*) *Assume* $q = 2$, *so* $\mathbf{C}l_2(\mathbb{L}_6)$ *and* $\mathbf{C}l_2(\mathbb{L}_7)$ *are of type* $(2, 2^{n+2})$ *if* $\left(\frac{p_1}{p_2}\right) = 1$, *otherwise we have:*

$$\mathbf{C}l_2(\mathbb{L}_6) \simeq \begin{cases} (4, 4) \text{ if } \left(\frac{\pi_1}{\pi_3}\right) = 1, \\ (2, 8) \text{ if } \left(\frac{\pi_1}{\pi_3}\right) = -1, \end{cases} \quad \mathbf{C}l_2(\mathbb{L}_7) \simeq \begin{cases} (2, 8) \text{ if } \left(\frac{\pi_1}{\pi_3}\right) = 1, \\ (4, 4) \text{ if } \left(\frac{\pi_1}{\pi_3}\right) = -1. \end{cases}$$

(*b*) *Assume* $q = 1$.

If $\left(\frac{1+i}{\pi_1}\right)\left(\frac{1+i}{\pi_3}\right) = 1$, *then* $\begin{cases} \mathbf{C}l_2(\mathbb{L}_6) \simeq (2^{m-1}, 2^{n+1}), \\ \mathbf{C}l_2(\mathbb{L}_7) \simeq (2^{\min(m-1,n)}, 2^{\max(m,n+1)}). \end{cases}$

If $\left(\frac{1+i}{\pi_1}\right)\left(\frac{1+i}{\pi_3}\right) = -1$, *then* $\begin{cases} \mathbf{C}l_2(\mathbb{L}_6) \simeq (2^{\min(m-1,n)}, 2^{\max(m,n+1)}), \\ \mathbf{C}l_2(\mathbb{L}_7) \simeq (2^{m-1}, 2^{n+1}). \end{cases}$

The theme of a vanishing period
Author(s): Daniel Barlet
Abh. Math. Semin. Univ. Hambg. (2014) 84:155–185

Proposition 3.1.6 *Let* E *be a* $[\lambda]$*-primitive theme of rank* $k \geq 1$.

1. *Then there exists* $\varphi \in \Xi_\lambda^{(k-1)} \setminus \Xi_\lambda^{(k-2)}$ *such that* E *is isomorphic to* $\tilde{\mathcal{A}}.\varphi \subset \Xi_\lambda^{(k-1)}$, *with the convention* $\Xi_\lambda^{(-1)} = \{0\}$.
2. *Conversely, for any such* φ, $\tilde{\mathcal{A}}.\varphi$ *is a* $[\lambda]$*-primitive theme of rank* k.
3. *In this situation, for any* $j \in [1, k]$ $F_j := \tilde{\mathcal{A}}.\varphi \cap \Xi_\lambda^{(j-1)}$ *is a rank* j *normal sub-module of* $\tilde{\mathcal{A}}.\varphi$ *contained in* $b^{k-j}.\Xi_\lambda^{(j-1)}$.

Advances in Mathematics

The Isbell monad
Richard Garner
Advances in Mathematics 274 (2015) 516–537

Proposition 8. *Every cylinder factorisation system* $(\mathcal{E}, \mathcal{M})$ *on* $\mathcal{C}$ *is the underlying cylinder factorisation system of a unique extended cylinder factorisation system* $(\bar{\mathcal{E}}, \overline{\mathcal{M}})$*; moreover, any morphism of cylinder factorisation systems* $F: \mathcal{C} \to \mathcal{D}$ *preserves these extended classes, in the sense that* $F(\bar{\mathcal{E}}) \subset \bar{\mathcal{E}}$ *and* $F(\overline{\mathcal{M}}) \subset \overline{\mathcal{M}}$.

Theorem 12. *The forgetful* 2*-functor* $\mathcal{I}$-**Alg** $\to$ **CAT** *has a (strictly commuting) factorisation*

$$\begin{array}{ccc} \mathcal{I}\text{-}\mathbf{Alg} & \xrightarrow{J} & \mathbf{CFS} \\ & \searrow \quad \swarrow & \\ & \mathbf{CAT} & \end{array}$$

wherein J is a biequivalence 2*-functor satisfying $JK = 1$; it follows that K is a biequivalence, and so that* **CFS** *is pseudomonadic over* **CAT**.

On Zippin's Embedding Theorem of Banach spaces into Banach spaces with bases
Th. Schlumprecht
Advances in Mathematics 274 (2015) 833–880

Theorem 1.1. *(See [27, Corollary].) Every separable and reflexive Banach space into a reflexive Banach space with a basis.*

Theorem 1.2. *(See [27, Theorem].) Every Banach space with a separable dual into a space with shrinking basis.*

Main Theorem. *Assume that X is a Banach space with separable dual. Then X embeds into a space W with a shrinking basis (w_i) so that*

a) $\mathrm{Sz}(W) = \mathrm{Sz}(X)$,
b) *if X is reflexive then W is reflexive and* $\mathrm{Sz}(X^*) = \mathrm{Sz}(W^*)$, *and*
c) *if X has the w^*-*Unconditional Tree Property, *then (w_i) is unconditional.*

Formality theorem for gerbesT
Paul Bressler, Alexander Gorokhovsky, Ryszard Nest,Boris Tsygan
Advances in Mathematics 273 (2015) 215–241

Theorem 1.1. *Let X be a C^∞-manifold. Then the DGLA $\mathfrak{g}_{\mathrm{DR}}(\mathcal{J}_X)_\omega$ is L_∞ quasi-isomorphic to the L_∞-algebra $\mathfrak{s}(\mathcal{O}_X)_H$.*

Theorem 1.2. *For any Artin algebra R with maximal ideal $\mathfrak{m}_R$ there is an equivalence of* 2*-groupoids*

$$\mathrm{Def}(\mathcal{S})(R) \cong \mathbf{Bic}\, \Pi_2\big(\Sigma\big(\mathfrak{s}(\mathcal{O}_X)_H \otimes \mathfrak{m}_R\big)\big)$$

natural in R.

Theorem 1.3. *Suppose that X is a C^∞ manifold equipped with a pair of complementary complex integrable distributions $\mathcal{P}$ and $\mathcal{Q}$, and $\mathcal{S}$ is a twisted form of $\mathcal{O}_{X/\mathcal{P}}$ (6.2). Let $H \in \Gamma(X; F_{-1}\Omega^3_X)$ be a representative of $[\mathcal{S}]$ (6.2). Then, for any Artin algebra R with maximal ideal $\mathfrak{m}_R$ there is an equivalence of bi-groupoids*

$$\mathbf{Bic}\,\Pi_2\big(\Sigma\big(\mathfrak{s}(\mathcal{O}_{X/\mathcal{P}})_H \otimes \mathfrak{m}_R\big)\big) \cong \mathrm{Def}(\mathcal{S})(R),$$

natural in R.

Annals of Functional Analysis

ON GENERALIZED BECKNER'S INEQUALITY
KICHI-SUKE SAITO AND RYOTARO TANAKA
Ann. Funct. Anal. 6 (2015), no. 1, 267–278

Theorem. *Let $1 < p \le q < \infty$, and let $\gamma_{p,q} = \sqrt{(p-1)/(q-1)}$. Then the inequality*

$$\left(\frac{|u+\gamma_{p,q}v|^q + |u-\gamma_{p,q}v|^q}{2}\right)^{1/q} \le \left(\frac{|u+v|^p + |u-v|^p}{2}\right)^{1/p}$$

holds for all $u, v \in \mathbb{R}$.

Banach Journal of Mathematical Analysis

ON E-FRAMES IN SEPARABLE HILBERT SPACES
GHOLAMREZA TALEBI, MOHAMMAD ALI DEHGHAN
Banach J. Math. Anal. 9 (2015), no. 3, 43–74

Theorem 2.3. *For a* E*−orthonormal system $\{g_k\}_{k=1}^\infty$, the following are equivalent:*

(i) $\{g_k\}_{k=1}^\infty$ *is an* E*−orthonormal basis.*

(ii) $f = \sum_{k=1}^\infty \left\langle f, \left(E\{g_j\}_{j=1}^\infty\right)_k \right\rangle \left(E\{g_j\}_{j=1}^\infty\right)_k, \ \forall f \in \mathcal{H}.$

(iii) $\langle f, g\rangle = \sum_{k=1}^\infty \left\langle f, \left(E\{g_j\}_{j=1}^\infty\right)_k \right\rangle \left\langle \left(E\{g_j\}_{j=1}^\infty\right)_k, g \right\rangle, \ \forall f, g \in \mathcal{H}.$

(iv) $\left\| \left\{ \left\langle f, \left(E\{g_j\}_{j=1}^\infty\right)_k \right\rangle \right\}_{k=1}^\infty \right\|_{\ell^2}^2 = \|f\|^2, \ \forall f \in \mathcal{H}.$

(v) $\{g_k\}_{k=1}^\infty$ *is an* E*−complete sequence.*

(vi) *If* $\left\langle f, \left(E\{g_j\}_{j=1}^\infty\right)_k \right\rangle = 0, \ \forall k \in \mathbb{N},$ *then* $f = 0.$

Bulletin AMS

Singular perturbations of complex polynomials
Robert L. Devaney.
Bull. Amer. Math. Soc. 50 (2013), 391-429

Theorem (The escape trichotomy). *Suppose the orbits of the free critical points tend to* ∞.

(1) *If* v_λ *lies in* B_λ, *the* $J(F_\lambda)$ *is a Cantor set.*
(2) *If* v_λ *lies in* T_λ, *then* $J(F_\lambda)$ *is a Cantor set of concentric simple closed curves, each one of which surrounds the origin.*
(3) *In all other cases,* $J(F_\lambda)$ *is a connected set, and if* $F_\lambda^k(v_\lambda) \in T_\lambda$ *where* $k \geq 1$, *then* $J(F_\lambda)$ *is a Sierpiński curve.*

Theorem (Escape time conjugacy). *Let*

$$F_\lambda(z) = z^n + \frac{\lambda}{z^n} \quad \text{and} \quad F_\mu(z) = z^n + \frac{\mu}{z^n},$$

where λ *and* μ *are parameters that lie in Sierpiński holes.*

(1) *If* λ *and* μ *lie in the same Sierpiński hole, then* F_λ *and* F_μ *are topologically conjugate on their Julia sets.*
(2) *If* λ *and* μ *lie in Sierpiński holes with different escape times, then* F_λ *and* F_μ *are not topologically conjugate on their Julia sets.*
(3) *Suppose* λ *and* μ *are centers of different Sierpiński holes that have the same escape time. Let* α *be a primitive* $(n-1)$*-st root of unity. Then* F_λ *and* F_μ *are topologically conjugate on their Julia sets if and only if, for some integer* j, *either*
   - $\mu = \alpha^{2j}\lambda$ *or*
   - $\mu = \alpha^{2j}\overline{\lambda}$.

   Therefore, if λ *and* μ *are parameters that lie in different Sierpiński holes whose escape times are the same, then* F_λ *and* F_μ *are topologically conjugate on their Julia sets if and only if the parameters corresponding to the centers of these Sierpiński holes are symmetrically located with respect to rotation by* α^{2j} *or by complex conjugation followed by such a rotation.*

Theorem (Rings around the McMullen domain). *F* Go to page 427 ≥ 3, *the M domain is surrounded by infinitely many "Mandelpinski necklaces"* $\mathcal{S}^k$ $1, 2, \ldots$. *These are simple closed curves that have the properties that:*

(1) *Each curve* $\mathcal{S}^k$ *surrounds* $\mathcal{M}$ *as well as* $\mathcal{S}^{k+1}$, *and the* $\mathcal{S}^k$ *accun the boundary of the McMullen domain as* $k \to \infty$.
(2) *The curve* $\mathcal{S}^k$ *meets the centers of* τ_k^n *Sierpiński holes, each wi time* $k+2$, *where*

$$\tau_k^n = (n-2)n^{k-1} + 1.$$

(3) *The curve* $\mathcal{S}^k$ *also passes through* τ_k^n *centers of baby Mandelbrot base period* k *(when* $k \neq 2$*), and these Mandelbrot sets and Sierpi alternate as the parameter winds around* $\mathcal{S}^k$.

Proposition 2.1. *For each* $k \in \mathbb{N}$, *let* $P_k := \left\{\sum_{i=0}^{k} a_i x^i : a_i \in \mathbb{R}\right\} \cong \mathbb{R}^{k+1}$. *Then for all* $n \in \mathbb{N}$, *given* $A \in P_k$ *there is a unique* $B \in P_k$ *such that* $\mathcal{S}_n(B) = A$.

Chern–Weil forms and abstract homotopy theory
Daniel S. Freed and Michael J. Hopkins.
Bull. Amer. Math. Soc. 50 (2013), 431-468

Definition 3.7. Let $\mathcal{F}', \mathcal{F}$ be presheaves on manifolds. Then a *map* $\varphi\colon \mathcal{F}' \to \mathcal{F}$ is a natural transformation of functors. Thus for each test manifold M there is a map $\mathcal{F}'(M) \xrightarrow{\varphi(M)} \mathcal{F}(M)$ of sets such that for every smooth map $M' \xrightarrow{f} M$ of test manifolds, the diagram

$$\begin{array}{ccc} \mathcal{F}'(M') & \xleftarrow{\mathcal{F}'(f)} & \mathcal{F}'(M) \\ {\scriptstyle \varphi(M')}\downarrow & & \downarrow{\scriptstyle \varphi(M)} \\ \mathcal{F}(M') & \xleftarrow{\mathcal{F}(f)} & \mathcal{F}(M) \end{array} \tag{3.8}$$

commutes.

Lemma 3.9 (Yoneda). *For any presheaf* $\mathcal{F}$, *evaluation on* X *determines an isomorphism* $\mathbf{Pre}(\mathcal{F}_X, \mathcal{F}) \cong \mathcal{F}(X)$.

Theorem 3.17. *The de Rham complex of* Ω^1 *is isomorphic to*

$$\mathbb{R} \xrightarrow{0} \mathbb{R} \xrightarrow{1} \mathbb{R} \xrightarrow{0} \mathbb{R} \xrightarrow{1} \cdots . \tag{3.18}$$

In particular, the de Rham cohomology of Ω^1 *is*

$$H^{\bullet}_{dR}(\Omega^1) \cong \begin{cases} \mathbb{R}, & \bullet = 0; \\ 0, & \bullet \neq 0. \end{cases} \tag{3.19}$$

Definition 3.22. Let $\mathcal{F}\colon \mathbf{Man}^{\mathrm{op}} \to \mathbf{Set}$ be a presheaf. Then $\mathcal{F}$ is a *sheaf* if for every manifold M and every open cover $\{U_\alpha\}$ of M

$$\mathcal{F}(M) \longrightarrow \prod_{\alpha_0} \mathcal{F}(U_{\alpha_0}) \rightrightarrows \prod_{\alpha_0,\alpha_1} \mathcal{F}(U_{\alpha_0} \cap U_{\alpha_1}) \tag{3.23}$$

is an equalizer diagram.

Definition 7.1. Let $\mathcal{F}_\bullet$ be a simplicial presheaf. The *de Rham complex* of $\mathcal{F}$

$$\begin{aligned} &\mathrm{ho}\,\mathbf{sPre}(\mathcal{F}_\bullet\ ,\ \Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \cdots) \\ &\quad \cong \mathrm{ho}\,\mathbf{sPre}(\mathcal{F}_\bullet, \Omega^0) \xrightarrow{d} \mathrm{ho}\,\mathbf{sPre}(\mathcal{F}_\bullet, \Omega^1) \xrightarrow{d} \cdots . \end{aligned} \tag{7.2}$$

Theorem 7.28.

(i) *For any smooth manifold X the de Rham complex of $X \times (\Omega^1 \otimes \mathfrak{g})$ is $\Omega(X; \mathrm{Kos}\,\mathfrak{g}^*)^\bullet$ with differential the sum of the de Rham differential d_X on X and the Koszul differential d_K in* (7.17).

(ii) *The de Rham complex of the simplicial Borel quotient $(X_G)_\nabla$ in* (7.23) *is the basic subcomplex of $\Omega(X; \mathrm{Kos}\,\mathfrak{g}^*)^\bullet$ with differential $d_X + d_K$.*

Counting problems in Apollonian packings
Elena Fuchs.
Bull. Amer. Math. Soc. 50 (2013), 229-266

Theorem Z (Bourgain and Kontorovich, 2011 [BK11]). *Almost every natural ber is the denominator of a reduced fraction whose partial quotients are bounc* 50.

Conjecture 2.14 (Hensley, 1996 [Hen96, Conjecture 3, p. 16]).

$$\mathscr{D}_\mathcal{A} \supset \mathbb{N}_{\gg 1} \qquad \Longleftrightarrow \qquad \delta_\mathcal{A} > 1/2. \tag{2.15}$$

Theorem 2.7 (Zaremba, 1966 [Zar66, Corollary 5.2]). *Fix $(b, d) = 1$ with b* $[a_1, a_2, \ldots, a_k]$ *and let $A := \max a_j$. Then for $\mathcal{Z}_{b,d}$ given in* (2.6),

$$\mathrm{Disc}(\mathcal{Z}_{b,d}) \le \left(\frac{4A}{\log(A+1)} + \frac{4A+1}{\log d}\right)\frac{\log d}{d}. \tag{2.8}$$

Complex analysis and operator theory

Complex Anal. Oper. Theory (2013) 7:519–528
The Generalized Schwarz–Pick Estimates of Arbitrary Order on the Unit Polydisk
Jianfei Wang · Yang Liu

Theorem A *Suppose $\varphi(z)$ is holomorphic mapping from D_n to B_N. Then for any multi-index $m = (m_1, \ldots, m_n)$ such that $m_j > 0$, $j = 1, \ldots, n$,*

$$|\langle \partial^m \varphi(z), \varphi(z)\rangle|^2 + (1 - |\varphi(z)|^2)|\partial^m \varphi(z)| \le \left(m!\frac{1 - |\varphi(z)|^2}{(1 - ||z||_\infty^2)^{|m|}}(1 + ||z||_\infty)^{|m|-n}\right)^2.$$

Lemma 2.1 ([13]) *If $f \in H(\Omega, \Omega)$, then*

$$F_c^\Omega(f(z), J_f(z)\xi) \le F_c^\Omega(z, \xi), \quad z \in \Omega, \ \xi \in \mathbb{C}^N.$$

Theorem 3.1 *If $\varphi : D_n \to \Omega$ is a holomorphic mapping, then*

$$F_c^{\Omega}(\varphi(z), J_{\varphi}(z)\zeta) \leq \max\left(\frac{|\zeta_1|}{1-|z_1|^2}, \ldots, \frac{|\zeta_n|}{1-|z_n|^2}\right) \tag{1}$$

holds for each $z \in D_n$ and $\zeta = (\zeta_1, \ldots, \zeta_n) \in \mathbb{C}^n$.

Complex Anal. Oper. Theory (2013) 7:623–634
Properties of ODEs and PDEs in Algebras Yakov Krasnov

Definition 1.1 The $\mathbb{A}$-valued function $u(x)$ is called $\mathbb{A}$-analytic ($\mathbb{A}$-monogenic) if $u(x)$ is the solution of the Dirac equation in algebra $\mathbb{A} = (\mathbb{R}^n, \circ)$:

$$D \circ u(x) = 0, \quad D = \nabla_x = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}\right) \tag{1.3}$$

Proposition 2.1 ([11]) *Let*

$$\frac{d^m z}{dt^m} = P\left(z, \frac{dz}{dt}, \ldots, \frac{d^{m-1}z}{dt^{m-1}}\right), \quad z \in \mathbb{R}^n \tag{2.1}$$

be a polynomial ODEs in $\mathbb{R}^n$; i.e. $P(z, z_1, \ldots, z_{m-1})$ is a polynomial in the z_i's. Then the solution to this equation may be obtained from the solution of a quadratic system (1.1) occurred in a suitable algebra $\mathbb{A} = (\mathbb{R}^n, \circ)$.

Theorem 2.4 *The formal solution of the initial value problem (IVP) for the Ricatti equation in the binary algebra $\mathbb{A}$ is given by*

$$x(t) = x_0 + x_0^2 t + \cdots + x_0^{[n+1]} t^n + \cdots \tag{2.4}$$

where $a^{[k]}$ is a k-th symmetric power defined recurrently

$$a^{[1]} := a, \quad a^{[2]} = a^2, \quad a^{[m+1]} = \frac{1}{m}\sum_{k=1}^{m} a^{[k]} \circ a^{[m-k+1]} \tag{2.5}$$

Theorem 2.6 *Assume that* $\mathbf{e}_0, \mathbf{e}_1, \ldots, \mathbf{e}_n$ *forms an orthogonal basis in unital associative algebra* $\mathbb{A} = (\mathbb{R}^{n+1}, \circ)$ *and let* $\mathbf{e}_0$ *be the two sided unit element in* $\mathbb{A}$. *Denote by* $z_i = x_0\mathbf{e}_i - x_i\mathbf{e}_0$, $i = 1, \ldots, n$. *By construction,* $D \circ z_i = 0$. *Then any polynomial solution to the Dirac equation in* $\mathbb{A}$ *may be represented by the superposition of the following homogenic monomials:*

$$z^{[m_1,m_2,\ldots,m_k]} = \frac{1}{k!} \sum_{\pi(m_1,\ldots,m_k)} z_{m_1} \circ z_{m_2} \circ \cdots \circ z_{m_k} \tag{2.6}$$

where the sum runs over all distinguishable permutations of $m_1, \ldots, m_k$.

Theorem 2.7 (Lyapunov function) [2] *Let the Riccati equation* (1.1) *occur in an algebra* $\mathbb{A}$. *Suppose, there exists the symmetric, positive definite, bilinear form* $b : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ *satisfies* $b(x, x^2) = 0$ *for all* $x \in \mathbb{A}$; *then the origin is stable.*

Theorem 2.8 (Boundness) [3] *Suppose,* $\mathbb{A}$ *is a rank three algebra. Then there exists bounded solution to* (1.1) *iff* $\mathbb{A}$ *has a complete complex structure. (The existence of complete complex structure equivalent to the non-trivial solubility of two equations:* $x^2 \circ x^2 = -x^2$ *and* $y \circ y^2 = -y$ *in the algebra* $\mathbb{A}$.)

Complex Anal. Oper. Theory (2013) 7:33–42
Chaos of the Differentiation Operator on Weighted Banach Spaces of Entire Functions
José Bonet · Antonio Bonilla

Corollary 2.4 *Let* $\varphi(r)$ *be a positive function with* $\lim_{r\to\infty} \varphi(r) = \infty$. *For each* $1 \le p \le \infty$ *there is an entire function* f *such that*

$$M_p(f,r) \le \varphi(r) \frac{e^r}{r^{\frac{1}{2p}}}$$

that is frequently hypercyclic for the differentiation operator D *on* $H(\mathbb{C})$.

Theorem 2.3 *Let* v *be a weight function such that* $\lim_{r\to\infty} v(r) \frac{e^r}{r^{\frac{1}{2p}}} = 0$ *for some* $1 \le p \le \infty$. *If the differentiation operator* $D : B_{p,0} \to B_{p,0}$ *is continuous, then* D *is frequently hypercyclic.*

Lemma 2.2 *The following conditions are equivalent for a weight* v *and* $1 \le p < \infty$:

(i) $\{e^{\theta z} : |\theta| = 1\} \subset B_{p,0}$.
(ii) *There is* $\theta \in \mathbb{C}$, $|\theta| = 1$, *such that* $e^{\theta z} \in B_{p,0}$.
(iii) $\lim_{r\to\infty} v(r) \frac{e^r}{r^{\frac{1}{2p}}} = 0$

Proposition 2.1 *Let v be a weight function such that* $\sup_{r>0} \frac{v(r)}{v(r+1)} < \infty$. *Then the differentiation operators* $D : B_{p,\infty} \to B_{p,\infty}$ *and* $D : B_{p,0} \to B_{p,0}$ *are continuous.*

Complex variables and elliptic equations

Entire functions that share a set with their derivatives

Jun-Fan Chen

Volume 58, Issue 6, 2013, pages 727-734

LEMMA 6 *Let $\mathcal{F}$ be a family of meromorphic functions in a domain D, let the set $S = \{a_1, a_2\}$, where a_1 and a_2 are distinct finite complex numbers, and let M be a positive number. If, for any $f \in \mathcal{F}$, f and f' share the set S, and $0 < |f''(z)| \leq M$ whenever $f(z) \in S$, then $\mathcal{F}$ is normal in D.*

LEMMA 1 (cf. [7,8]) *Let k be a positive integer and let $\mathcal{F}$ be a family of func meromorphic on the unit disc, all of whose zeros have multiplicity at least k, suppose that there exists $A \geq 1$ such that $|f^{(k)}(z)| \leq A$ whenever $f(z) = 0, f \in \mathcal{F}$. Th $\mathcal{F}$ is not normal, there exist, for each $0 \leq \alpha \leq k$,*

(a) *a number* $0 < r < 1$,
(b) *points* z_n, $|z_n| < r$,
(c) *functions* $f_n \in \mathcal{F}$, *and*
(d) *positive numbers* $\rho_n \to 0$

such that

$$\frac{f_n(z_n + \rho_n \zeta)}{\rho_n^{\alpha}} = g_n(\zeta) \to g(\zeta)$$

locally uniformly with respect to the spherical metric, where g is a noncon meromorphic function on $\mathbf{C}$ such that $g^{\sharp}(\zeta) \leq g^{\sharp}(0) = kA + 1$. Moreover, g has ord most two.

THEOREM A *Let f be a nonconstant entire function, and let a be nonzero finite complex number. If f and f' share a CM, and if $f''(z) = a$ whenever $f(z) = a$, then $f \equiv f'$.*

Integration of vector hydrodynamical partial differential equations over octonions

S.V. Ludkovsky

Volume 58, Issue 6, 2013, pages 579-609

PROPOSITION 3.1 *A family $\mathcal{D}_r$ of all differential operators with constant $\mathcal{A}_r$ coefficients is a power associative real algebra with a centre $Z(\mathcal{D}_r)$ consisting of all differential operators with real coefficients and with a unit element I.*

PROPOSITION 3.4 *Let*

$$\lim_{z\to\infty} {}^1\sigma_z^{k\,2}\sigma_x^{s\,2}\sigma_z^n \mathsf{F}(z,y)\mathsf{K}(x,z) = 0 \tag{3.20}$$

for each x, y in a domain U satisfying conditions D1 and D2 (see Section 2.1) with $\infty \in U$ and every non-negative integers $0 \le k, s, n \in \mathbf{Z}$ such that $k+s+n \le m$. Suppose also that ${}_\sigma\int_x^\infty \partial_x^\alpha \partial_y^\beta \partial_z^\omega[\mathsf{F}(z,y)\mathsf{K}(x,z)]\mathrm{d}z$ converges uniformly by parameters x, y on each compact subset $W \subset U \subset \mathcal{A}_r^2$ for each $|\alpha|+|\beta|+|\omega| \le m$, where $\alpha = (\alpha_0, \dots, \alpha_{2^r-1})$, $|\alpha| = \alpha_0 + \dots + \alpha_{2^r-1}$, $\partial_x^\alpha = \partial^{|\alpha|}/\partial x_0^{\alpha_0} \cdots \partial x_{2^r-1}^{\alpha_{2^r-1}}$. Then the non-commutative line integral ${}_\sigma\int_x^\infty \mathsf{F}(z,y)\mathsf{K}(x,z)\mathrm{d}z$ from Section 2 satisfies the identities:

$$\sigma_x^m {}_\sigma\int_x^\infty \mathsf{F}(z,y)\mathsf{K}(x,z)\mathrm{d}z =^2 \sigma_x^m {}_\sigma\int_x^\infty \mathsf{F}(z,y)\mathsf{K}(x,z)\mathrm{d}z + A_m(\mathsf{F},\mathsf{K})(x,y), \tag{3.21}$$

$${}^1\sigma_z^m {}_\sigma\int_x^\infty \mathsf{F}(z,y)\mathsf{K}(x,z)\mathrm{d}z = (-1)^{m\,2}\sigma_z^m {}_\sigma\int_x^\infty \mathsf{F}(z,y)\mathsf{K}(x,z)\mathrm{d}z + B_m(\mathsf{F},\mathsf{K})(x,y), \tag{3.22}$$

where

$$A_m(\mathsf{F},\mathsf{K})(x,y) = -{}^2\sigma_x^{m-1}[\mathsf{F}(x,y)\mathsf{K}(x,z)]|_{z=x} + \sigma_x A_{m-1}(\mathsf{F},\mathsf{K})(x,y) \tag{3.23}$$

for $m \ge 2$,

$$B_m(\mathsf{F},\mathsf{K})(x,y) = (-1)^{m\,2}\sigma_z^{m-1}\mathsf{F}(x,y)\mathsf{K}(x,z)\big|_{z=x} + \left[{}^1\sigma_z B_{m-1}(\mathsf{F}(z,y),\mathsf{K}(x,z))\right]\big|_{z=x} \tag{3.24}$$

for $m \ge 2$,

$$A_1(\mathsf{F},\mathsf{K})(x,y) = B_1(\mathsf{F},\mathsf{K})(x,y) = -\mathsf{F}(x,y)\mathsf{K}(x,x) \tag{3.25}$$

σ_x is an operator σ acting by the variable $x \in U \subset \mathcal{A}_r$.

THEOREM 4.5 *Partial differential Equation* (4.76) *with $\psi_0 = {}_1\psi_0 = 0$ over the Cayley–Dickson algebra $\mathcal{A}_r$, $2 \le r \le 3$, has a solution given by Formulas* (4.38)–(4.40), (4.65), (4.66), *when the appearing integrals uniformly converge by parameters as in Proposition* 3.4 *and the operator* $(I - A_x)$ *is invertible and* $\mathsf{F} \in Mat_s(\mathbf{R})$ *and* $\mathsf{K} \in Mat_s(\mathcal{A}_r)$ *with $s \in \mathbf{N}$ for $r = 2$ and $s = 1$ for $r = 3$.*

Application of the argument principle to Maxwell's Conjecture for three point charges
Ronen Peretz
Volume 58, Issue 6, 2013, pages 715-725

THEOREM (The topological argument principle) *Let $g(Z)$ be a complex valued function defined in a domain $\Omega \subseteq \mathbb{C}$ ($Z = X + iY$). Suppose that $g(Z)$ is continuous non-zero in Ω except on a set $E \subseteq \Omega$ consisting of isolated points $\{a_j\}$ having no accumulation point in Ω. If γ is a zero cycle in Ω and $\gamma \subseteq D = \Omega - E$, then $d(g,\gamma) = \Sigma_j n(\gamma, a_j)m(g, a_j)$.*

$$f(Z) = \sum_{j=1}^{n} \frac{\xi_j (Z - Z_j)}{|Z - Z_j|^p},$$

PROPOSITION 2.1 *Let $f(Z)$ be as above, then the zeros of $f(Z)$ lie in the convex hull of the set $\{Z_1, \ldots, Z_n\}$.*

Proof Let $f(\xi) = 0$, then $\xi = \sum_{j=1}^{n} \alpha_j Z_j$ where

$$\alpha_j = \left(\frac{\xi_j}{|\xi - Z_j|^p} \right) \Big/ \left(\sum_{k=1}^{n} \frac{\xi_k}{|\xi - Z_k|^p} \right), \quad 1 \le j \le n.$$

PROPOSITION 3.1 *Let the function $f(Z)$ be defined as follows:*

$$f(Z) = \sum_{k=1}^{n} \frac{\xi_k (Z - Z_k)}{|Z - Z_k|^p}, \quad n \in \mathbb{Z}^+,\ \xi_k \in \mathbb{R}^\times,\ Z_k \in \mathbb{C},\ 1 \le k \le n,\ p \in \mathbb{R}.$$

Then we have

$$m(f, Z_j) = \begin{cases} 1 & \textit{if } p > 1 \\ 0 & \textit{if } p < 1, \quad \sum_{k \ne j} (\xi_k (Z_j - Z_k))/|Z_j - Z_k|^p \ne 0 \\ 0 & \textit{if } p = 1, \quad \left| \sum_{k \ne j} (\xi_k (Z_j - Z_k))/|Z_j - Z_k|| > |\xi_j \right| \\ 1 & \textit{if } p = 1, \quad \left| \sum_{k \ne j} (\xi_k (Z_j - Z_k))/|Z_j - Z_k|| < |\xi_j \right| \end{cases}.$$

Proposition 3.3 *Let the function* $f(Z)$ *be defined as follows*:

$$f(Z) = \sum_{k=1}^{n} \frac{\xi_k (Z - Z_k)}{|Z - Z_k|^p}, \quad n \in \mathbb{Z}^+,\ \xi_k \in \mathbb{R}^+,\ Z_k \in \mathbb{C},\ 1 \le k \le n,\ p \in \mathbb{R}.$$

Then we have

(1)

$$J(f) = \left(1 - \frac{p}{2}\right)^2 \left(\sum_{j=1}^{n} \frac{\xi_j}{|Z - Z_j|^p} \right)^2 - \left(\frac{p}{2}\right)^2 \left| \sum_{j=1}^{n} \frac{\xi_j (Z - Z_j)^2}{|Z - Z_j|^{p+2}} \right|^2 .$$

(2) *If* $0 < p < 1$, f *is a local-diffeomorphism, orientation preserving* ($J(f) > 0$) *and so for each zero* w_j *of* f, $m(f, w_j) > 0$. *In particular, if* m *is the total number of zeros of* $f(Z)$ *in* $\mathbb{C}$ *and if*

$$\sum_{k \neq j} \frac{\xi_k (Z_j - Z_k)}{|Z_j - Z_k|^p} \neq 0, \quad 1 \le j \le n,$$

then $m \le d(f(Z), \gamma)$.

(3) *If* $1 < p$, $J(f)$ *might be negative, zero or positive.*

(4) *If* $p = 2$ *then* $J(f) \le 0$, *and so* $f(Z)$ *is orientation reversing. So at each zero* w_j *of* f, $m(f, w_j) \le 0$.

Conformal geometry and dynamics

Volume 17, Pages 1–5 (January 9, 2013)

CONFORMAL AUTOMORPHISMS OF COUNTABLY CONNECTED REGIONS

IAN SHORT

Theorem 1.1. *The conformal automorphism group of a countably connecte cular region of connectivity at least three is either a Fuchsian group or a dis elementary group of Möbius transformations. Furthermore, each Fuchsian and discrete elementary group arises as the conformal automorphism group countably connected circular region.*

Corollary 1.2. *Each countably connected region of connectivity at least thr conformally equivalent to a region whose conformal automorphism group is e a Fuchsian group or a discrete elementary group of Möbius transformations.*

Theorem 3.1. *The conformal automorphism group of a countably connected tured sphere of connectivity at least three is a discrete elementary group of M transformations.*

Lemma 4.1. *Let G be a non-elementary subgroup of $\mathcal{M}$. There exists invariant disc if and only if G contains no strictly loxodromic elements.*

Volume 17, Pages 39–46 (February 28, 2013)
COMPACT KLEIN SURFACES OF GENUS 5 WITH A UNIQUE EXTREMAL DISC
GOU NAKAMURA

give all the groups of automorphisms of the other [illegible] surfaces.

Theorem 2.2. *The groups of automorphisms of the non-orientable surfaces of genus 5 with a unique extremal disc are classified as follows:*

(1) D_3 : **803, 2765, 3431, 3509**.
(2) $\mathbb{Z}_3$: **3436, 3486**.

We obtained the following result via the use of a computer.

Theorem 2.1. *There exist 71 trivalent graphs with 8 vertices and 12 edges (Figures 3 and 4). There exist 3627 side-pairing patterns for the regular 24-gon to be a non-orientable extremal surface of genus 5. The surfaces obtained from these side-pairings are not isomorphic to each other.*

Volume 17–118 (June 6, 2013)
BOUNDARY VALUES OF THE THURSTON PULLBACK MAP
RUSSELL LODGE

Theorem. *Let $\frac{p}{q}$ be a reduced fraction. Then under iteration of σ_f, $\frac{p}{q}$ lands either on the two-cycle $\frac{0}{1} \leftrightarrow \frac{1}{0}$ or on the fixed point $-\frac{1}{1}$. More precisely, $\frac{p}{q}$ lands on $-\frac{1}{1}$ if and only if p and q are odd.*

Theorem. *For any $g \in PMCG(\widehat{\mathbb{C}}, P_f)$ there is a positive number N so that $\overline{\psi}^{\circ n}(g)$ $\in \mathfrak{M}$ for all $n > N$, where*

$$\mathfrak{M} = \{e, \beta, \alpha^{-1}, \alpha^2\beta^{-1}, \alpha^{-1}\beta\alpha^{-1}, \alpha\beta^{-1}, \beta^2\} \cup \{\alpha(\beta\alpha)^k : k \in \mathbb{Z}\}.$$

Theorem 2.2. *Let F be a Thurston map not equivalent to a Lattès map. Then F is Thurston equivalent to a rational function if and only if there are no obstructions. If this rational function exists, it is unique up to Möbius conjugation.*

Definition 2.3. The Teichmüller space for a Thurston map F is defined to be

$$\mathscr{T}_F = \{\phi : (S^2, P_F) \longrightarrow \widehat{\mathbb{C}}\}/\sim,$$

where $\phi_1 \sim \phi_2$ if and only if there is a Möbius transformation M so that ϕ_2 is isotopic to $M \circ \phi_1$ rel P_F.

$$\begin{array}{ccc} (S^2, P_F) & \xrightarrow{\tilde{\phi}} & (\widehat{\mathbb{C}}, \tilde{\phi}(P_F)) \\ {\scriptstyle F}\downarrow & & \downarrow{\scriptstyle F_\tau} \\ (S^2, P_F) & \xrightarrow{\phi} & (\widehat{\mathbb{C}}, \phi(P_F)) \end{array}$$

Definition 2.4. The *Thurston pullback map* $\sigma_F : \mathscr{T}_F \longrightarrow \mathscr{T}_F$ is defined by $\sigma_F(\tau) = [\tilde{\phi}]$.

Theorem 2.5. *The action of* $\pi_1(S^2 \setminus P_F, z_0)$ *on* X^* *is the action associated with* $\Phi : \pi_1(S^2 \setminus P_F, z_0) \to \pi_1(S^2 \setminus P_F, z_0) \wr S_d$ *given by*

$$\Phi(\gamma) = \langle\langle \ell_1 \gamma_1 \overline{\ell}_{k_1}, \ell_2 \gamma_2 \overline{\ell}_{k_2}, ..., \ell_d \gamma_d \overline{\ell}_{k_d} \rangle\rangle \rho,$$

where $\gamma_i = F^{-1}(\gamma)[z_i]$, z_i *is the endpoint of* ℓ_i, k_i *is the element of* X *corresponding to* z_i, *and* ρ *is the permutation defined by* $i \mapsto k_i$ *for all* $i \in X$.

Proposition 2.6. *Let* $\Phi : G \to G \wr S_d$ *be a wreath recursion, and let* ϕ *be an associated virtual endomorphism. If* Φ *is contracting, then* $\rho_\phi < 1$. *If the action of* G *is transitive on every level* X^n *and* $\rho_\phi < 1$, *then the wreath recursion* Φ *is contracting.*

Volume 17, Pages 68–76 (May 6, 2013)

CLASSIFICATION OF QUATERNIONIC HYPERBOLIC ISOMETRIES

KRISHNENDU GONGOPADHYAY AND SHIV PARSAD

Theorem 2.1 (see [16, Theorem 1]). *Given a polynomial* $f(x)$ *with real coefficients,*

$$f(x) = a_0 x^n + a_{n-1} x^{n-1} + \cdots + a_n,$$

if the number of the sign changes of the revised sign list of

$$\{\Delta_1(f), \Delta_2(f), \cdots, \Delta_n(f)\}$$

is p, *then the pairs of distinct conjugate imaginary roots of* $f(x)$ *equal* p. *Furthermore, if the number of non-vanishing members of the revised sign list is* q, *then the number of distinct real roots of* $f(x)$ *equals* $q - 2p$.

Theorem 2.2 (see [9, Number of Roots Theorem]). *Let*

$$D_n = (-1)^{\frac{n(n-1)}{2}} a_0^{n-2} n^{-n} \Delta_n.$$

Suppose the roots of $f(x)$ *are distinct. Then the number of real roots of* $f(x)$ *is:*

(1) *if* n *is odd, congruent to* 1 *or* 3 *modulo* 4 *according to whether* $D_n > 0$ *or* $D_n < 0$;

(2) *if* n *is even, congruent to* 0 *or* 2 *modulo* 4 *according to whether* D_n *and the leading coefficient of* $f(x)$ *have the same or opposite signs.*

Theorem 3.1. *Let A be an element in* $\mathrm{Sp}(n,1)$. *Suppose $A_{\mathbb{C}}$ is the corresponding element in* $\mathrm{GL}(2(n+1),\mathbb{C})$. *Let $S_A = \{\Delta_1, \cdots, \Delta_{n+1}\}$ be the discriminant sequence of $g_A(t)$, where $\Delta_{n+1} = \Delta$ is the usual algebraic discriminant of $g_A(t)$. Let D be the discriminant of the minimal polynomial of $A_{\mathbb{C}}$. Then the following holds.*

(1) *A is regular hyperbolic if and only if $\Delta < 0$.*
(2) *A is regular elliptic if and only if $\Delta > 0$.*
(3) *A is semi-regular hyperbolic if and only if $\Delta = 0$ and the number of sign changes of the revised sign list of S_A is exactly one.*
(4) *A is screw hyperbolic if and only if $\Delta = 0$ and $g_A(t)$ has a real root λ such that $|\lambda| > 2$.*
(5) *A is strictly hyperbolic if and only if $g_A(t)$ has a real root λ such that $|\lambda| > 2$ and for all $m \leq n-2$, $g_A^{(m)}(2) = 0$.*
(6) *A is elliptic or parabolic if and only if $\Delta = 0$ and there is no sign change in the number of revised sign list of S_A. Further, A is parabolic if $D = 0$; otherwise it is elliptic. Further, A is simple elliptic if the number of non-vanishing members of the revised sign list is exactly one.*

Duke Mathematical Journal

Volume 165, Pages 2809 - 2895 (1 December 2015)
The geometry of Newton strata in the reduction modulo p of Shimura varieties of PEL type
Paul Hamacher

THEOREM 1.2
Let $\mathcal{M}_G(b,\mu)$ be the underlying reduced subscheme of the Rapoport–Zink space associated to an unramified Rapoport–Zink datum (cf. Definition 4.8).

(1) *The dimension of $\mathcal{M}_G(b,\mu)$ equals*

$$\langle \rho, \mu - \nu_G(b) \rangle - \frac{1}{2}\,\mathrm{def}_G(b). \tag{1.2}$$

(2) *If b is superbasic, then the connected components of $\mathcal{M}_G(b,\mu)$ are projective.*

Volume 164, Pages 235 - 275 (1 February 2015)
Detecting squarefree numbersAndrew
R. Booker, Ghaith A. Hiary, and Jon P. Keating

PROPOSITION 3.2

Let $Y_1, Y_2, \ldots$ be independent random variables such that $\mathbb{P}(Y_j = 1) = \mathbb{P}(Y_j = -1) = \frac{1}{2}$, and put $Y := 2\sum_{p_j \le e^X} \frac{Y_j \log p_j}{\sqrt{p_j}}(1 - \frac{\log p_j}{X})$, where p_j denotes the jth prime number. Then, for each n satisfying $3 \le n < e^X$, we have

$$\mathbb{P}(Y \ge v_n) \ge 2^{-22} \exp\Big(-\frac{30 v_n^2}{c_n}\Big), \qquad \mathbb{P}(Y \ge u_n) \le \exp\Big(-\frac{u_n^2}{32 c_n}\Big),$$

where $v_n := \sum_{p_j \le n} \frac{\log p_j}{\sqrt{p_j}}(1 - \frac{\log p_j}{X})$, $u_n := 4 v_n$, and $c_n := \sum_{n < p_j \le e^X} \frac{\log^2 p_j}{p_j}(1 - \frac{\log p_j}{X})^2$.

Journal of Mathematical Physics

Hölder continuity of the solution map for the Novikov equation
A. Alexandrou Himonas and John Holmes
J. Math. Phys. 54, 061501 (2013)

Theorem 1. *If $s > 3/2$ and $0 \le r < s$, then the data-to-solution map for the NE Cauchy problem* (1.1) and (1.2), *on both the line and the circle, is Hölder continuous on the space H^s equipped with the H^r norm. More precisely, for initial data $u(0)$, $w(0)$ in a ball $B(0, \rho) = \{\varphi \in H^s : \|\varphi\|_{H^s} \le \rho\}$ of H^s, the corresponding NE solutions $u(t)$, $w(t)$ satisfy the inequality*

$$\|u(t) - w(t)\|_{C([0,T];H^r)} \le c\|u(0) - w(0)\|_{H^r}^{\alpha}, \tag{1.9}$$

where the exponent α is given by

$$\alpha = \begin{cases} 1 & \text{if } (s,r) \in A_1 \\ 2(s-1)/(s-r) & \text{if } (s,r) \in A_2 \\ s - r & \text{if } (s,r) \in A_3 \end{cases}, \tag{1.10}$$

and the regions A_1, A_2, and A_3 in the sr-plane are defined by

$$A_1 = \{(s,r) : s > 3/2,\ 0 \le r \le s - 1, r + s \ge 2\},$$

$$A_2 = \{(s,r) : 2 > s > 3/2, 0 \le r \le 2 - s\},$$

$$A_3 = \{(s,r) : s > 3/2, s - 1 \le r \le s\}.$$

The lifespan T and the constant c depend on s, r, and ρ.

Identities from infinite-dimensional symmetries of Herglotz variational functional
Bogdana Georgieva and Theodore Bodurov
J. Math. Phys. 54, 062901 (2013)

Theorem 1. *If $s > 3/2$ and $0 \le r < s$, then the data-to-solution map for the NE Cauchy problem* (1.1) and (1.2), *on both the line and the circle, is Hölder continuous on the space H^s equipped with the H^r norm. More precisely, for initial data $u(0)$, $w(0)$ in a ball $B(0,\rho) = \{\varphi \in H^s : \|\varphi\|_{H^s} \le \rho\}$ of H^s, the corresponding NE solutions $u(t)$, $w(t)$ satisfy the inequality*

$$\|u(t) - w(t)\|_{C([0,T];H^r)} \le c\|u(0) - w(0)\|_{H^r}^{\alpha}, \tag{1.9}$$

where the exponent α is given by

$$\alpha = \begin{cases} 1 & \text{if } (s,r) \in A_1 \\ 2(s-1)/(s-r) & \text{if } (s,r) \in A_2 \\ s-r & \text{if } (s,r) \in A_3 \end{cases}, \tag{1.10}$$

and the regions A_1, A_2, and A_3 in the sr-plane are defined by

$$A_1 = \{(s,r) : s > 3/2,\ 0 \le r \le s-1, r+s \ge 2\},$$

$$A_2 = \{(s,r) : 2 > s > 3/2, 0 \le r \le 2-s\},$$

$$A_3 = \{(s,r) : s > 3/2, s-1 \le r \le s\}.$$

The lifespan T and the constant c depend on s, r, and ρ.

Theorem 3.3. *Let the infinite-dimensional group of transformations (6), which depends on the arbitrary function $p(t) \in C^{r+2}$ and its derivatives $p^{(i)} = d^i p/dt^i$, subject to the conditions $\bar{t} = t$, $\bar{x}^k = x^k$ and $\bar{u} = u$ when $p(t) = p^{(1)}(t) = \ldots = p^{(r)}(t) = 0$, be a symmetry group of the functional z defined by the differential equation (1). Then the identity*

$$\int_\Omega \left(\tilde{U}^i(E\,Q_i) - \tilde{T}(E\,Q_i\,u_t^i) - \tilde{X}^k(E\,Q_i\,u_{x^k}^i)\right)d^n x = 0 \tag{9}$$

Theorem 3.4. *Let the infinite-dimensional group of transformations (7) which depends on the arbitrary function $p(t, x)$ and its derivatives up to some order r, subject to the conditions $\bar{t} = t$, $\bar{x}^k = x^k$, and $\bar{u} = u$ when p and all its derivatives up to order r are zero, be a symmetry group of the functional z defined by the integro-differential equation (1). Then the identity*

$$\tilde{U}^i(E\,Q_i) - \tilde{X}^k(E\,Q_i\,u_{x^k}^i) = 0\,, \qquad i = 1,\ldots,m,\ \ k = 1,\ldots,n \tag{22}$$

holds. Here $\tilde{U}^i$ and $\tilde{X}^k$ are the adjoints of the linear differential operators

$$X^k = \frac{\partial\varphi^k}{\partial p} + \frac{\partial\varphi^k}{\partial p_t}\frac{\partial}{\partial t} + \frac{\partial\varphi^k}{\partial p_{x^j}}\frac{\partial}{\partial x^j} + \ldots + \sum_J \frac{\partial\varphi^k}{\partial p^{(J)}}\frac{\partial^r}{\partial t^{j_0}\partial x^{j_1}\ldots\partial x^{j_r}}, \qquad k = 1,\ldots,n$$

$$U^i = \frac{\partial\psi^i}{\partial p} + \frac{\partial\psi^i}{\partial p_t}\frac{\partial}{\partial t} + \frac{\partial\psi^i}{\partial p_{x^j}}\frac{\partial}{\partial x^j} + \ldots + \sum_J \frac{\partial\psi^i}{\partial p^{(J)}}\frac{\partial^r}{\partial t^{j_0}\partial x^{j_1}\ldots\partial x^{j_r}}, \qquad i = 1,\ldots,m$$

evaluated with $p(t, x)$ and all its partial derivatives up to order r set to zero. Here the multi-index $J = (j_0, j_1, \ldots, j_r)$ is a r-tuple of $j_0 = 0, 1$; $1 \le j_q \le n$ and

$$p^{(J)} = \frac{\partial^r p(t,x)}{\partial t^{j_0}\partial x^{j_1}\ldots\partial x^{j_r}}.$$

Q_i denotes the generalized Euler-Lagrange expressions (11) and E is defined by (12).

Scattering theory for graphs isomorphic to a regular tree at infinity
Yves Colin de Verdière and Françoise Truc
J. Math. Phys. 54, 063502 (2013)

Definition 2.1. Let $q \geq 2$ be a fixed integer. We say that the infinite connected graph Γ is asymptotic to a regular tree of degree $q+1$ *if there exists a finite sub-graph Γ_0 of Γ such that $\Gamma' := \Gamma \backslash \Gamma_0$ is a disjoint union of a finite number of trees $T_l, l = 1, \cdots, L$, rooted at a vertex x_l linked to Γ_0 and so that all vertices of T_l different from x_l are of degree $q+1$. The trees $T_l, l = 1, \cdots, L$, are called the* ends of Γ. *(See Fig. 1.)*

Equivalently, Γ is infinite, has a finite number of cycles and a maximal sub-tree of Γ has all vertices of degree $q+1$ except a finite number of them.

Definition 2.2. We define the edge boundary $(\partial_e \Gamma_0)$ *of Γ_0 as the set of edges of Γ connecting a vertex of Γ_0 to a vertex of Γ', namely, one of the x_l's. We denote by $|x|_{\Gamma_0}$ the combinatorial distance of $x \in V_\Gamma$ to Γ_0.*

In particular, for $l = 1, \cdots, L$, $|x_l|_{\Gamma_0} = 1$.

Proposition 3.1. The map $\tilde{\Lambda} : s \to \lambda_s$ is holomorphic from S to $\mathbb{C}$. It maps bijectivel sheet $S^+ = \{s \in S | \Im s > 0\}$ onto $\mathbb{C} \setminus I_q$. By this map the circle S^0 is a double coverin

$$q^{\frac{1}{2}-is} - q^{-\frac{1}{2}+is}$$

Theorem 3.1. *The spectrum of A_0 is the interval $I_q = [-2\sqrt{q}, +2\sqrt{q}]$. The Green's function of the tree $\mathbb{T}_q$ is given, for $s \in S^+$ by*

$$G_0(\lambda_s, x, y) = C(s) q^{(-\frac{1}{2}+is)d(x,y)} = \frac{q^{(-\frac{1}{2}+is)d(x,y)}}{q^{\frac{1}{2}-is} - q^{-\frac{1}{2}+is}}.$$

As a function of s, the Green's function extends meromorphically to S with two poles $+ \tau/2$.

Moreover, we have, for any $x \in V_q$ and any y belonging to the ray from x_ω to ω,

$$G_0(\lambda_s, x, y) = G_{rad}(\lambda_s, y) q^{(\frac{1}{2}-is) b_\omega(x)}$$

with

$$G_{rad}(\lambda_s, y) = C(s) q^{(-\frac{1}{2}+is)|y|}.$$

Theorem 3.2. *(See, for example, Ref. 3): The spectral measure de_x of $\mathbb{T}_q$ is independent of the vertex x and is given by*

$$de_x(\lambda) := de(\lambda) = \frac{(q+1)\sqrt{4q - \lambda^2}}{2\pi \left((q+1)^2 - \lambda^2\right)} d\lambda. \quad (7)$$

Journal of Theoretical Probability

Representations of the Absolute Value Function and Applications in Gaussian Estimates
Ang Wei
J Theor Probab (2014) 27:1059–1070

Proposition 2.1 *Assume that* $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ *is a non-degenerate Gaussian random vector with covariance matrix* Σ. *When* $\tau \in (0, 2)$, *for a*
symmetric matrix A, *n-dimensional column vector* $\mathbf{b}$, *and constant* c *we h*

$$\mathbb{E}\,|\langle \mathbf{X}, A\mathbf{X}\rangle + \langle \mathbf{b}, \mathbf{X}\rangle + c|^\tau = C_\tau \int_0^\infty t^{-\tau-1}\left(1 - F(t) - \overline{F(t)}\right) dt$$

where

$$C_\tau = \frac{\tau 2^\tau \Gamma(1/2 + \tau/2)}{\sqrt{\pi}\,\Gamma(1 - \tau/2)} \text{ and } F(t) = \frac{\exp\left(itc - \frac{1}{2}t^2\langle \mathbf{b}, \left(\Sigma^{-1} - 2itA\right)\right.}{2[\det(I - 2it\Sigma A)]^{1/2}}$$

Theorem 2.2 *Suppose* $E = (X_{j,k})$ *is a symmetric Gaussian matrix, where* $X_{j,k} = X_{k,j} \sim N(0, \sigma_{j,k}^2)$ *and they are independent for any* $1 \le j \le k \le n$. *Then for* $\tau \in (0, 2)$,

$$\mathbb{E}\,|\det(E)|^\tau \ge c_\tau^n \left(\frac{\pi \Gamma(\tau+1)}{2^{\tau+1}\Gamma^2(\frac{1}{2} + \frac{\tau}{2})} \prod_{j=1}^n \sigma_{j,j}^\tau + 2^{\tau-1} \prod_{1 \le j < k \le n} \sigma_{j,k}^{4\tau/(n^2-n)} \prod_{j=1}^n \sigma_{j,j}^{(1-2/n)\tau} \right), (5)$$

where $c_\tau = 2^{\tau/2}\pi^{-1/2}\Gamma(1/2 + \tau/2)$ *is the value of the* τ*-th absolute moment of a standard Gaussian random variable.*

Journal of Number Theory

Prime polynomial values of linear functions in short intervals
Efrat Bank, Lior Bary-Soroker
Journal of Number Theory 151 (2015) 263–275

$$\mathbb{1}(h) = \begin{cases} 1, & h \text{ is prime} \\ 0, & \text{otherwise.} \end{cases}$$

Theorem 1.1. *Let $B > 0$ and $1 > \epsilon > 0$ be fixed real numbers. Then the asymptotic formula*

$$\sum_{f \in I(f_0,\epsilon)} \mathbb{1}(L_1(f)) \cdots \mathbb{1}(L_n(f)) = \frac{\#I(f_0,\epsilon)}{\prod_{i=1}^{n} \deg(L_i(f_0))}(1 + O_B(q^{-1/2}))$$

holds uniformly for all odd prime powers q, $1 \le n \le B$, distinct primitive linear functions $L_1(X), \ldots, L_n(X)$ defined over $\mathbb{F}_q[t]$ each of height at most B, and monic $f_0 \in \mathbb{F}_q[t]$ of degree in the interval $B \ge \deg f_0 \ge \frac{2}{\epsilon}$.

Theorem 3.2. *(See [1, Theorem 3.1].) Let $A = (A_0, \ldots, A_m)$ be an $(m+1)$-tuple of variables over $\mathbb{F}_q$, let $\mathcal{F}(t) \in \mathbb{F}_q[A][t]$ be monic and separable in t, let L be a splitting field of $\mathcal{F}$ over $K = \mathbb{F}_q(A)$, and let $G = \mathrm{Gal}(\mathcal{F}, K) = \mathrm{Gal}(L/K)$. Assume that $\mathbb{F}_q$ is algebraically closed in L. Then there exists a constant $c = c(m, \mathrm{tot.deg}(\mathcal{F}))$ such that for every conjugacy class $C \subseteq G$ we have*

$$\left| \#\{\boldsymbol{a} \in \mathbb{F}_q^{m+1} : \mathrm{Fr}_{\boldsymbol{a}} = C\} - \frac{|C|}{|G|} q^{m+1} \right| \le c q^{m+1/2}.$$

Decomposition of products of Riemann zeta values
Chan-Liang Chung, Minking Eie,, Wen-Chin Liaw, Yao Lin Ong
Journal of Number Theory 150 (2015) 1–20

Main Theorem. *Suppose that S_n is the symmetric group of n objects,*

$$x_1(x_1 + x_2) \cdots (x_1 + x_2 + \cdots + x_n) = \sum_{|\boldsymbol{b}|=n} m_{\boldsymbol{b}} \boldsymbol{x}^{\boldsymbol{b}} = \sum_{|\boldsymbol{b}|=n} m_{\boldsymbol{b}} x_1^{b_0} x_2^{b_1} \cdots x_n^{b_{n-1}},$$

and

$$T_n = \left\{ \boldsymbol{b} = (b_0, b_1, \ldots, b_{n-1}) \;\middle|\; m_{\boldsymbol{b}} > 0 \text{ in the above product} \right\}.$$

Then for an n-tuple $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ *of nonnegative integers with* $|\mathbf{d}| = k$*, we have*

$$\zeta(d_1+2)\zeta(d_2+2)\cdots\zeta(d_n+2) = \sum_{\sigma\in S_n}\sum_{|\mathbf{a}|=k}\sum_{\mathbf{b}\in T_n} I(\mathbf{a},\mathbf{b}) m_{\mathbf{b}}\, \mathbf{b}!\, \sigma_{\mathbf{d}}\left\{\binom{g_n}{d_n}\binom{g_{n-1}}{d_{n-1}}\cdots\binom{g_2}{d_2}\right\}$$

with

$$g_j = a_j + \sum_{i=j+1}^{n}(a_i - d_i), \quad j = 1, 2, \ldots, n,$$

and

$$I(\mathbf{a},\mathbf{b}) = \sum_{\substack{|\boldsymbol{\alpha}_j|=a_j+b_j+1\\ 1\le j\le n-1}} \zeta\big(\{1\}^{b_0-1}, \alpha_{1,0}+1, \alpha_{1,1}, \ldots, \alpha_{1,b_1}+\alpha_{2,0}, \ldots, \alpha_{n-1,b_{n-1}}+a_n+1\big).$$

Journal of Fourier Analysis and Applications

Poisson Wavelets on *n*-Dimensional Spheres
Ilona Iglewska-Nowak
J Fourier Anal Appl (2015) 21:206–227

Proposition 4.1 *Poisson wavelets* g_ρ^m, $m \in \mathbb{N}$, *can be uniquely harmonically continued to functions over* $\mathbb{R}^{n+1} \setminus \{r\hat{e}\}$. *They are given by*

$$g_\rho^m(x) = \frac{\rho^m}{\Sigma_n}\sum_{l=0}^{m+1} l!\left(\alpha_l^m + \frac{\alpha_l^{m+1}}{\lambda}\right)e^{-\rho l}\,\frac{C_l^\lambda(\cos\chi)}{|x-r\hat{e}|^{l+2\lambda}}, \tag{9}$$

where $r = e^{-\rho}$,

$$\cos\chi = \frac{x - r\hat{e}}{|x-r\hat{e}|}\cdot\hat{e}$$

and the coefficients α_l^m *are recursively given by*

$$\begin{aligned} \alpha_0^0 &= 1, \\ \alpha_0^m &= 0 \quad \text{for } m \ge 1, \\ \alpha_m^l &= 0 \quad \text{for } l > m, \\ \alpha_l^{m+1} &= l\alpha_l^m + \alpha_{l-1}^m. \end{aligned}$$

A Note on Spaces of Absolutely Convergent Fourier Transforms
Björn G. Walther
J Fourier Anal Appl (2014) 20:1328–1337

Theorem 3.1 *Let $\mathcal{Y}$ be a closed subspace of $\overline{\mathcal{C}_0}(\mathbf{R}^n)$ such that $\mathcal{Y}$ is a subset of $\left[\mathcal{F}L^1\right](\mathbf{R}^n)$. If $\mathcal{Y}$ is reflexive then it is of finite dimension.*

Growth and Integrability of Fourier Transforms on Euclidean Space
William O. Bray
J Fourier Anal Appl (2014) 20:1234–1256

Theorem 1.1 *Let $1 \le p \le 2$. Then there is a constant $c_p > 0$ such that for all $f \in L^p(\mathbb{R})$,*

- *when $p = 1$,*

$$\sup_\lambda \left[\min\{1, (\lambda t)^2\}|\widehat{f}(\lambda)|\right] \le c_1 \Omega_1[f](t);$$

- *when $1 < p \le 2$,*

$$\left[\int_{\mathbb{R}} \min\{1, (\lambda t)^{2p'}\}|\widehat{f}(\lambda)|^{p'} d\lambda\right]^{1/p'} \le c_p \Omega_p[f](t).$$

Theorem 1.1 *Let $1 \le p \le 2$. Then there is a constant $c_p > 0$ such that for all $f \in L^p(\mathbb{R})$,*

- *when $p = 1$,*

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- *when $1 < p \le 2$,*

$$\left[\int_{\mathbb{R}} \min\{1, (\lambda t)^{2p'}\}|\widehat{f}(\lambda)|^{p'} d\lambda\right]^{1/p'} \le c_p \Omega_p[f](t).$$

Letters in mathematical physics

Lett Math Phys (2013) 103:843–849
On the Maximal Excess Charge
of the Chandrasekhar–Coulomb Hamiltonian in Two Dimension
MICHAEL HANDREK and HEINZ SIEDENTOP

THEOREM 1. *Assume $d \ge 2, \mathfrak{A} \in L^2_{\mathrm{loc}}(\mathbb{R}^d : \mathbb{R}^d)$, $\mathbf{p} := -\mathrm{i}\nabla$, and $T_{\mathfrak{A}} := |\mathbf{p} + \mathfrak{A}|$, then*

$$|\mathbf{x}|T_{\mathfrak{A}} + T_{\mathfrak{A}}|\mathbf{x}| \ge 0 \tag{8}$$

on $C_0^\infty(\mathbb{R}^d)$.

THEOREM 2. *Assume* $L^\infty(\mathbb{R}^2 : \mathbb{R}^2)$ *and* $|\mathbf{A}(\mathbf{x})| \leq e\delta/|\mathbf{x}|, \varphi(\mathbf{x}) \leq eZ/|\mathbf{x}|, e^2 Z \in [0, 4\pi^2/\Gamma(1/4)^4]$. *Assume that* $C_{\mathbf{A},\varphi,N}$ *has a ground state with ground state energy* E_N *below the saturation threshold, i.e.,* $E_N < E_{N-1}$. *Then*

$$N < 2(\delta + Z) + 1.$$

Lett Math Phys (2013) 103:865–879
Quantizing the Discrete Painlevé VI Equation: The Lax Formalism
KOJI HASEGAWA

PROPOSITION 1. *We have*

$$L_z^+(\Delta^+) = \frac{(q^4 z^{-1}\Delta^+, q^4)_\infty}{(q^2 z^{-1}\Delta^+, q^4)_\infty} \begin{bmatrix} 1 & \frac{1}{z}E_0^+ \\ E_1^+ & 1 \end{bmatrix} \begin{bmatrix} k^{-\frac{1}{2}} & 0 \\ 0 & k^{\frac{1}{2}} \end{bmatrix} q^{-c^+ d}, \quad (10)$$

$$L_z^-(\Delta^-) = \frac{(q^4 z\Delta^-, q^4)_\infty}{(q^2 z\Delta^-, q^4)_\infty} \begin{bmatrix} 1 & E_1^- \\ zE_0^- & 1 \end{bmatrix} \begin{bmatrix} k^{-\frac{1}{2}} & 0 \\ 0 & k^{\frac{1}{2}} \end{bmatrix} q^{-c^- d}, \quad (11)$$

where we used the standard notation for the infinite product: $(x, Q)_\infty := \prod_{n=0}^\infty (1 - xQ^n)$.

PROPOSITION 2.

$$L_z^+(\Delta^+)R(\Delta^+, \Delta^-)L_z^-(\Delta^-) = L_z^-(\Delta^-)R(\Delta^+, \Delta^-)L_z^+(\Delta^+). \quad (16)$$

THEOREM 1. *We have*

$$\mathcal{T}(w_0(1^+1^-)) = \frac{w_0(1^+0^-) - q\Delta(1^+0^-)}{w_0(1^+0^-) - q} \cdot \frac{w_0(2^+1^-) - q\Delta(2^+1^-)}{w_0(2^+1^-) - q} w_0(2^+0^-)^{-1}, \quad (21)$$

$$\mathcal{T}^{-1}(w_0(2^+1^-)) = \frac{w_0(1^+1^-) - q^{-1}\Delta(1^+1^-)}{w_0(1^+1^-) - q^{-1}} \cdot \frac{w_0(2^+2^-) - q^{-1}\Delta(2^+2^-)}{w_0(2^+2^-) - q^{-1}} w_0(1^+2^-)^{-1}. \quad (22)$$

Mathematics in Computer Science

Antimagicness of Generalized Corona and Snowflake Graphs
Jacqueline W. Daykin · Costas S. Iliopoulos · Mirka Miller · Oudone Phanalasy
Math.Comput.Sci. (2015) 9:105–111

Theorem 3.1 *Let G be a connected or disconnected graph with p vertices. Then the sequential gener graph* $seqG \odot \mathcal{H}$ *is antimagic.*

Theorem 3.3 *Let G be a connected or disconnected graph with p vertices and let H_j, $1 \le j \le m$, for $m \ge 2$, be the connected or disconnected k_j-regular graphs with n_j vertices such that for $m = 2$, $\delta(G) + n_2 > k_2 + n_1$, and for $m \ge 3$, $\delta(G) + n_m > k_j + n_{j-1} \ge k_h + n_{h-1}$, $2 \le h < j \le m$. Then the generalized snowflake graph $Sf(H_1, H_2, \ldots, H_m, G)$ is antimagic.*

Probability Theory and Related Fields

Sequential complexities and uniform martingale laws of large numbers
Alexander Rakhlin · Karthik Sridharan · Ambuj Tewari
Probab. Theory Relat. Fields (2015) 161:111–153

Theorem 1 *Let $\mathcal{F}$ be a class of $[-1, 1]$-valued functions. Then the following statements are equivalent.*

1. *$\mathcal{F}$ satisfies Sequential Uniform Convergence.*
2. *For any $\alpha > 0$, the sequential fat-shattering dimension $\mathrm{fat}_\alpha(\mathcal{F})$ is finite.*
3. *Sequential Rademacher complexity $\mathfrak{R}_n(\mathcal{F})$ satisfies $\lim_{n\to\infty} \mathfrak{R}_n(\mathcal{F}) = 0$.*

Theorem 2 *The following relation holds between the empirical process with dependent random variables and the sequential Rademacher complexity:*

$$\mathbb{E} \sup_{f\in\mathcal{F}} \mathbb{M}_n(f) \le 2\,\mathfrak{R}_n(\mathcal{F}). \tag{5}$$

Furthermore, this bound is tight, as we have

$$\frac{1}{2}\left(\mathfrak{R}_n(\mathcal{F}) - \frac{B}{2\sqrt{n}}\right) \le \sup_{\mathbb{P}} \mathbb{E} \sup_{f\in\mathcal{F}} \mathbb{M}_n(f) \tag{6}$$

where $B = \inf_{z\in\mathcal{Z}} \sup_{f,f'\in\mathcal{F}}(f(z) - f'(z)) \ge 0$.

Proceedings of the American Mathematical Society

A GENERAL FORM OF GREEN'S FORMULA AND THE CAUCHY INTEGRAL THEOREM
JULIACÚFI AND JOAN VERDERA
Volume 143, Number 5, May 2015, Pages 2091–2102

$$\mathrm{Ind}(\gamma, z) = \frac{1}{2\pi i}\int_\gamma \frac{dw}{w - z}.$$

Set

$$D = \{z \in \mathbb{C} : \mathrm{Ind}(\gamma, z) \neq 0\}$$

and

$$D_0 = \{z \in \mathbb{C} : \mathrm{Ind}(\gamma, z) = 0\}.$$

Theorem. *Let γ be a closed rectifiable curve and let f be a continuous function on $D \cup \gamma$ such that the $\bar{\partial}$ derivative of f in D, in the sense of distributions, belongs to $L^2(D)$. Then*

$$\int_\gamma f(z)\,dz = 2i \int_D \bar{\partial} f(z)\,\mathrm{Ind}(\gamma, z)\,dA(z). \tag{1}$$

Theorem (pointwise version). *Let γ be a closed rectifiable curve. Let f be a continuous function on $D \cup \gamma$ whose partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ exist at each point of $D \setminus E$, where E is a countable union of closed sets of finite length (one dimensional Hausdorff measure), and such that $\bar{\partial} f \in L^2(D)$, where $\bar{\partial} f$ is defined pointwise almost everywhere on D. Then*

$$\int_\gamma f(z)\,dz = 2i \int_D \bar{\partial} f(z)\,\mathrm{Ind}(\gamma, z)\,dA(z).$$

THE GRAPHIC NATURE OF GAUSSIAN PERIODS
WILLIAM DUKE, STEPHAN RAMON GARCIA, AND BOB LUTZ
Volume 143, Number 5, May 2015, Pages 1849–1863

Theorem 2.1. *Suppose that $\sigma_{\langle\omega\rangle r}$ is a cyclic supercharacter on $\mathbb{Z}/mn\mathbb{Z}$, where $(m,n) = 1$, and let $\rho : \mathbb{Z}/mn\mathbb{Z} \to \mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$ be the natural isomorphism. If $\rho(\omega) = (\omega_m, \omega_n)$, $\rho(r) = (r_m, r_n)$, and $a, b \in \mathbb{Z}$ with $mb + na = 1$, then for all $y \in \mathbb{Z}/mn\mathbb{Z}$ we have*

$$\sigma_{\langle\omega\rangle r}(y) = \sigma_{\langle\omega_m\rangle r_m}(ay)\sigma_{\langle\omega_n\rangle r_n}(by).$$

Theorem 6.3. *Let σ_X be a cyclic supercharacter of $\mathbb{Z}/q\mathbb{Z}$, where $q = $ 1 nonzero power of an odd prime. If $X = A1$ and $|X| = d$ divides $p - 1$, tl image of σ_X is contained in the image of the function $g : \mathbb{T}^{\varphi(d)} \to \mathbb{C}$ definec*

$$g(z_1, z_2, \ldots, z_{\varphi(d)}) = \sum_{k=0}^{d-1} \prod_{j=0}^{\varphi(d)-1} z_{j+1}^{b_{k,j}} \tag{6}$$

where the integers $b_{k,j}$ are given by

$$t^k \equiv \sum_{j=0}^{\varphi(d)-1} b_{k,j} t^j \pmod{\Phi_d(t)}. \tag{7}$$

For a fixed d, as q becomes large, the image of σ_X fills out the image of g, in the sense that, given $\epsilon > 0$, there exists some $q \equiv 1 \pmod d$ such that if $\sigma_X : \mathbb{Z}/q\mathbb{Z} \to \mathbb{C}$ is a cyclic supercharacter with $|X| = d$, then every open ball of radius $\epsilon > 0$ in the image of g has nonempty intersection with the image of σ_X.

Proceedings London Mathematical Society

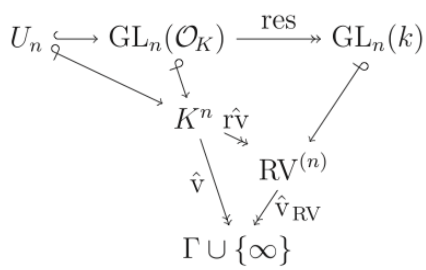
Non-Archimedean Whitney stratifications

Immanuel Halupczok
Proc. London Math. Soc. (3) 109 (2014) 1304–1362

Theorem 1.1. *For every set $X \subseteq K^n$ in the class $\mathcal{C}$, there exists a 't-stratifica reflecting X', that is, a partition $(S_i)_{0 \leqslant i \leqslant n}$ of K^n with $S_i \in \mathcal{C}$ such that for each have the following:*

(i) $\dim S_d = d$ or $S_d = \emptyset$;
(ii) *for any ball $B \subseteq S_d \cup \cdots \cup S_n$, the family $(S_d, \ldots, S_n, X)$ is d-translatable o*

Lemma 2.8. *Let U_n be the kernel of the map $\mathrm{res}\colon \mathrm{GL}_n(\mathcal{O}_K) \twoheadrightarrow \mathrm{GL}_n(k)$. Then we have the following (commutative) diagrams, where $G \curvearrowright X$ means that G acts on X, and each straight line $G \curvearrowright X \twoheadrightarrow Y$ is exact in the sense that Y is the quotient of X by the action of G.*



$$U_n \curvearrowright \mathbb{G}_{n,d}(\mathcal{O}_K) \overset{\mathrm{res}}{\twoheadrightarrow} \mathbb{G}_{n,d}(k)$$

Quarterly Journal of Mathematics

ERROR TERM IMPROVEMENTS FOR VAN DER CORPUT TRANSFORMS
JOSEPH VANDEHEY
Quart. J. Math. 65 (2014), 1461–1502; doi:10.1093/qmath/hat040

Theorem 1.1 [14, Lemma 5.5.3] *Suppose that $f(x)$ is real and four times continuously differentiable on $[a, b]$. Suppose that there are positive parameters M and T, with $M \geq b - a$, such that, for $x \in [a, b]$, we have*

$$f''(x) \asymp T/M^2, \quad f^{(3)}(x) \ll T/M^3 \quad \text{and} \quad f^{(4)}(x) \ll T/M^4.$$

Let $g(x)$ be a real function, and let V be the bounded variation of g on $[a, b]$ plus $g(a)$. Then

$$\sum_{a \leq n \leq b} g(n) e(f(n)) = \sum_{f'(a) \leq r \leq f'(b)} \frac{g(x_r) e(f(x_r) - r x_r + 1/8)}{\sqrt{f''(x_r)}} + O\left(V\left(\frac{M}{\sqrt{T}} + \log(f'(b) - f'(a) + 2)\right)\right),$$

where x_r is the unique solution in $[a, b]$ to $f'(x_r) = r$. The implicit constant in the big-O term depends on the implicit constants in the relations between T, M and the derivatives of $f(x)$.

THEOREM 1.3 *Suppose that $f(x)$ and $g(x)$ are real-valued functions with $f \in C^5[a,b]$ $C^2[a,b]$. Suppose that there are positive constants M, T, U and c, with $M \geq b-a, c \geq 1$, $T \leq cM^3$, such that, for $x \in [a,b]$,*

$$f''(x) \geq c^{-1}TM^{-2}, \quad |f^{(r)}(x)| \leq cTM^{-r} \text{ for } r = 2,3,4,5,$$
$$|g^{(r)}(x)| \leq cUM^{-r} \quad \text{for } r = 0,1,2.$$

Then,

$$\sum_{a\leq n\leq b}^{*} g(n)e(f(n)) = \sum_{r=\lfloor f'(a)\rfloor+1}^{\lfloor f'(b)\rfloor} \frac{g(x_r)e(f(x_r)-rx_r+1/8)}{\sqrt{f''(x_r)}} + \mathcal{R}(b) - \mathcal{R}(a) + O(U),$$

where

$$R(\mu) = g(\mu)e\left(f(\mu)+\frac{1}{4}\right)\int_0^\infty \frac{\sinh(2\pi s(f'(\mu))z)}{\sinh(\pi z)} e\left(-\frac{f''(\mu)}{2}z^2\right) \mathrm{d}z,$$

$s(\cdot)$ is the sawtooth function, and the implicit constant depends only on c.

A RECURSION FORMULA FOR MOMENTS OF DERIVATIVES OF RANDOM MATRIX POLYNOMIALS

S. ALˈI ALTUG

Quart. J. Math. 65 (2014), 1111–1125; doi:10.1093/qmath/hat054

$$g_m(u) = \frac{1}{2\pi \mathrm{i}} \oint_{|w|=1} \frac{\mathrm{e}^{w+u/w^2}}{w^{m+1}} \mathrm{d}w$$
$$= \frac{1}{\Gamma(m+1)} {}_0F_2\left(\frac{m}{2}+1, \frac{m+1}{2}; \frac{u}{4}\right),$$

$$M_k(\mathrm{G}(2N), m) := \int_{\mathrm{G}(2N)} \left(\Lambda_A^{(m)}(1)\right)^k \mathrm{d}A,$$

$$\mathcal{T}_{k,\ell}(u) := \det_{k\times k}(g_{2\mathrm{i}-j+\ell}(u)),$$

THEOREM 1.1 *We have*

$$M_k(\mathrm{USp}(2N),2) = b_k(\mathrm{USp}(2N),2)\cdot(2N)^{(k^2+5k)/2} + O(N^{(k^2+3k)/2}),$$

where

$$b_k(\mathrm{USp}(2N),2) = 2^{-(k^2+5k)/2}\frac{d^k}{\mathrm{d}u^k}(\mathrm{e}^u\mathcal{T}_{k,0}(2u))|_{u=0}.$$

The Ramanujan Journal

Ramanujan J (2013) 31:53–66 DOI
Generalized hypergeometric functions: product identities and weighted norm inequalities
Arcadii Z. Grinshpan

Theorem 2 *Given* $\alpha, \beta > 0$, *let* $f(z) = a_0 + a_1 z + \cdots$ *and* $g(z) = b_0 + b_1 z + \cdots$ *be power series such that convolutions* $f_{*\alpha}(z)$ *and* $g_{*\beta}(z)$ *are analytic in a disk* $D_r = \{z : |z| < r\}$. *Then for any* $\lambda > 0$, *real* x, $p > 1$ $(1/p + 1/q = 1)$, $\tau \in (0, \min(p, q)]$, *and nonzero complex* $\zeta \in D_r$, *the following inequality holds*:

$$\left\|(fg)_{*(\alpha+\beta)}(\zeta t)\right\|_{[\tau;\alpha+\beta,\lambda,\tau x]} \leq \left\|f_{*\alpha}(\zeta t)\right\|_{[p;\alpha,\beta+\lambda,px]}\cdot\left\|g_{*\beta}(\zeta t)\right\|_{[q;\beta,\alpha+\lambda,qx]}. \tag{29}$$

The equality in (29), *provided that* f *and* g *are not identically* 0, *holds if and only if*

$$f(z) = f(0)\left[1 + (x + i\theta)z/\zeta\right]^{-\alpha} \quad \textit{and} \quad g(z) = g(0)\left[1 + (x + i\theta)z/\zeta\right]^{-\beta}$$

(θ *is a real number*).

Theorem 1 [14] *Let $\phi(x)$ and $\psi(x)$ be complex-valued continuous functions on $[0,1]$. Then for any numbers $\alpha,\beta,\lambda>0$, $p>1$ $(1/p+1/q=1)$, and $\tau\in(0,\min(p,q)]$, the following inequality holds:*

$$\left[\int_0^1 x^{\alpha+\beta-1}(1-x)^{\lambda-1}\left|\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\phi(xt)\psi\big(x(1-t)\big)\,dt\right|^{\tau}dx\right]^{1/\tau}$$
$$\le K\left[\int_0^1 x^{\alpha-1}(1-x)^{\beta+\lambda-1}|\phi(x)|^p\,dx\right]^{1/p}$$
$$\times\left[\int_0^1 x^{\beta-1}(1-x)^{\alpha+\lambda-1}|\psi(x)|^q\,dx\right]^{1/q}, \qquad (27)$$

where

$$K=\big[\Gamma(\lambda)\big]^{1/\tau}\cdot\left[\frac{\Gamma(\alpha)}{\Gamma(\alpha+\lambda)}\right]^{1/q}\cdot\left[\frac{\Gamma(\beta)}{\Gamma(\beta+\lambda)}\right]^{1/p}\cdot\left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+\lambda)}\right]^{1/\tau-1}.$$

The equality in (27), provided that ϕ and ψ are not identically 0, holds if and only if $\phi(x)=\phi(0)e^{i\theta x}$ and $\psi(x)=\psi(0)e^{i\theta x}$ for $x\in[0,1]$ (θ is real).

Lemma 2 [12, 13] *Let $f_{*\alpha}(z)$ and $g_{*\beta}(z)$ be analytic in a disk $D_r=\{z:|z|<r\}$, where f and g are some power series and $\alpha,\beta>0$. Then the $(\alpha+\beta)$-convolution $(fg)_{*(\alpha+\beta)}(z)$ is analytic in D_r and the integral formula*

$$B(\alpha,\beta)(fg)_{*(\alpha+\beta)}(z)=\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}f_{*\alpha}(zt)g_{*\beta}\big(z(1-t)\big)\,dt \qquad (5)$$

holds for any $z\in D_r$.

Lemma 1 *(i) Let $f(z)={}_jF_k(\omega z)$. Then $f_{*\alpha}(z)={}_jF_{k+1}(\omega z)$, where the additional parameter equals α.*

(ii) Let $F(z)={}_jF_k(\omega z^2)$. Then $F_{*\alpha}(z)={}_jF_{k+2}(\omega z^2/4)$, where the additional parameters are equal to $\alpha/2$ and $(\alpha+1)/2$.

(iii) Let $F(z)=z\,{}_jF_k(\omega z^2)$. Then $F_{*\alpha}(z)=z\,{}_jF_{k+2}(\omega z^2/4)/\alpha$, where the additional parameters are equal to $(\alpha+1)/2$ and $(\alpha+2)/2$.

Ramanujan J (2013) 30:399–402
Pollaczek polynomials and hypergeometric representation
Jamel Benameur · Mongi Blel

Proposition 1 (Pfaff's transformation formula) *Let $b, c, z \in \mathbb{C}$ such that $z \neq 1$, c is not a negative integer. Then*

$$(1-z)^n {}_2F_1\left(\begin{matrix}-n, b \\ c\end{matrix}; \frac{-z}{1-z}\right) = {}_2F_1\left(\begin{matrix}-n, c-b \\ c\end{matrix}; z\right).$$

Theorem 1 *Let $\Phi(\theta) = \frac{a\cos\theta + b}{\sin\theta}$ and*

$$F(\theta) = e^{in\theta}\frac{(\lambda - i\Phi(\theta))_n}{n!}{}_2F_1\left(\begin{matrix}-n, \lambda + i\Phi(\theta) \\ -\lambda + i\Phi(\theta) - n + 1\end{matrix}; e^{-2i\theta}\right).$$

Then, F is a polynomial of degree n of $\cos(\theta)$ and $F(\theta) = P_n^\lambda(\cos(\theta), a, b)$.

Transactions of the American Mathematical Society

Multiplicity on a Richardson variety in a cominuscule G/P
Michaël Balan.
Trans. Amer. Math. Soc. 365 (2013), 3971-3986

Theorem 0.1. *Assume P is cominuscule. Let $m \in X_w^v$ be arbitrary, ar by μ_w (resp. μ^v, μ_w^v) the multiplicity of m on X_w (resp. X^v, X_w^v). Then*

$$\mu_w^v = \mu_w\ \mu^v. \tag{1}$$

Lemma 1.1. *Let $\beta \in R$, and $\tau \in W^P$. Then U_β fixes e_τ if and only if $-\beta \notin \tau(R^+ \setminus R_P^+)$.*

Lemma 1.2. *The Schubert cell C_τ is the affine subspace of $\mathcal{O}_\tau$ defined by vanishing of the coordinates $x_{-\beta}$ with $\beta \in R^+$.*

Proposition 1.10. *Assume Y^v is a cone over m. Let μ_w (resp. μ^v, μ_w^v) be the multiplicity of m on X_w (resp. X^v, X_w^v). Then*

$$\mu_w^v = \mu_w\ \mu^v. \tag{4}$$

Proposition 2.1.

(a) $\deg Z_w^v = \deg Z_w \deg Z^v$.
(b) Z_w^v *is not a cone over* m.
(c) $\deg(p_m)_{|Z_w^v} = \deg(p_m)_{|Z^v}$.
(d) $\deg(p_m Z_w^v) = \deg Z_w \deg(p_m Z^v)$.

The automorphism group of a simple $\mathcal{Z}$-stable C^{*}-algebra
Ping Wong Ng and Efren Ruiz.
Trans. Amer. Math. Soc. 365 (2013), 4081-4120

Lemma 2.2. *Let $\mathfrak{A}$ be a separable, simple, unital C^*-algebra and let $\mathfrak{C}$ be a UHF algebra. Then $\mathfrak{A} \otimes \mathfrak{C}$ is $\mathcal{Z}$-stable and hence either purely infinite or stably finite. Moreover, if $\mathfrak{A} \otimes \mathfrak{C}$ is (stably) finite, then it has the following properties:*

(1) *Stable rank one.*
(2) *Cancellation of projections.*
(3) *Strict comparison of positive elements when $\mathfrak{A}$ is, additionally, exact.*
(4) *Weak unperforation.*
(5) *K_1-injectivity.*
(6) *The (SP) property.*
(7) *For every nonzero projection $p \in \mathfrak{A}\otimes\mathfrak{C}$, for every $n \geq 2$, $p(\mathfrak{A}\otimes\mathfrak{C})p$ contains a unital sub-C^*-algebra which is isomorphic to $\mathsf{M}_n \oplus \mathsf{M}_{n+1}$.*
(8) *If p, q are nonzero projections in $\mathfrak{A}\otimes\mathfrak{C}$, then there exist nonzero projections p', q' in $p(\mathfrak{A} \otimes \mathfrak{C})p$ and $q(\mathfrak{A} \otimes \mathfrak{C})q$, respectively, such that $p' \sim q'$.*

Lemma 2.7. *Consider the supernatural numbers $\mathfrak{p} = 2^\infty$ and $\mathfrak{q} = 3^\infty$. Let $\mathfrak{A}$ be a simple unital C^*-algebra. Let G be a closed normal subgroup of $U(\mathfrak{A} \otimes \mathcal{Z}_{\mathfrak{p},\mathfrak{q}})_0$ that contains $CU(1_{\mathfrak{A}} \otimes \mathcal{Z}_{\mathfrak{p},\mathfrak{q}})_0$ and let $u_i, v_i : [0,1] \to U(\mathfrak{A} \otimes 1_{\mathcal{Z}_{\mathfrak{p},\mathfrak{q}}})_0$ $(1 \leq i \leq n)$ be norm-continuous paths. Define w by*

$$w = \prod_{i=1}^{n} (u_i, v_i) = (u_1, v_1)(u_2, v_2) \cdots (u_n, v_n).$$

Note that $w \in CU(C[0,1] \otimes \mathfrak{A} \otimes 1_{\mathsf{M}_\mathfrak{p} \otimes \mathsf{M}_\mathfrak{q}})_0 \subseteq CU(\mathfrak{A} \otimes \mathcal{Z}_{\mathfrak{p},\mathfrak{q}})_0$. If $w(0) = 1$, then $w \in G$.

Theorem 2.20. *Let $\mathfrak{A}$ be an exact, separable, simple, unital $\mathcal{Z}$-stable C^*-alg Suppose that either*

(1) *$\mathfrak{A}$ is nuclear and quasidiagonal or*
(2) *$\mathfrak{A}$ has unique tracial state.*

Then we have the following:

(a) *$CU(\mathfrak{A})_0/\mathbb{T}$ is a simple topological group.*
(b) *Every automorphism in $\overline{\mathrm{Inn}_0}(\mathfrak{A})$ can be realized using unitaries in CU*
(c) *$\overline{\mathrm{Inn}_0}(\mathfrak{A})$ is a simple topological group.*

String connections and Chern-Simons theory

Konrad Waldorf.
Trans. Amer. Math. Soc. 365 (2013), 4393-4432

Definition 2.1 ([27, Definition 6.4.2]). Let P be a principal $\mathrm{Spin}(n)$-bundle over M. A *string class* on P is a class $\xi \in \mathrm{H}^3(P, \mathbb{Z})$, such that for every point $p \in P$ the associated inclusion

$$\iota_p : \mathrm{Spin}(n) \longrightarrow P : g \longmapsto p.g$$

pulls ξ back to the standard generator of $\mathrm{H}^3(\mathrm{Spin}(n), \mathbb{Z})$.

Theorem 2.2 ([36, Section 5]). *Let $\pi : P \longrightarrow M$ be a principal $\mathrm{Spin}(n)$-bundle over M.*

(a) *P admits string classes if and only if $\frac{1}{2}p_1(P) = 0$.*
(b) *If P admits string classes, the possible choices form a torsor over the group $\mathrm{H}^3(M, \mathbb{Z})$, where the action of $\eta \in \mathrm{H}^3(M, \mathbb{Z})$ takes a string class ξ to the string class $\xi + \pi^*\eta$.*

Theorem 2.4. *The bundle P admits string classes if and only if the Chern-Simons 2-gerbe $\mathbb{CS}_P$ has a trivialization. In that case, the assignment $\mathbb{T} \longmapsto \xi_{\mathbb{T}}$ establishes a bijection,*

$$\left\{ \begin{array}{c} \textit{isomorphism classes of} \\ \textit{trivializations of } \mathbb{CS}_P \end{array} \right\} \cong \{ \textit{ string classes on } P \}.$$

Theorem 2.9. *Let P be a principal $\mathrm{Spin}(n)$-bundle over M with connection A, and let $Z_{P,A}$ be the extended Chern-Simons theory associated to (P, A). Then, the map $\mathscr{S}$ is injective. Moreover, under the assumption that the cobordism hypothesis holds for $Z_{P,A}$, it is also surjective.*

Theorem 2.12. *Let $\pi : P \longrightarrow M$ be a principal $\mathrm{Spin}(n)$-bundle over M with a connection A. Let $(\mathbb{T}, \blacktriangledown)$ be a geometric string structure on (P, A). Then, there exists a unique 3-form $H_{\blacktriangledown} \in \Omega^3(M)$ such that*

$$\pi^* H_{\blacktriangledown} = K_{\blacktriangledown} + TP(A), \tag{2.1}$$

where $K_{\blacktriangledown}$ is the 3-form that represents the string class $\xi_{\mathbb{T}} \in \mathrm{H}^3(P, \mathbb{Z})$, and $TP(A)$ is the Chern-Simons 3-form associated to the connection A. Moreover, $H_{\blacktriangledown}$ has the following properties:

(a) *Its derivative $\mathrm{d}H_{\blacktriangledown}$ is one-half of the Pontryagin 4-form of A.*
(b) *It depends only on the isomorphism class of $(\mathbb{T}, \blacktriangledown)$.*
(c) *For $\kappa \in \hat{\mathrm{H}}^3(M, \mathbb{Z})$ we have*

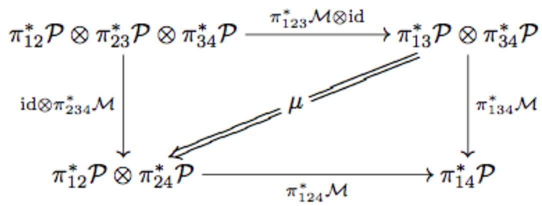
$$H_{\blacktriangledown . \kappa} = H_{\blacktriangledown} + \Omega(\kappa).$$

under the action of Corollary 2.11.

Definition 3.1 ([34, Definition 5.3]). A *bundle 2-gerbe* over M is a covering $\pi : Y \longrightarrow M$ together with a bundle gerbe $\mathcal{P}$ over $Y^{[2]}$, an isomorphism

$$\mathcal{M} : \pi_{12}^*\mathcal{P} \otimes \pi_{23}^*\mathcal{P} \longrightarrow \pi_{13}^*\mathcal{P}$$

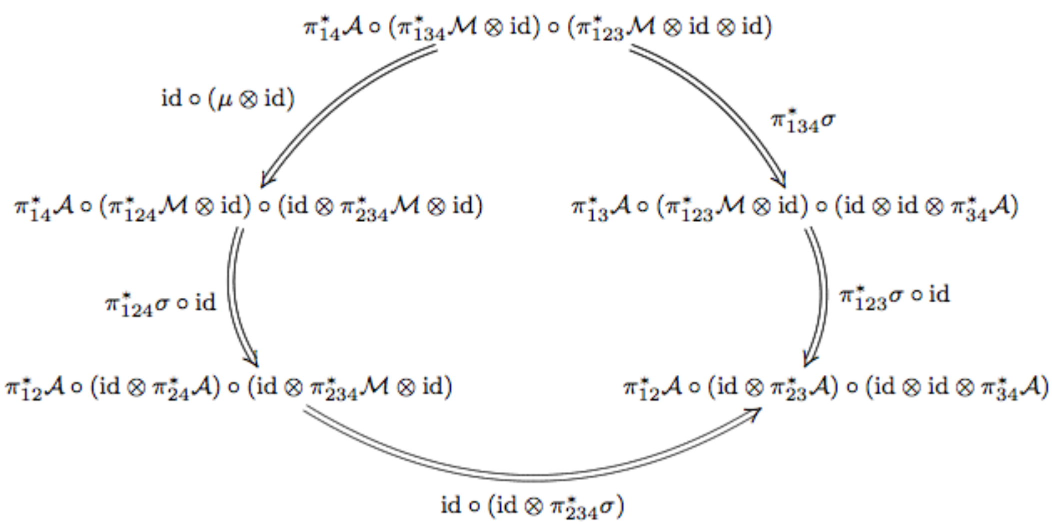
of bundle gerbes over $Y^{[3]}$, and a transformation



over $Y^{[4]}$ that satisfies the pentagon axiom shown in Figure 1.

Definition 3.5 ([34, Definition 11.1]). Let $\mathbb{G} = (Y, \mathcal{P}, \mathcal{M}, \mu)$ be a bundle 2-gerbe over M. A *trivialization* of $\mathbb{G}$ is a bundle gerbe $\mathcal{S}$ over Y, together with an isomorphism

$$\mathcal{A} : \mathcal{P} \otimes \pi_2^*\mathcal{S} \longrightarrow \pi_1^*\mathcal{S}$$



String connections and Chern-Simons theory
Konrad Waldorf.
Trans. Amer. Math. Soc. 365 (2013), 4393-4432

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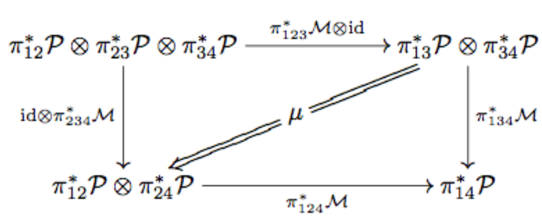

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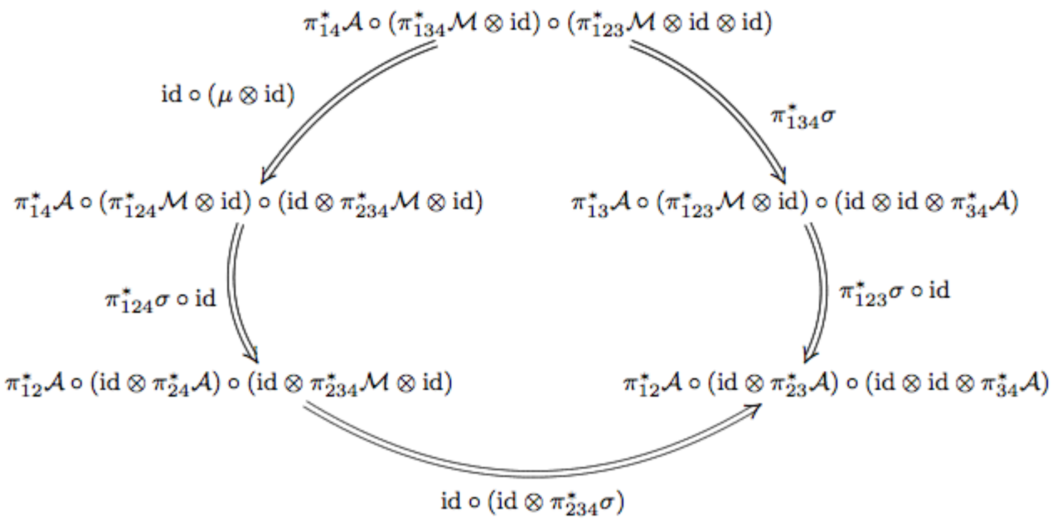

Short geodesic loops on complete Riemannian manifolds with a finite volume
Regina Rotman.
Trans. Amer. Math. Soc. 365 (2013), 2881-2894

Theorem 0.3. *Let M^n be a complete noncompact Riemannian manifold of a finite volume V. Then given a point $p \in M^n$ there exists $T > 0$, such that for all $t > T$ there exists a geodesic loop of length at most ε at the distance t from p.*

Theorem 0.5 ([G]). *Let M^n be an n-dimensional manifold. Then* Fill Rad$M$ $k(n)\mathrm{vol}(M^n)^{\frac{1}{n}}$, *where $k(n)$ is an explicit function of the dimension of a manif*

Lemma 1.1. *Let M^n be a complete noncompact Riemannian manifold of a volume V, $p \in M^n$. Let $\sigma(t)$ be a geodesic ray, starting at a point p. Then give 0, there exists a set $A = A(\tilde{\varepsilon}) \subset (0, \infty)$ of measure at most $\frac{16V}{\tilde{\varepsilon}}$, such that for t^* in A^c (the complement of A in $(0, \infty)$), and for every $0 < \delta < \min\{1, \frac{\tilde{\varepsilon}}{2}\}$ exists an $(n-1)$-dimensional submanifold $Z_{\tilde{\varepsilon}}^{\delta}$ of M^n with the following prope*

(1) $\mathrm{vol}_{n-1}(Z_{\tilde{\varepsilon}}^{\delta}) < \tilde{\varepsilon}$;

(2) $Z_{\tilde{\varepsilon}}^{\delta}$ does not bound in $M^n \setminus \{p\}$;

(3) the distance between $Z_{\tilde{\varepsilon}}^{\delta}$ and the geodesic sphere $\tilde{S}_{t^*}(p) = \{x \in M^n | \mathrm{dist}(x,$ $t^*\}$ is at most δ.

Lemma 1.2. *Let M^n be a complete Riemannian manifold. Let $q \in M^n$. Suppose that the length of a shortest geodesic loop $l_q(M^n)$ based at q is greater than L. Then, given any piecewise differentiable loop $\gamma : [0,1] \longrightarrow M^n$ of length $\leq L$ such that $\gamma(0) = \gamma(1) = q$, there exists a length decreasing path homotopy connecting this curve with q that depends continuously on the initial loop γ.*

Lemma 2.4. *Let M^n be a complete Riemannian manifold, $p \in M^n$. Let $\varepsilon, \tau, \varrho$ be positive numbers, such that $\varrho < \frac{\varepsilon}{100^n}$. Define $\tilde{\varepsilon}$ by the equation*

$$6 \cdot 4^{n-1}(k(n-1)\tilde{\varepsilon}^{\frac{1}{n-1}} + 2\varrho + 3\tau) = \varepsilon,$$

where $k(n-1) = 27^{n-1}n!$, and τ is sufficiently small for $\tilde{\varepsilon}$ to exist and to be positive. Suppose that given $t > \frac{\varepsilon}{2}$, there exists an $(n-1)$-dimensional submanifold Z that lies within the ϱ-tubular neighborhood of the geodesic sphere $\tilde{S}_t(p)$ centered at p of radius t, such that

(1) *Z does not bound in $M^n - p$;*

(2) $\mathrm{vol}(Z) < \tilde{\varepsilon}$. *Then there exists a geodesic loop of length at most ε based at a distance t from the point p.*

The topology of spaces of polygons
Michael Farber and Viktor Fromm.
Trans. Amer. Math. Soc. 365 (2013), 3097-3114

Definition 1.1. A length vector ℓ is called *generic* if there is no subset $J \subset \{1, \ldots, n\}$ so that $\sum_{j \in J} l_j = \sum_{j \notin J} l_j$.

Theorem 1.3. *Let $\ell, \ell' \in \mathbb{R}^n_>$ be two generic length vectors and let $d \geq 3$. The following conditions are equivalent:*

(a) *The manifolds $E_d(\ell)$ and $E_d(\ell')$ are $\mathrm{O}(d)$-equivariantly diffeomorphic.*

(b) *The cohomology rings $H^*(E_d(\ell); \mathbb{Z}_2)$ and $H^*(E_d(\ell'); \mathbb{Z}_2)$ are isomorphic as graded rings.*

(c) *The rings $H^{(d-1)*}(E_d(\ell); \mathbb{Z}_2)$ and $H^{(d-1)*}(E_d(\ell'); \mathbb{Z}_2)$ are isomorphic.*

(d) *For some permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$, the length vectors ℓ and $\sigma(\ell')$ lie in the same chamber.*

Proposition 2.2. *Let M be a smooth compact manifold, possibly with boundary. Let $f : M \to \mathbb{R}$ be a smooth function which is nondegenerate in the sense of Bott. If $\partial M \neq \emptyset$ we will additionally assume that ∂M coincides with the set of points where f achieves its maximum and $df \neq 0$ on ∂M. Suppose that for some $k \geq 2$, each connected critical submanifold $C \subset M$ of f is k-lacunary (see Definition 2.1) and the Morse-Bott index $\mathrm{ind}_f(C)$ of C is divisible by k. Then f is perfect, i.e.*

$$H_*(M; \mathbb{Z}) \simeq \bigoplus_{C \subset \mathrm{Crit}(f)} H_{*-\mathrm{ind}_f(C)}(C; \mathbb{Z}), \tag{2}$$

where C runs over the connected components of the set of critical points of f.

Proposition 2.3. *Suppose that in addition to the assumptions of Proposition 2.2, for each critical submanifold $C \subset M$ of f we are given a closed submanifold $W_C \subset M$ and a finite collection of closed submanifolds $\mathcal{W}_C = \{Z; Z \subset W_C\}$ such that the following conditions are satisfied:*

(1) *$C \subset W_C$ and $\dim W_C = \operatorname{ind}_f(C) + \dim C$.*
(2) *The function $f|W_C$ is nondegenerate in the sense of Bott and achieves its maximum on C.*
(3) *Each $Z \in \mathcal{W}_C$ is transversal to C as a submanifold of W_C.*
(4) *The set of homology classes $[Z \cap C] \in H_*(C; \mathbb{Z}_2)$, for all $Z \in \mathcal{W}_C$, forms a basis of $H_*(C; \mathbb{Z}_2)$.*

Then the set of the homology classes $[Z] \in H_*(M; \mathbb{Z}_2)$, for all $Z \in \mathcal{W}_C$ and for all critical submanifolds $C \subset \operatorname{Crit}(f)$, forms a basis of $H_*(M; \mathbb{Z}_2)$.

Proposition 2.4. *Suppose that in addition to the assumptions of Proposition 2.3, each of the submanifolds $Z \in \mathcal{W}_C$ is oriented. Fix an orientation of the normal bundle to C in W_C. Then each intersection $Z \cap C$ is canonically oriented and the symbol $[Z \cap C] \in H_*(C; \mathbb{Z})$ will denote the homology class of C realized by $Z \cap C$. Assume that for each critical submanifold $C \subset \operatorname{Crit}(f)$ the collection of classes $[Z \cap C] \in H_*(C; \mathbb{Z})$, where $Z \in \mathcal{W}_C$, forms a free basis of $H_*(C; \mathbb{Z})$. Then the collection of the homology classes $[Z] \in H_*(M; \mathbb{Z})$, for all $Z \in \mathcal{W}_C$ and for all critical submanifolds $C \subset \operatorname{Crit}(f)$, forms a free basis of $H_*(M; \mathbb{Z})$.*

Proposition 3.1. *A vector $\ell \in \mathbb{R}^n_>$ is a regular value of $F|\Omega$ if and only if ℓ is generic, i.e. $\sum_{i=1}^n \epsilon_i l_i \neq 0$ for $\epsilon_i = \pm 1$. Thus, for a generic $\ell \in \mathbb{R}^d_>$ the preimage $F^{-1}(\ell) = E_d(\ell)$ is a smooth closed manifold of dimension $d(n-1) - n$.*

The American Mathematical Monthly

Euler-Boole Summation Revisited
Author(s): Jonathan M. Borwein, Neil J. Calkin and Dante Manna
Source: The American Mathematical Monthly, Vol. 116, No. 5 (May, 2009), pp. 387-412

Proposition 2.1. *For each $k \in \mathbb{N}$, let $P_k := \left\{\sum_{i=0}^k a_i x^i : a_i \in \mathbb{R}\right\} \cong \mathbb{R}^{k+1}$. Then for all $n \in \mathbb{N}$, given $A \in P_k$ there is a unique $B \in P_k$ such that $\mathcal{S}_n(B) = A$.*

Proposition 2.2. *Let g be a probability density function whose absolute moments exist. For all $h \in P_k$, there is a unique $f \in P_k$ so that $\mathcal{S}_g(f) = h$.*

Theorem 2.3. *For each n in $\mathbb{N}_0$, let $P_n^g(x)$ be the* Strodt polynomial *associated with a given density $g(x)$; that is, for all $x \in \mathbb{R}$, $P_n^g(x)$ is defined implicitly by the relation*

$$\mathcal{S}_g(P_n^g(x)) = x^n \quad \text{for all } n \in \mathbb{N}_0, \tag{21}$$

where $\mathcal{S}_g$ is a Strodt operator. Then

$$\frac{d}{dx} P_n^g(x) = n P_{n-1}^g(x) \quad \text{for all } n \in \mathbb{N}. \tag{22}$$

Corollary 3.1. *For each positive integer k, if a degree-n polynomial $B_n^{(k)}(x)$ satisfies*

$$\mathcal{S}_B^{(k)}[B_n^{(k)}(x)] = x^n \quad \text{for } n \in \mathbb{N}_0, \tag{41}$$

A Proof of the Cayley-Hamilton Theorem
Author(s): Chris Bernhardt
Source: The American Mathematical Monthly, Vol. 116, No. 5 (May, 2009), pp. 456-457

Theorem 1. *Let $A \in M(n, n)$ with characteristic polynomial*

$$\det(tI - A) = c_0 t^n + c_1 t^{n-1} + c_2 t^{n-2} + \cdots + c_n.$$

Then

$$c_0 A^n + c_1 A^{n-1} + c_2 A^{n-2} + \cdots + c_n I = 0.$$

A New Constructive Proof of the Malgrange-Ehrenpreis Theorem
Author(s): Peter Wagner
Source: The American Mathematical Monthly, Vol. 116, No. 5 (May, 2009), pp. 457-462

Lemma 1. *If $\lambda_0, \ldots, \lambda_m \in \mathbf{C}$ are pairwise different, then the unique solution of the linear system of equations*

$$\sum_{j=0}^{m} a_j \lambda_j^k = \begin{cases} 0, & \text{if } k = 0, \ldots, m-1, \\ 1, & \text{if } k = m, \end{cases}$$

is given by $a_j = \prod_{k=0, k \neq j}^{m} (\lambda_j - \lambda_k)^{-1}$.

Proposition 1. *Let* $P(\xi) = \sum_{|\alpha| \le m} c_\alpha \xi^\alpha \in \mathbf{C}[\xi] \setminus \{0\}$ *be a not identically vanishing polynomial on* $\mathbf{R}^n$ *of degree* m. *If* $\eta \in \mathbf{R}^n$ *with* $P_m(\eta) \neq 0$, *the real numbers* $\lambda_0, \ldots, \lambda_m$ *are pairwise different, and* $a_j = \prod_{k=0, k \neq j}^{m} (\lambda_j - \lambda_k)^{-1}$, *then*

$$E = \frac{1}{\overline{P_m(2\eta)}} \sum_{j=0}^{m} a_j \, e^{\lambda_j \eta x} \mathcal{F}_\xi^{-1} \left(\frac{\overline{P(i\xi + \lambda_j \eta)}}{P(i\xi + \lambda_j \eta)} \right)$$

is a fundamental solution of $P(\partial)$, *i.e.,* $P(\partial)E = \delta$.

Curves in Cages: An Algebro-Geometric Zoo
Author(s): Gabriel Katz
Source: The American Mathematical Monthly, Vol. 113, No. 9 (Nov., 2006), pp. 777-791

Theorem 2.3 (Cage Theorem for Cubics). *Any cubic curve* $\mathcal{C}$ *that passes through eight nodes of a* (3×3)*-cage must pass through the ninth node.*

Theorem 3.1 (Cage Theorem for Plane Curves).

1. *If a curve in* $\mathbb{P}^2$ *of degree* d *passes through a supra-quasi-triangular set* $\mathcal{A}$ *of nodes of a* $(d \times e)$*-cage with* $d \ge e$, *then it passes through all the nodes of the cage.*
2. *No curve of degree less than* e *can pass through a quasi-triangular set of nodes of a* $(d \times e)$*-cage when* $d \ge e$.

A Short Proof for the Krull Dimension of a Polynomial Ring
Author(s): Thierry Coquand and Henri Lombardi
Source: The American Mathematical Monthly, Vol. 112, No. 9 (Nov., 2005), pp. 826-829

Theorem 1. *Let* R *be a commutative ring, and let* ℓ *be a nonnegative integer. The following statements are equivalent:*

1. *The Krull dimension of* R *is at most* ℓ.
2. *For each* x *in* R *the Krull dimension of* $R_{\{x\}}$ *is at most* $\ell - 1$.

Corollary 2. *Let* ℓ *be a nonnegative integer. The Krull dimension of* R *is at most* ℓ *if and only if for any given* $x_0, \ldots, x_\ell$ *in* R *there exist* $a_0, \ldots, a_\ell$ *in* R *and* $m_0, \ldots, m_\ell$ *in* $\mathbb{N}$ *such that*

$$x_0^{m_0}\big(\cdots(x_\ell^{m_\ell}(1 + a_\ell x_\ell) + \cdots) + a_0 x_0\big) = 0. \qquad (1)$$

Corollary 3. *Let* K *be a field, and let* R *be a commutative* K*-algebra. If any sequence* $x_0, \ldots, x_\ell$ *in* R *is algebraically dependent over* K, *then the Krull dimension of* R *is at most* ℓ.

Zeitschrift für angewandte Mathematik und Physik

Lp-convergence rates to nonlinear diffusion waves for quasilinear equations with nonlinear dampingAuthor(s): Shifeng Geng and Lina Zhang
Z. Angew. Math. Phys. 66 (2015), 31–50

In this paper, we consider the following model of hyperbolic equations with nonlinear dampin

$$\begin{cases} v_t - (h(v)p)_x = 0, \\ p_t + \sigma(v)_x = f(v)p, \end{cases}$$

where $\sigma'(v) < 0, h(v) > 0, f(v) < 0$ and $v > 0$. This system derived in [16,17] describes the p of heat wave for rigid solids at very low temperature, below about 20 K.

In [7], Li and Saxton proved that the Cauchy problem (1.1) with

$$(v,p)(x,0) = (v_0,p_0)(x) \to (v_\pm, 0),$$

Theorem 1.2. *Assume that* $\sigma \in C^3, \sigma' < 0, h \in C^2, h > 0, f \in C^2, f < 0$ *and* $(V_0(x), z_0(x)) \in (H^3 \times H^2)(\mathbb{R})$. *Then, there exists a* $\delta > 0$ *such that if* $\|V_0\|_3 + \|z_0\|_2 + |v_+ - v_-| \leq \delta$, *the Cauchy problem* (1.1) *and* (1.2) *admits a unique global smooth solution* (v,p) *which satisfies*

$$\begin{cases} \|\partial_x^k \partial_t^l V(t)\| \leq C\delta(1+t)^{-\frac{k}{2}-l}, \ 0 \leq k+l \leq 3, \ 0 \leq l \leq 2, \\ \|\partial_t^3 V(t)\| \leq C\delta(1+t)^{-\frac{5}{2}}, \end{cases} \tag{1.18}$$

Furthermore, under the additional assumption that $(V_0, z_0) \in (L^1 \times L^1)(\mathbb{R})$, *the following* $L^p(2 \leq p \leq \infty)$ *decay rates are true*

$$\|\partial_x^k (v - \bar{v})(t)\|_{L^p} \leq C\delta(1+t)^{-\frac{1}{2}(1-\frac{1}{p})-\frac{k+1}{2}} \log(1+t), \tag{1.19}$$

$$\|\partial_x^k (p - \bar{p})(t)\|_{L^p} \leq C\delta(1+t)^{-\frac{1}{2}(1-\frac{1}{p})-\frac{k+2}{2}} \log(1+t), \tag{1.20}$$

for any $k \leq 2$ *if* $p = 2$ *and* $k \leq 1$ *if* $(2, \infty]$.